Analytical Study of CLOS Guidance Law against Head-on High-Speed Maneuvering Targets

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A command to line-of-sight (CLOS) guidance law is developed against head-on high-speed maneuvering targets. Preliminary studies have shown that the aspect angle of the interceptor at lock-on near 180 deg is a fundamental requirement for achieving small miss distance against a very high-speed incoming target. The solutions of missile trajectory obtained before under this guidance scheme seemed tedious and incomplete. Now, in this study, exact and complete solutions which are more general and comprehensive than those obtained before are derived for head-on high-speed maneuvering targets. Some related important characteristics such as lateral acceleration demand and normalized missile acceleration are investigated and discussed. Additionally, illustrated examples of target maneuvering are introduced to easily describe the trajectory of the target. The results obtained in this study are very significant and practical, and will be useful in actual application.

Key Words: Guidance Law, CLOS, Analytical Solution

1. Introduction

In a CLOS trajectory, the pursuer always lies on the line between the target tracker and target. If the pursuer is always on the LOS, then the pursuer will surely hit the target.1,3,7,11-14,16,18 In other words, in CLOS guidance, the pursuer maneuvers so as to be on the LOS between the target tracker and target.2,5,8,10 A closed-form solution of a CLOS trajectory for maneuvering target and a pursuer with constant velocity is not available. In 1955, Locke8) presented a 10-term CLOS series solution with simplified assumptions. In addition, Macfadzean10) claimed that there is no closed-form solution for LOS trajectory, even with simplifying assumptions such as those made by Locke. In 2000, Jalali-Naimi and Esfahanian4) claimed a closed-form solution for LOS trajectory for non-maneuvering targets. In 2001, Shoucri15) showed that some scattered published results in the guidance theory field were similar to the generalized proportional navigation (GPN) guidance law. In this paper, a closed-form solution of CLOS trajectory for maneuvering targets is proposed. Here, the total pursuer acceleration is assumed to be equal to the required acceleration in the direction normal to LOS, whereas Locke assumed the pursuer velocity is constant, that is, the pursuer acceleration is restricted in the direction normal to pursuer velocity. Shoucri’s solution seems tedious and incomplete. Now, in this paper, we try to derive an exact and complete closed-form solution for a maneuvering target. Some important and significant characteristics related to the system performance, such as lateral acceleration and normalized missile acceleration, are investigated and discussed in detail.

2. Equations of Motion

An important assumption for successful engagement is that the missile intercepts the target at a near head-on geometry. Based on this assumption, the LOS rotational rates are minimized and unsuitable command signals are avoided at the terminal phase. The guidance scheme proposed here possesses three phases: midcourse, shaping and terminal. Assume that an ideal midcourse guidance law navigates the missile first. As the seeker “sees” the incoming target, the missile then enters the terminal phase. The shaping phase is between the midcourse and terminal phases. The geometric relationship between a defense missile and an incoming target is illustrated in Fig. 1.6)

In Fig. 1, $Z_M$ is the axis of the missile system along a theoretical missile inertial reference unit lock-on point. $O_M$ is defined as the missile forward direction (i.e., the direction the missile is required to go for a direct-hit). $X_M$ is the lateral axis of the missile system, which is perpendicular to $Z_M$, and is positive in the missile travel direction. The planar missile-target engagement geometry motion is shown in Fig. 2, where M and T are the missile and target, respectively. Point O is fixed and represents the “lock point” or the reference axis at the lock-on point, where $r$ is the vector corresponding to the line-of-sight (LOS) distance, $r_M$ is the missile distance and $r_T$ is the target distance from the origin.

