H^∞ Feedback-Compensated Minimum-Bending-Moment Control for Flexible Space Structures

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1. Introduction

The vibration of flexible space structures is undesired when conducting precise attitude control, and is unlikely to be suppressed naturally once it has been excited in space because of the vacuum feature of the space environment and low structural damping characteristics of flexible space structures. Therefore, rest-to-rest motion is desired when maneuvering flexible space structures. Many control methods have been proposed to meet these objectives. Input shaping is one such control method, and there are various versions of input shaping control techniques, such as the Zero Vibration (ZV), Zero Vibration and Derivative (ZVD),¹⁾ Specified-Insensitivity (SI)²⁾ and Extra-Insensitivity (EI) approaches.³⁾ Adaptive input shaping approaches^{4–8)} have also been proposed to successfully reduce the vibration of a system with unknown or varying frequencies. Inevitably excited flexible motion during slew maneuvering could be a serious factor to assure the stringent high performance of large space structures with very low bending rigidity and damping characteristics. In order to overcome this problem, minimum-bending moment control (MBMC)⁹⁾ and deflection-limiting input shaping control (DLC)¹⁰⁾ have been investigated to reduce excited deflection or limit it within a specific level during maneuvers.

These optimal control algorithms, however, do not take into consideration constant or varying disturbances acting on the system. It is usually difficult to predict the properties and magnitudes of external disturbances. Therefore, robust feedback controllers should be employed to ensure the good performance of systems, such as precise attitude tracking and attitude stabilization, which can be achieved by rejecting or compensating external disturbances acting on systems. The H^∞ control method is one such controller, and has been studied experimentally for many applications,^{11–13)} such as an experimental attitude control of the ETS-VI currently in orbit.¹²⁾ For the case of the ETS-VI, a step input used as the reference attitude angle and an impulsive disturbance torque were employed to study the performance of the H^∞ controller designed.

In the present study, contrary to some other studies mea-

suring the tip acceleration of the beam, the bending moment at the root of the beam is used as a measurement and optimal trajectories are generated by MBMC,⁹⁾ which takes the bending moment on the flexible appendage into consideration in order to suppress the maximum induced deflection and make the input profile smoother. The H^∞ control method is employed to reduce errors between the current output and the optimal output obtained by the MBMC, and to achieve a final attitude that is the directional attitude. The advantage of MBMC is that it generates a smoother reference trajectory than the one generated by minimum time control,¹³⁾ and the undesired residual vibrations are smaller.

The control algorithm has been implemented and its validity has been verified experimentally, employing a model that consists of a rigid body rotating around a shaft and a flexible beam attached to the rigid body.

2. System Model

Figure 1 shows a model of a space structure consisting of a rigid body equipped with a flexible appendage. The appendage is a cantilever beam with one end fixed to the rigid body and the other end free. A circular plate is attached between the root and the tip of the beam. The rotational and vibrational motions are assumed to be planar two-dimensional motions. The control torque is applied only on the center of the rigid body. In the present analyses, for the sake of simplicity, the structural damping and the air drag are neglected.

Considering the boundary conditions at the root and tip-end of the beam, and expanding the flexible vibration of the appendage by the vibrational mode, the equations of motion can be written in the form of a linear time invariant (LTI) system, as follows:

$$\{I_r + m_d(L + l_d)^2\}\ddot{\theta} - EI \sum_{i=1}^n \alpha_i \beta_i^4 q_i = T, \quad (1)$$

$$EI \beta_i^4 q_i + \rho(\alpha_i \ddot{\theta} + \ddot{q}_i) = 0 \quad (2)$$

where I_r is the moment of inertia of the main body with radius l , L is the length of the flexible appendage, EI and

$$\dot{\mathbf{x}}_c = \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_c \mathbf{m}_e, \quad (13)$$

$$\mathbf{u} = \mathbf{u}_{\text{opt}} + \mathbf{C}_c \mathbf{x}_c, \quad (14)$$

where

$$\mathbf{A}_c = \mathbf{A} + \gamma^{-2} \mathbf{B}_w \mathbf{B}_w^T \mathbf{P} - \mathbf{B}_u \mathbf{B}_u^T \mathbf{P} - (\mathbf{I} - \gamma^{-2} \mathbf{Q} \mathbf{P})^{-1} \mathbf{Q} \mathbf{C}_m^T \mathbf{C}_m, \quad (15)$$

