A Particle Filter Approach to DGNSS Integrity Monitoring
—Consideration of Non-Gaussian Error Distribution—

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For more accurate and reliable aviation navigation systems which can be used for civil and military aircraft or missiles, researchers have employed various filtering methods to reduce the measurement noise level, or to integrate sensors such as global navigation satellite system/inertial navigation system (GNSS/INS) integration. Most GNSS applications including Differential GNSS assume that the GNSS measurement error follows a Gaussian distribution, but this is not true. Therefore, we propose an integrity monitoring method using particle filters assuming non-Gaussian measurement error. The performance of our method was contrasted with that of conventional Kalman filter methods with an assumed Gaussian error. Since the Kalman filters presume that measurement error follows a Gaussian distribution, they use an overbounded standard deviation to represent the measurement error distribution, and since the overbound standard deviations are too conservative compared to actual deviations, this degrades the integrity monitoring performance of the filters. A simulation was performed to show the improvement in performance provided by our proposed particle filter method, which does not use sigma overbounding. The results show that our method can detect about 20% smaller measurement biases and reduce the protection level by 30% versus the Kalman filter method based on an overbound sigma, which motivates us to use an actual error model instead of overbounding, or to improve the overbounding methods.

Key Words: DGNSS, Non-Gaussian Error Distribution, Integrity Monitoring, Particle Filter, Sigma Overbounding

Nomenclature

AAD-A: airborne accuracy designator A
AL: alert limit
b: bias
c_tr: constant used in GL model
C_r: covariance matrix of the residual vector
DGNSS: differential GNSS
e_i: line of sight vector from the user to the i-th satellite
f: time propagation function
FDE: fault detection and exclusion
GAD-B: ground accuracy designator B
GBAS: ground-based augmentation system
GL: Gaussian-core Laplacian-tail model
GNSS: global navigation satellite system
GPS: global positioning system
h: observation function
H: observation matrix
H_w: weighted pseudo inverse of the observation matrix
I: identity matrix
INF: inflation factor
INS: inertial navigation system
k_ffmd: fault-free missed detection multiplier
k_md: missed detection multiplier
K: standard deviation multiplier
KFIR: Kalman filter individual residual method
KFX2: Kalman filter \( \chi^2 \) method
LAAS: local area augmentation system
M: inflation factor estimation parameter
n: measurement error
n_e: measurement error vector
n_tr: transition point from the Gaussian to the Laplacian distribution in GL model
N_MD: number of measurements
N_S: number of particles
OG: overbounded Gaussian model
p_a: constant used in GL model
P_{true}(n): observed true PDF of measurement error
pr: one-sided tail probability of a residual
PDF: probability density function
PL: protection level
P_{fa}: individual false alarm rate
P_{ffmd}: fault-free missed detection probability
P_{FA}: total false alarm rate
PFIR: particle filter individual residual method
P_{md}: missed detection probability
P_{N(m,\sigma)}: Gaussian PDF, which has mean m and standard deviation \( \sigma \)
PUF: probability density uncertainty factor
P_{\chi}(x): probability density function
r: residual
r: residual vector
RAIM: receiver autonomous integrity monitoring
R_i: i-th satellite’s position
SBAS: space-based augmentation system
SIS: sequential importance sampling
SIR: sampling importance resampling
SSE: squared sum of errors
SSX2: snapshot \( \chi^2 \) method

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1. Introduction

For safety-critical applications of global navigation satellite systems (GNSSs), such as aircraft and missile navigation systems, it is important to be able to detect and exclude faults that could cause risks to accuracy and integrity, so that the navigation system can operate continuously without any degradation in performance. For high-accuracy systems, the fault detection and exclusion (FDE) function needs to be able to detect and exclude smaller and multiple biases. It is difficult to detect small errors and simultaneous multiple faults using conventional snapshot receiver autonomous integrity monitoring (RAIM) algorithms, and therefore various filtering methods have been studied for reducing the measurement noise level or integrating a GNSS with other sensors so that the navigation system can estimate its position more accurately and reliably. Because filters reduce the noise level of measurements using previous information, they can provide better integrity monitoring performance than snapshot algorithms can. However, since most filters, such as Kalman filters, presume that the measurement error and disturbance follow a Gaussian distribution, their performance can degrade if this assumption is not correct. Because GNSS measurement error does not follow a Gaussian distribution perfectly, the Kalman filter approach has to use an inaccurate error model that may cause performance degradation.