The equation of motion is depicted by point mass dynamics. A particle $p$ (M or T) in the polar coordinates ($r$, $\theta$), according to Fig. 2, can be written in the following form:

$$v_p = \dot{r}_r + r_\theta \dot{\theta}_r$$  \hspace{1cm} (1)

$$a_p = (\ddot{r} - \dot{r} \dot{\theta}) r_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \dot{\theta}_r$$  \hspace{1cm} (2)
where, \( e_r \) and \( e_\theta \) are all unit vectors. If the missile is always on the LOS, the missile will hit the target. Therefore, the basic guidance law is defined as:

\[
\theta = \theta_M(t) = \theta_T(t)
\]  
(3)

where, \( \theta_M \) and \( \theta_T \) are the LOS angles from the reference point to the missile and to the target, respectively. We assume an ideal case in which the missile is always on the line between the reference point and the target without error. Based on this point, the required lateral acceleration for the missile on LOS is:

\[
a_M = r_M \dot{\theta_T} + 2r_M \dot{\theta_T}
\]  
(4)

Let \( r_M = r \), and using the theorem, \( a_M \) is perpendicular to the LOS. Therefore, if the missile is initially aimed at the target and accelerated according to Eq. (4), it will always lie on the reference-target LOS line. We assume that the total lateral acceleration is equal to the required acceleration, thus from Eq. (2), we have:

\[
\ddot{r} - r \dot{\theta}^2 = 0
\]  
(5)

\[
\dot{\theta} + 2 \dot{\theta} \dot{r} = a_M
\]  
(6)

Using the relationship,

\[
\dot{r} = \theta \frac{dr}{d\theta}
\]  
(7)

Equation (5) can be rewritten in the following form:

\[
\frac{d^2 r}{d\theta^2} + \frac{\dot{\theta}}{r} \frac{d r}{d\theta} - r = 0
\]  
(9)

By differentiating \( z = r \cos \theta \) and \( x = r \sin \theta \), one can obtain \( z (\dot{z} = v \cos \gamma) \) and \( x \) components (\( \dot{x} = v \sin \gamma \)) for the velocity vector \( \dot{r} \). This can also be written as:

\[
v \cos \gamma = \dot{r} \cos \theta - r \dot{\theta} \sin \theta
\]  
(10)

\[
v \sin \gamma = \dot{r} \sin \theta + r \dot{\theta} \cos \theta
\]  
(11)

with

\[
\tan \gamma = \frac{\dot{r} \sin \theta + r \dot{\theta} \cos \theta}{\dot{r} \cos \theta - r \dot{\theta} \sin \theta}
\]  
(12)

where \( \gamma \) is the angle between the direction of velocity \( v \) and the \( Z_M \) axis. By differentiating Eq. (12) with respect to \( \theta \), one can obtain the following equation:

\[
\frac{1}{\cos^2 \gamma} \frac{d \theta}{d\theta} = \frac{r^2 + 2 \frac{d r}{d\theta} \frac{d^2 r}{d\theta^2}}{\left[ \frac{d r}{d\theta} \cos \theta - r \sin \theta \right]^2}
\]  
(13)

Noting that \( \tan^2 \gamma + 1 = \sec^2 \gamma = \frac{1}{\cos^2 \gamma} \), we have

\[
\frac{1}{\cos^2 \gamma} = \frac{\left[ \frac{d r}{d\theta} \right]^2}{\left[ \frac{d r}{d\theta} \cos \theta - r \sin \theta \right]^2}
\]  
(14)

and finally derive at:

\[
\frac{d \gamma}{d\theta} = 2 - \frac{r^2 + \left( \frac{d r}{d\theta} \right)^2}{r^2 + \left( \frac{d r}{d\theta} \right)^2}
\]  
(15)

The LOS trajectory geometry can also be shown using another form in Fig. 3, where \( s \) is the arc length of trajectory. Referring to Fig. 3, we have,

\[
(\Delta s)^2 = (\Delta r)^2 + r^2 (\Delta \theta)^2,
\]  

and

\[
\left( \frac{dr}{d\theta} \right)^2 + r^2 = \left( \frac{ds}{d\theta} \right)^2,
\]  
(16)

Now, using the sine function (found in Fig. 3), Eq. (16) can be written as:

\[
\sin^2 \psi = \frac{r^2}{r^2 + \left( \frac{dr}{d\theta} \right)^2}
\]  
(17)