$$\mathbf{B}_c = (\mathbf{I} - \gamma^{-2} \mathbf{Q} \mathbf{P})^{-1} \mathbf{Q} \mathbf{C}_m^T, \quad (16)$$

$$\mathbf{C}_c = -\mathbf{B}_u^T \mathbf{P}, \quad (17)$$

$\mathbf{m}_e$ is the difference between the current output $\mathbf{m}_m$ and optimal output $\mathbf{m}_{\text{opt}}$ and is represented as $\mathbf{m}_e := \mathbf{m}_m - \mathbf{m}_{\text{opt}}$. γ is a positive constant scalar bounding the norm of the transfer function from the disturbance to the control variables, and $\mathbf{P}$ and $\mathbf{Q}$ are positive definite matrix solutions of the Riccati equations, respectively:

$$\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P} - \mathbf{P} (\mathbf{B}_u \mathbf{B}_u^T - \gamma^{-2} \mathbf{B}_w \mathbf{B}_w^T) \mathbf{P} + \mathbf{C}_y^T \mathbf{C}_y = \mathbf{O}, \quad (18)$$

$$\mathbf{A} \mathbf{Q} + \mathbf{Q} \mathbf{A}^T - \mathbf{Q} (\mathbf{C}_m^T \mathbf{C}_m - \gamma^{-2} \mathbf{C}_y^T \mathbf{C}_y) \mathbf{Q} + \mathbf{B}_w \mathbf{B}_w^T = \mathbf{O}. \quad (19)$$

Note that the spectral radius of the product of $\mathbf{P}$ and $\mathbf{Q}$ must satisfy

$$\rho(\mathbf{P} \mathbf{Q}) < \gamma^2. \quad (20)$$

5. Experimental Results

5.1. Experimental setup

An experimental apparatus was constructed to test the controller described above. The flexible appendage used in this experiment is an aluminum beam. This beam is clamped to a rigid main body that is rotated by a direct-current (dc) torque motor. This experimental setup is set in the horizontal plane in order to eliminate the effect of gravity. The necessary data for the control is provided by two full-bridge strain gauges located at the root of the beam and a tachometer at the rotary shaft of the torque motor. The strain gauges are used to sense the bending moment induced at the root of the beam, and the tachometer is used to measure the angular velocity of the central rigid body about the rotary axis. The angular velocity is integrated by an analog circuit to obtain the slewing angle of the central rigid body. A computer is used to process the control input torque in accordance with the control algorithm and outputs the control signal to drive the dc torque motor through a digital/analog (D/A) converter. The control signal is converted to electric current by a current amplifier. The control software is implemented in C language. The sample interval for the control signal is set to be 50 ms. The data is stored in the computer and saved to a text file for analysis of the experimental results.

The robustness of the H^∞ compensated optimal controller is investigated. The parameters of the experimental setup are as follows: $I_r = 3.7 \times 10^{-2} \text{ kgm}^2$, $l = 0.0338 \text{ m}$, $L = 0.914 \text{ m}$, $EI = 0.1516 \text{ Nm}^2$, $m_d = 0.13 \text{ kg}$, $l_d = 0.1 \text{ m}$, and

$\rho = 8.157 \times 10^{-2} \text{ kg/m}$. Using these parameters, the first and second modal frequencies of the system, which are treated as the dominant vibrations, are analytically obtained as 7.23 rad/s and 36.2 rad/s, respectively, and higher-order modal frequencies are ignored in the present study. The objective attitude angle of the main body is set to 60 deg, and the vector of fictitious disturbances $\boldsymbol{\varepsilon}$ is set to $\boldsymbol{\varepsilon} = [0.01 \ 0.01 \ 0.01]^T$. The minimum value of the positive scalar γ is obtained as 1.4 so that the stability of the controller is guaranteed and the performance of the controller is effective.