To address this problem, we propose a new integrity monitoring algorithm using particle filters. Particle filters have been researched over the last few years\(^2\) as an alternative for solving nonlinear/non-Gaussian problems. The proposed algorithm estimates a distribution of a measurement residual from the posterior density and detects exceedingly large residuals to satisfy a false alarm rate. This algorithm can detect faults based on an accurate estimation of the posterior distribution, and it can detect and exclude faults almost simultaneously, so that the system can exclude a fault measurement easily.

With non-Gaussian measurement error, a particle filter can estimate the distribution of the state more accurately than a Kalman filter can, and therefore it has better integrity monitoring performance. In addition, if the system is also highly nonlinear, then the performance will also be better. Particle filters for nonlinear systems have been researched heavily. However, the advantage of these filters with non-Gaussian error distribution has not been investigated. Therefore, our work focused on the effect of non-Gaussian error distribution of the DGNSS measurement on integrity monitoring performance, which will be assessed by a simulation.

2. Non-Gaussian DGNSS Error Distribution

This section describes a true error model and the overbounding method used in the simulations, which are based on the work of Shively and Braff.\(^3\)

2.1. DGNSS pseudorange error model

In most GNSS applications, including the Space-Based Augmentation System (SBAS) and the Ground-Based Augmentation System (GBAS), the pseudorange measurement error is assumed to follow a Gaussian distribution. However, this is not true. In general, the core part of the error distribution can be characterized well using a Gaussian distribution, but the tail part of the distribution is heavier than that of the Gaussian distribution. The heavier tail is due to ground-reflected multipaths or to systematic receiver/antenna errors.\(^4\)

Our simulation used a Gaussian core-Laplacian (GL) tail probability density function (PDF) as the receiver true error distribution to simulate a heavy-tailed model, which will be applied to our particle filter method. Shively and Braff\(^3\) used the GL model to represent the actual Local Area Augmentation System (LAAS) pseudorange error data collected at the ground facility.
\[ p(n) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{n^2}{2\sigma^2} \right), & |n| \leq n_{tr} \\ p_{tr} \times \exp \left( -\frac{p_{tr}}{c_{tr}} |n - n_{tr}| \right), & |n| > n_{tr} \end{cases} \]  

where \( n_{tr} = 2.58 \sigma \) is the transition point from the Gaussian to the Laplacian distribution. The other constants are defined as follows:

\[ p_{tr} = \mathcal{N}(0,\sigma)(n_{tr}) \times \text{PUF}, \quad c_{tr} = \int_{n_{tr}}^{\infty} \mathcal{N}(0,\sigma)(n)dn \times \text{TUF}. \]  

Originally, PUF and TUF provide confidence in consideration of the volume of data, with the PUF having a value below 1 and the TUF value being over 1. However, the purpose of this paper is not estimating the confidence level with actual data, but investigating the performance degradation of the previous methods. For this, the simulation sets \( \text{PUF} = 0.52 \) and \( \text{TUF} = 1 \) arbitrarily follow the convoluted three-receiver error model that has an inflation factor of 2.07, shown in Shively and Braff’s paper. The narrower curve shown in Fig. 1 represents the PDF.