Equation (17) is the same as that of Eq. 15 in the report by Shoucri,\(^{15}\) where \( \psi = \gamma - \theta \) is the angle between \( v \) and \( r \). This development only utilizes the fundamental guidance equation concept. Shoucri utilized the GPN concept
treated by Yang et al.\textsuperscript{17} and the result derived by Lu.\textsuperscript{9} This is the major difference between the two approaches. Note that, in the derivation of Eqs. (16) and (17), no restriction is placed on the target and missile time variation in the velocities $v_T$ and $v$.

3. Solutions for a Non-Maneuvering Target

In the case of a non-maneuvering straight-line target, the target velocity $v_T$ is assumed to be constant. It is easily shown that:

\begin{align}
\dot{\theta} &= \frac{v_T \sin^2 \theta}{\Delta x} \quad (18) \\
\dot{\theta} &= \frac{2v_T^2 \sin^3 \theta \cos \theta}{\Delta x^2} \quad (19) \\
\dot{\theta}/\dot{\theta}^2 &= 2 \cot \theta \quad (20)
\end{align}

where, $\Delta x$ is the constant horizontal distance of the target from the reference axis.\textsuperscript{6}

Substituting $\dot{\theta}$ from Eq. (20), Eq. (9) can be rewritten as:

\begin{equation}
\frac{d^2 r}{d\theta^2} + 2 \cot \theta \frac{dr}{d\theta} - r = 0
\end{equation}

This is the case treated by Jalali-Naini and Esfahanian.\textsuperscript{4} The solution in this case is given by $r \sin \theta = A_1(\theta - \theta_0)$, and one can easily derive the transverse acceleration as:

\begin{equation}
a_r = \frac{2A_1v_T^2}{\Delta x} \sin^3 \theta
\end{equation}

which is also shown as Eq. (22) in Jalali-Naini and Esfahanian.\textsuperscript{5}

4. Solutions for Maneuvering Target

There are two techniques for deriving the CLOS trajectory differential equation. By assuming an ideal case in which the pursuer is always on the line between the reference point and the target without any lateral error; if the pursuer is always on the LOS, the pursuer will surely hit the target. Therefore, the basic guidance law for CLOS is $\theta(t) = \theta(t)$ and

$$\frac{d\theta(t)}{dr} = \frac{d\theta_T(t)}{dr}.$$  

Define two cases to examine the robustness of the trajectory solution as depicted in methods A and B.

Method A: The trajectory approach with LOS angle measurement only.

From the observation of $\theta$ and $\gamma$, Eq. (9) is employed. Because Eq. (9) is a homogeneous equation, no particular solution can be produced. We assume that the missile trajectory is simple and has a continuous first derivative $r'(s)$, which is different from the zero vector for all $s$ under consideration. Then missile trajectory is a smooth curve; that is, it has a unique tangent at each of its points whose direction varies continuously as we traverse the curve. We subdivide missile trajectory into $n$ portions by the LOS measurement; let $P_1(\theta_1, r_1), P_2(\theta_1, r_1), P_2(\theta_2, r_2), \ldots, P_n(\theta_n, r_n)$ be the end points of these portions, and let $\Delta r_{i} = r_i - r_{i-1}$ and $\Delta \theta_{i} = \theta_i - \theta_{i-1}$. Defining $B_i = \frac{\theta_i}{\dot{\theta}_i}$, we have the homogeneous equation:

\begin{equation}
\frac{d^2 \Delta r_i}{d\theta_i^2} + B_i \frac{d\Delta r_i}{d\theta_i} = \Delta r_i = 0
\end{equation}

The roots of the characteristic equation are $\frac{1}{2}(-B_i \pm \sqrt{B_i^2 + 4})$, and we obtain the solution $\Delta r_{a,b} = e^{-\frac{1}{2}(B_i \pm \sqrt{B_i^2 + 4})\Delta \theta_i}$.