5.2. Results

Three controllers were investigated: MBMC without feedback control, H^∞ feedback-compensated MBMC with $k = 3.0$ and H^∞ feedback-compensated MBMC with $k = 5.0$. The time responses of the attitude angle and the bending moment at the root of the flexible appendage are shown in Fig. 2. When the feedback control is not used, the attitude angle of the main body does not reach the desired angle and the residual vibrations are not quickly damped out after the maneuver. This is because there are modeling errors, the

Fig. 2. Time responses of attitude angle (a) and bending moment at the root of the beam (b).

air drag affecting the flexible appendage during the slew maneuver prevents the attitude angle of the main body from reaching the desired angle and the structural damping effect is low. In addition, the peak of the bending moment is greater than that of the reference bending moment obtained by MBMC. On the other hand, when the H^∞ feedback controller is employed, the desired attitude angle is achieved, residual vibrations are more quickly damped out and the induced maximum bending moment is smaller compared to the case without employment of H^∞ feedback control. In addition, it can be seen that, in accordance with the increase of the value of k , the residual vibrations are more quickly damped out, but the settling time of the attitude angle becomes longer. This is because when the greater value is used for the weighting coefficient k , the vibrations are treated as a relative important index compared to the attitude angle.

6. Conclusion

H^∞ feedback tracking control has been studied for a flexible space structure consisting of a main rigid body equipped with a flexible beam. The reference trajectory, which is followed by the H^∞ feedback, is generated by the minimum bending moment control, which evaluates the bending moment at the root of the beam in order to avoid a large deflection induced on the flexible appendage. The experimental results show that the proposed control scheme is effective in tracking the reference trajectory, suppressing residual vibrations, and avoiding large bending moments that might damage the flexible appendage.

References

- 1) Singer, N. C. and Seering, W. P.: Preshaping Command Inputs to Reduce System Vibration, *J. Dynam. Syst. Meas. Control*, **112** (1990), pp. 76–82.
- 2) Singhose, W. E., Seering, W. P. and Singer, N. C.: Input Shaping for Vibration Reduction with Specified Insensitivity to Modeling Errors, Japan/USA Symposium on Flexible Automation, Boston, MA., 1996, pp. 307–319.
- 3) Singhose, W. E., Derezinski, S. and Singer, N. C.: Extra-Insensitive Input Shaping for Controlling Flexible Spacecraft, *J. Guid. Control Dynam.*, **19** (1996), pp. 385–391.
- 4) Tzes, A. and Yurkovich, S.: An Adaptive Input Shaping Control Scheme for Vibration Suppression in Slewing Flexible Structures, *IEEE Trans. Control Syst. Technol.*, **1** (1993), pp. 114–121.
- 5) Khorrami, F., Jain, S. and Tzes, A.: Adaptive Nonlinear Control and Input Preshaping for Flexible-Link Manipulators, American Control Conf., San Francisco, CA., 1993, pp. 2705–2709.
- 6) Bodson, M.: An Adaptive Algorithm for the Tuning of Two Input Shaping Methods, *Automatica*, **34** (1998), pp. 771–776.
- 7) Magee, D. and Book, W.: The Application of Input Shaping to a System with Varying Parameters, Japan/USA Symposium on Flexible Automation, 1992, pp. 519–525.
- 8) Cutforth, F. C. and Pao, Y. L.: Adaptive Input Shaping for Maneuvering Flexible Structures, *Automatica*, **40** (2004), pp. 685–693.
- 9) Fujii, A. H., Kojima, H. and Nakajima, N.: Slew Maneuver of a Flexible Space Structure with Constraint on Bending-Moment, *J. Guid. Control Dynam.*, **26** (2003), pp. 259–266.
- 10) Robertson, M. and Singhose, W.: Closed-Form Deflection-Limiting Commands, Proceedings of the American Control Conference, **3** (2005), pp. 2104–2109.
- 11) Wie, B., Liu, Q. and Bauer, F.: Classical and Robust H^∞ Control Redesign for the Hubble Space Telescope, *J. Guid. Control Dynam.*, **16** (1993), pp. 1069–1077.
- 12) Kida, T., Yamaguchi, I., Chida, Y. and Sekiguchi, T.: On-Orbit Attitude Control Experiment of ETS-VI by H^∞ Controller, *J. Jpn. Soc. Aeronaut. Space Sci.*, **45** (1997), pp. 284–294.
- 13) Kojima, H., Fujiki, T. and Fujii, A. H.: “ H_∞ Tracking Control for Minimum Time Slewing of Flexible Space Structure, *J. Jpn. Soc. Aeronaut. Space Sci.*, **53** (2005), pp. 482–489.