### 2.2. Sigma overbounding

Since the measurement error does not follow a Gaussian distribution, sigma overbounding methods are used to simplify implementation and to prevent integrity risks. Overbounding is a procedure used to find the inflation factor \( (\text{INF}) \), which is the ratio of the overbounded \( (\sigma_{over}) \) and observed \( (\sigma) \) standard deviations:

\[ \sigma_{over} = \text{INF} \cdot \sigma. \]  

The inflation factor is determined using Eqs. (4) and (5):

\[ \int_{M}^{\infty} \mathcal{P}_{\text{true}}(n)dn = \int_{0.581\sigma}^{\infty} \mathcal{N}(0,1)(n)dn \]

\[ = \frac{P_{\text{ffmd}}}{1 - 2 \times 3.12 \times 10^{-9}} \]  

\[ \text{INF} = \frac{M}{5.81 \sigma}, \]  

where \( \mathcal{P}_{\text{true}}(n) \) is the observed true PDF, which was taken to represent the GL distribution in our work. In the Gaussian measurement systems, we can obtain the probability information of random variables by using multiples of standard deviations, while we must use the random variable values and the probability distributions directly in general systems. The denominator value of 5.81 is a multiplier used for GBAS users to calculate a defined fault-free protection level that bounds the actual position error with a required fault-free missed detection probability \( (P_{\text{ffmd}}) \). In a general system, \( M \) is the counterpart of the 5.81 \( \sigma \). This method provides users with an overbounded broadcast sigma that they can use to estimate the protection levels meeting the probability requirement.

Figure 2 shows a pictorial representation of this method, which shows that the one-sided tail probabilities of the Gaussian and GL distributions have the same observed standard deviation. Using the above method, the value of the INF was determined to be INF = 2.07. Figure 1 shows the PDF values of the true error model (GL) and the overbounded Gaussian (OG) model. The OG model had a broader distribution, so that the tail was conservative enough to overbound the heavy-tailed GL model.
3. Particle Filters

Particle filters can deal with nonlinear/non-Gaussian dynamic systems as well as linear/Gaussian systems. They use a sequential Monte Carlo approach, and are based on a sequential importance sampling (SIS) procedure. We have used the sampling importance resampling (SIR) algorithm for the simulations discussed in this paper. This section briefly introduces the basic SIR algorithm to provide a background for our proposed integrity monitoring algorithm. A more detailed derivation of this filter is described in other literature.8–11

Let us define state–space equations for a dynamic system as follows:

\[ x_k = f(x_{k-1}, v_{k-1}), \quad z_k = h(x_k, n_k), \]

where the subscript \( k \) stands for the \( k \)-th epoch.

In the above system, the propagation and observation functions \( f_k \) and \( h_k \) do not need to be linear, and the noise vectors, \( v_{k-1} \) and \( n_k \), can follow an arbitrary probability distribution, not necessarily a Gaussian distribution, whose probability density function must be known to implement the filter. In this paper, the state vector \( x \) consists of a user’s position and a clock bias of the user receiver. In this case, the observation vector \( z \) is obtained from the pseudo-range measurements. For more extended systems such as GNSS/INS (Inertial Navigation System) integrated systems or attitude determination systems, we can add velocity, acceleration and attitude parameters to the state vector. In those cases, the observation vector might have doppler, angular rate and acceleration measurements.

The expectation and the variance of the states are obtained from the posterior PDF, which is expressed using a mass function of random samples \( x(k,i) \) and associated normalized weights \( \omega_k^{(i)} \), as shown in Eq. (7). The random samples are known as ‘particles’:

\[
\hat{p}(x_k | z_{1:k}) = \sum_{i=1}^{N_\nu} \omega_k^{(i)} \delta(x_k - x(k,i)), \tag{7}
\]

where \( z_{1:k} = \{ z_i, i = 1, \ldots, k \} \).

First, the particles of the previous epoch \( x(k-1,i) \) are derived from the prior distribution (Eq. 8), and the filter propagates them with process noise samples drawn from the predefined noise distribution (Eq. 9):

\[
x(k-1,i) \sim \hat{p}(x_{k-1} | z_{1:k-1}) \tag{8}
\]

\[
x(k,i) = f(x(k-1,i), v(k,i)). \tag{9}
\]

Then, the associated weights \( \omega_k^{(i)} \) of the particles are updated sequentially based on the prior weights \( \omega_{k-1}^{(i)} \) and current observation vector using

\[
\omega_k^{(i)} = \omega_{k-1}^{(i)} \frac{p(z_k | x(k,i)) p(x(k,i) | x_{k-1,i})}{q(x(k,i) | x_{k-1,i}, z_k)}. \tag{10}
\]