A general solution of Eq. (23) is:

\begin{equation}
\Delta r_i(\theta_i) = c_1 e^{-\frac{1}{2}(B_i - \sqrt{B_i^2 + 4})\Delta \theta_i} + c_2 e^{-\frac{1}{2}(B_i + \sqrt{B_i^2 + 4})\Delta \theta_i}
\end{equation}

Hence, we yield $\Delta r_{a}(\theta_{b}) = c_1 + c_2 = 0$ by the first initial condition. Differentiation gives:

\begin{equation}
\Delta r_{a}'(\theta_{b}) = -\frac{1}{2} c_1 \left(B_i - \sqrt{B_i^2 + 4}\right) e^{-\frac{1}{2}(B_i - \sqrt{B_i^2 + 4})\Delta \theta_i} \\
- \frac{1}{2} c_2 \left(B_i + \sqrt{B_i^2 + 4}\right) e^{-\frac{1}{2}(B_i + \sqrt{B_i^2 + 4})\Delta \theta_i}
\end{equation}

Equation (16) leads to another type of initial condition. Letting

\begin{equation}
K = \frac{d\Delta r}{d\theta} = \pm \sqrt{\left(\frac{d\Delta x}{d\theta}\right)^2 - \Delta r^2}
\end{equation}

we have

\begin{equation}
K_i = \pm \sqrt{\left(\frac{v_i}{\dot{\theta}_i}\right)^2 - r_i^2}
\end{equation}

Hence, $\Delta r_i'(\theta_i) = K_i$ is the second initial condition. Together:

\begin{equation}
c_{1i} = K_i \sqrt{B_i^2 + 4}
\end{equation}
\[ c_{2i} = -\frac{K_i}{\sqrt{B_i^2 + 4}} \]  
\[ \Delta r_i(\theta_i) = \frac{K_i}{\sqrt{B_i^2 + 4}} e^{-\frac{1}{2}(B_i + \sqrt{B_i^2 + 4})\Delta \theta} - \frac{K_i}{\sqrt{B_i^2 + 4}} e^{-\frac{1}{2}(B_i - \sqrt{B_i^2 + 4})\Delta \theta} \]

(28) \[ \frac{dr_T}{dr} = v_T \cos(\gamma_T - \theta) \]  
\[ r_T \frac{d\theta}{dr} = v_T \sin(\gamma_T - \theta) \]

(39) \[ (40) \]

Rewrite the differential Eq. (38) in the form of:
\[ \frac{dr}{dt} + \frac{d^2\theta}{dr^2} = v \cos(\gamma - \theta) \left( \frac{dy}{dr} - \frac{d\theta}{dr} \right) + \frac{dv}{dr} \sin(\gamma - \theta) \]

(41) By taking the time derivative of Eq. (40), \((d^2\theta/dr^2)\) can be obtained as follows:
\[ \frac{d^2\theta}{dr^2} = \frac{1}{r_T^2} \left[ -2v_T^2 \cos(\gamma_T - \theta) \sin(\gamma_T - \theta) + r_T \cos(\gamma_T - \theta) v_T \frac{dy_T}{dr} + r_T \sin(\gamma_T - \theta) \frac{dv_T}{dr} \right] \]

(43) By assuming that the missile has a constant velocity (nominal atmospheric flight):
\[ a_{NT} = v_T \frac{dy_T}{dr} \]

(44) where, \(a_{NT}\) represents lateral target acceleration.
\[ a_N = v \frac{dy}{dr} \]

(45) \[ a_N \] represents the lateral acceleration of the missile as it follows the CLOS guidance trajectory. Substituting Eqs. (42), (40) and (43) into Eq. (45), it can be written as:
\[ a_N = \frac{r}{r_T} \left[ -v_T \sin(\gamma_T - \theta) \frac{\cos(\gamma - \theta)}{\sin(\gamma - \theta)} + \frac{2v_T \cos(\gamma - \theta)}{r_T} + \left( \frac{1}{v_T} \frac{dy_T}{dr} + \frac{1}{v} \frac{dv}{dr} \right) a_{NT} \frac{\cos(\gamma_T - \theta)}{\sin(\gamma - \theta)} \right] \]

(46) \(v_N\) represents the lateral missile velocity and \(v_s\) represents the tangent flight path velocity.
\[ v_N = \int a_N dt \]

(47) \[ v_s = \sqrt{v^2 - v_N^2} \]

(48) The missile trajectory is obtained as follows:
\[ s_z = \int (v_s \cos \gamma - v_N \sin \gamma) dt \]

(49) \[ s_x = \int (v_s \sin \gamma + v_N \cos \gamma) dt \]

(50)
5. Quantitative Studies

The CLOS guidance scheme is one in which the missile is guided on a LOS course in an attempt to remain on line, joining the target and the point of control. To do so, the missile requires a velocity component \( v_{M0} \) that is equal to the LOS velocity and perpendicular to it:

\[
v_{M0} = r \dot{\theta}
\]  
(51)

In general, the missile flies a pursuit guidance course at initiation of the entry the beam and flies on an approximately constant bearing course (CBC) near impact. This is observed from the velocity equation:

\[
v_{M0} = \frac{r}{rt} v_T
\]  
(52)

From performance analysis, a zero-lag guidance loop can be developed to remove heading error using a fixed beam \( \psi = \gamma - \theta \) and \( \dot{\psi} = \dot{\gamma} - \dot{\theta} \) to give a guidance law between \( \psi \) and \( \theta \) corresponding to the trajectory described by the differential equation given by Eq. (15).

\[
r^2 + r \left( \frac{d^2 r}{d\theta^2} \right) r = 2 = 0
\]  
(53)

Equation (53) can be rewritten as:

\[
\frac{d^2 r}{d\theta^2} - \frac{2}{r} \left( \frac{dr}{d\theta} \right)^2 - r = 0
\]  
(54)

By noting that \( \dot{\psi}/\dot{\theta} = -1 \),

\[
\frac{d^2 r}{d\theta^2} - \left( 1 - \frac{\dot{\psi}}{\dot{\theta}} \right) \left( \frac{1}{r} \right) \left( \frac{dr}{d\theta} \right)^2 + \frac{\dot{\psi}}{\dot{\theta}} r = 0.
\]  
(55)

When the proportional navigation (PN) law \( \dot{\psi}/\dot{\theta} = k \) is used, where \( k \) is a constant,\(^{18} \) PN is known to effectively engage an evasive target; \( a_c = N/\dot{\theta} = v_T \). PN is also a guidance law in which the angular rate of the missile flight path is directly proportional to the angular LOS rate of change;

\[
\gamma = \theta + \psi = N\dot{\theta}.
\]

So, it get \( \dot{\psi}/\dot{\theta} = k \).

\( k = 1 \), this is pursuit guidance.

In the CLOS case where \( k = -1 \), which is Eq. (19) of Shourci,\(^{15} \) the cumulative velocity increment is:

\[
\Delta V = \int_{t_0}^{t_f} |\dot{gN}| \, dt
\]  
(56)

where, \( t_f \) is the time of flight and \( t \) the time, which is related to the corresponding propellant mass required for effective intercept in exoatmospheric flight.

6. Illustrative Example

In this section, two target trajectories are used as case studies. Through these examples, the superior property of the proposed analytical methods (solution), A and B, are illustrated. The results from methods A and B are compared with a numerical solution.