After normalizing the weights as follows, we can obtain the posterior distribution using Eq. (7):

\[
\omega_k^{(i)} = \omega_k^{(i)} / \sum_{j=1}^{N_\nu} \omega_k^{(j)}. \tag{11}
\]

The conditional probabilities, \( p(x(k,i) | x_{k-1,i}) \) and \( p(z_k | x(k,i)) \), which are known as the transition prior density and likelihood density, respectively, are determined by the system definition. \( p(x(k,i) | x_{k-1,i}) \) can be calculated from the previous particles \( x_{k-1,i} \), the current particles \( x(k,i) \) and the process noise distribution. Similarly, \( p(z_k | x(k,i)) \) is calculated from the current particles, the observation vector and the error distribution.

The importance density \( q(x(k,i) | x_{k-1,i}, z_k) \), which is not defined by the problem, is a design parameter chosen by the designer. A popular choice of the importance density in the SIS algorithms is the transition prior density:

\[
q(x(k,i) | x_{k-1,i}, z_k) = p(x(k,i) | x_{k-1,i}). \tag{12}
\]

Then, Eq. (10) becomes a simple equation:

\[
\omega_k^{(i)} = \omega_{k-1}^{(i)} p(z_k | x(k,i)). \tag{13}
\]

This method is very simple and easy to implement. However, it has a degeneracy problem which can be mitigated by using a resampling (or selection) procedure. This algorithm is called the SIR filter.

4. Integrity Monitoring

GNSS-based navigation systems must have an integrity monitoring system that contains two main functions: 1) detection and exclusion of satellite faults, and 2) estimation of the uncertainty of the position solutions. First, the system calculates decision variables and compares them to thresholds that have been set to satisfy the integrity requirements for the desired operation. If any decision variable exceeds the threshold, then the system concludes that the corresponding measurement has a fault, and excludes the detected fault. Second, because the system cannot know the true position estimation errors, it estimates the uncertainty of the solution, which is known as the protection level (PL). The system compares the value of the PL to a predefined alert limit (AL) for an expected operation (e.g., precision approach, non-precision approach and en-route) to decide whether or not the navigation system will use the solution.

We implemented four monitoring algorithms in this study: the snapshot \( \chi^2 \) method (SSX2), the Kalman filter \( \chi^2 \) method (KF2), the Kalman filter individual residual (KFIR) method, and the particle filter individual residual (PFIR) method. Our proposed algorithms for the two integrity monitor functions based on particle filters, which are based on the PFIR method, are described in the following Subsections 4.1 and 4.2, and the other methods will be discussed briefly.

4.1. Fault detection and exclusion algorithm

The proposed algorithm is a residual method that uses the posterior density of a state, which is estimated from the particle filters and the measurement error distribution. Residual is defined as the difference between an expected
measurement and an actual measurement, which will be described in detail in the following subsections. Residual methods using the snapshot algorithm (SSX2) and the Kalman filter (KFX2) calculate the decision variables using the squared sum of the residuals \((r)\), which is known as the weighted squared sum of the error (WSSE) that follows a \(\chi^2\) distribution whose degrees of freedom are determined by the number of measurements \((N_m)\), as in Eq. (14):

\[
\text{WSSE} = \frac{r^T r}{C_r} \sim \begin{cases} \chi^2(0, N_M - 4), & \text{SSX2} \\ \chi^2(0, N_M), & \text{KFX2} \end{cases},
\]

where \(C_r\) is the covariance matrix of the residuals.

This type of algorithm will not be described here, but can be found in other literature.\(^{12,13}\)

The Kalman filter or the particle filter approach can examine the residuals individually, but not combined residuals, because it has a previously estimated position. In the Kalman filter approach, KFIR, the algorithm compares a measurement residual to the multiple of the residual standard deviation. Since our proposed method, PFIR, assumes a non-Gaussian error, it cannot use the residual standard deviation. Instead, it estimates the residual distribution, and then decides whether or not the residual is located in a normal region. These individual residual algorithms have to estimate the residual distribution and determine the individual detection threshold for each residual.