6.1. Missile trajectory model

Here, the trajectory model of missile interception using Tracking Via Missile (TVM) is applied. It uses the modified CLOS in the terminal phase. The major difference compared with conventional CLOS is the observation (reference) point, which is changed from the ground control station to seeker lock-on point in the collision course. The recently developed modified CLOS guidance law with an advanced inertial measurement unit (IMU) system serves as a possible approaches for solving TVM interception control problems.\(^{19} \) There are continuing efforts to improve the performance of modified CLOS for intercepting maneuvering targets. More detailed models have been investigated to derive modified CLOS. The inclusion of LOS angle measurement only lead to the introduction of Method A into guidance. When the target acceleration is unknown, Method B is used to account for target maneuvering. The missile acceleration demand has been dealt with in Method B. For the missile, \( v = 1500 \text{ m/s}, \) at ‘lock-on’ point \( x = 0 \text{ m} \) and \( z = 0 \text{ m}, \) \( \theta = 95 \text{ deg}, \gamma = 95 \text{ deg} \) (Fig. 1).

6.2. Simplified ballistic target model

Based on earlier assumptions,\(^{18} \) suppose that only drag and gravity act on the endoatmospheric ballistic target. Let the target have velocity \( v_T \) and initial reentry angle \( \gamma_T \). The downrange target is \( x_T \). The altitude is \( z_T \). Note that the drag force \( F_{\text{drag}} \) acts in a direction opposite the velocity vector, and gravity \( g \) always acts downward. Therefore, if the drag effect is greater than that of gravity, the target will decelerate. The target reentry angle can be computed using the two inertial target velocity components:

\[
\gamma_T = \tan^{-1} \left( \frac{-v_{Tz}}{v_{Tx}} \right)
\]  
(57)

The simple ballistic target acceleration components in the inertial downrange and altitude directions can be expressed in terms of the target’s weight \( W \), reference area \( A_{\text{ref}} \), zero lift drag \( C_{\text{D0}} \) and gravity \( g \), or more simply, in terms of the ballistic coefficient \( \beta \) according to the following equation:

\[
\frac{dv_T}{dt} = -\frac{F_{\text{drag}}}{m_T} \cos(\gamma_T)
\]
\[
= -\frac{Qg}{\beta} \cos(\gamma_T)
\]  
(58)

\[
\frac{dv_{Tz}}{dt} = \frac{F_{\text{drag}}}{m_T} \sin(\gamma_T) - g
\]
\[
= \frac{Qg}{\beta} \sin(\gamma_T) - g
\]  
(59)

where, \( m_t \) is the target mass and

\[
\beta = \frac{W_t}{C_{\text{D0}}A_{\text{ref}}}
\]  
(60)

The air density \( \rho \) is measured in kg/m\(^3 \) and is approximated as
The dynamic pressure $Q$ is defined as:

$$Q = 0.5 \rho v_t^2$$  \hspace{1cm} (62)

the total target velocity $v_t$ is obtained from:

$$v_t = \sqrt{v_{tx}^2 + v_{tz}^2}$$  \hspace{1cm} (63)

Because the acceleration equations are in a fixed or inertial frame, they can be integrated directly to yield the velocity and position. The initial condition is as follows. For the target, $v_t = 2700 \text{ m/s}$, $v = 1500 \text{ m/s}$. Figure 4 shows the simulated engagement trajectory for the missile guided by CLOS command against a simplified ballistic target. Note that these trajectories were plotted in rectangular coordinates with the reference point at the origin. Figures 5 and 6 show the CLOS engagement velocity and lateral acceleration profile.

6.3. Sinusoidal maneuvering target model

Ballistic targets may sometimes have maneuvering capabilities to avoid a counter-attack. This simulation used a simple target model. The sinusoidal target maneuver satisfies $v_{tx} = -1800 \text{ m/s}$, and $v_{tz} = 50 \sin 2 \pi t \text{ m/s}$. Figure 7 shows the CLOS trajectory for a sinusoidal target. Note that these trajectories were plotted in rectangular coordinates with the reference point at the lock-on point. Figures 8 and 9 show the CLOS engagement velocity and lateral acceleration profile.

6.4. Missile trajectory of CLOS course

To demonstrate how the analytical solution is presented in methods A and B, the solution was programmed on a digital computer and subsequently compared with a numerical simulated trajectory. The discussion is divided into two parts: 1) simplified ballistic target, and 2) sinusoidal target.

1) Figure 10 shows the defensible trajectory against a bal-
listic target with the missile guided using simulated CLOS data. The trajectory is in good comparison with the trajectories from analytical methods A and B. The results show that the trajectories obtained using methods A and B were close to that obtained using the numerical simulated data. The Method A trajectory could be obtained on a digital computer using a stepwise accumulation algorithm such as Eqs. (30) and (31). It is worth noting that Eqs. (30) and (31) depend on $K_i$ entirely. When $\theta$ is very close to zero, a singular point problem occurs and Eqs. (30) and (31) become anomalous. The error of $B_i$ is also caused mainly by $B_i$ approaching zero, which is especially sensitive to $\theta$. Consequently, each time a new engagement geometry is acquired, $\theta$ and $\theta$ must be solved in advance. The analytical solution accuracy using Method A is limited by these two conditions.

2) Figure 11 shows the defensible trajectory against a weaving target with the missile guided using simulated CLOS data. The target has the same velocity (1800 m/s) but also has maneuvering capability. This trajectory is in good comparison with the trajectories from analytical methods A and B. The results show that the trajectories obtained using methods A and B were close to that obtained using the numerical simulated data. An examination of Fig. 11 reveals that methods A and B exhibit analytical solutions with characteristics exactly like those derived using theory. Such behavior is not unexpected and, in fact, was manifested in all of the geometries characterized with a high LOS rate. From the plot, the error made by Method B is lower than that by Method A. The Method B scheme is more complex than Method A because the missile and target variables are included.

6.5. Lateral acceleration in CLOS course

The missile acceleration saturation problem is the most important guidance system nonlinearity in the performance comparison. Because the CLOS guide uses LOS, less lateral acceleration is required than the PN case when a maneuvering target must be destroyed. In Figs. 6 and 9, where lateral acceleration is plotted versus the time for the missile and target, it is evident that the CLOS law requires a lower acceleration advantage than the PN law to complete the mission. This is the major advantage of CLOS: it can use reasonable lateral acceleration in the last phase when a maneuvering target must be destroyed. The lateral missile acceleration for the CLOS course is produced using Eq. (46). As shown in Fig. 12, the missile approaches the target at a lower speed and velocity ratio. For a sinusoidal target, the lateral acceleration required for the missile is the same as the target acceleration, as shown in Fig. 13. The maximum acceleration during flight can be found using Eq. (46). The results presented here demonstrate that Eq. (46) is both a stable and predictable estimation algorithm for the CLOS course.
6.6 Normalized missile acceleration due to target maneuvering

The approximate solution for the missile acceleration response due to two example target maneuvers are displayed in normalized form in Figs. 14 and 15, respectively. We can see that the CLOS guidance scheme inhibits the acceleration requirements at the end of the flight. From a system sizing point of view, the designer must ensure that the acceleration capability of the missile is adequate at the end of the flight so that saturation can be avoided and the missile can hit the target.

6.7 Energy cost due to target maneuvering

A high energy cost is generated with extensive target maneuvering. It is simply illustrated with a typical example as described in this section. Thus, target maneuvering increases the energy cost required to effectively intercept a target, as depicted in Figs. 16 and 17, respectively. The energy cost is mainly affected by $\gamma_T$ and $\omega_{NT}$.

7. Conclusion

A CLOS guidance scheme concept was proposed for guiding a missile engaging an incoming target at very high speed. An analytic solution for the differential equations describing the missile LOS trajectory for a head-on high-speed maneuvering target was derived, which is more general and comprehensive than those obtained before. The CLOS trajectory problem was described and solutions for methods A and B, and a simulated trajectory were compared. The general equations for the two methods and derivations were given for comprehensive analysis and application to the CLOS guidance issue. Analytical expressions were obtained for method A using only the LOS angle measurement. Numerical results were given for two examples with a maneuvering target. Additionally, some significant characteristics such as missile acceleration demand, normalized acceleration and energy cost were investigated and discussed in detail.
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