(1) Calculation of the residuals and the distribution

The expected measurement was calculated from the propagation of the previous epoch state vector estimation, \(\hat{x}_k\), as

\[
\hat{x}_k = E[f_k(\hat{x}_{k-1}, v_{k-1})] = f_k(\hat{x}_{k-1}, 0).
\]

(15)

The expected measurement is a projection of the propagated state from the position domain to the measurement domain. Under normal conditions, the expected and the actually measured distributions lie close to each other, but when there is a measurement fault, they are separated by a distance. In our work, the distance between the two measurements is referred to as the residual:

\[
r_k = h_k(\hat{x}_k, 0) - z_k.
\]

(16)

The distribution for the \(j\)-th measurement residual at the \(k\)-th epoch \((r_k^j)\) is estimated by a form of a convolution sum as follows:

\[
p_{r_k^j}(r_k^j) = \int_{-\infty}^{\infty} p_{X_k}(\hat{x}_k')p_{x_k}(h_k^j(\hat{x}_k') - r_k^j)d\hat{x}_k'.
\]

\[
= \int_{-\infty}^{\infty} p_{X_k}(\hat{x}_k')p_{\text{norm}}(h_k^j(\hat{x}_k') - c_k^j - r_k^j)d\hat{x}_k'.
\]

\[
\approx \sum_{i=1}^{N_r} \hat{p}_{r_k}(r_k^j)\exp\left(-\frac{1}{2} r_k^j C_{r}^{-1} r_k^j\right).
\]

A sample of a residual distribution under nominal conditions is shown in Fig. 3. The expectation is near zero, and the large residuals have low probabilities. When the residual is in the region between the two thresholds, known as the normal region, the system declares that there is no fault in the measurement. Otherwise, it determines the measurement as having a fault, and excludes it for continuous operation.

In real implementation, the expectation of the PDF is located at the calculated residual and not zero. Therefore, we calculate a one-sided tail probability \((p_{r_k^j})\) below or above zero as a decision variable for the \(j\)-th measurement at the current epoch \(k\) using Eq. (18) and compare it to the individual false alarm rate that will be defined in the next subsection for each measurement:

\[
p_{r_k^j} = \begin{cases} \int_{-\infty}^{r_k^j} p_{r_k^j}(r_k^j)dr_k^j, & r_k^j \geq 0 \\ \int_{r_k^j}^{\infty} p_{r_k^j}(r_k^j)dr_k^j, & r_k^j < 0 \end{cases}.
\]

(18)

Since the Kalman filter individual residual method assumes a Gaussian error, the PDFs are all Gaussian, and the residual distribution can be expressed simply by a standard deviation so that the thresholds are determined as a multiple of the standard deviations.

(2) Set thresholds

The threshold is set to satisfy a specified false alarm rate \((P_{FA})\) when a \(\chi^2\) method based on the Gaussian distribution, SSX2 or KFX2, is employed:

\[
\int_{T_{kr}}^{\infty} p_{r_k^j}(x)dx = P_{FA}.
\]

(19)

where \(T_{kr}\) is the FDE threshold of the \(\chi^2\) methods.

However, the individual residual test requires a threshold for each measurement, and therefore an individual false alarm rate \((P_{r_k^j})\) needs to be determined first.

We used the multivariate normal distribution and the residual covariance matrix estimated from the Kalman filter. Assuming the individual false alarm rates for all residuals are same, we have to solve the following equation:

\[
P_{FA} = 2 \left(1 - \frac{1}{\sqrt{|C_r|/(2\pi)^{N_r}}} \int_{-\infty}^{K_{r1}} \int_{-\infty}^{K_{r2}} \cdots \int_{-\infty}^{K_{rN_r}} \exp\left(-\frac{1}{2} r^T C_r^{-1} r\right)dr\right).
\]

(20)
where $\sigma_i = C_r(i,i)$ and $K$ is a standard deviation multiplier. Because $K$ is the only unknown in this equation, we can calculate the unknown by iteration. With the determined $K$, the individual false alarm rate is calculated as follows:

$$ P_{fa} = 2 \int_{-\infty}^{\infty} P_{N(0,1)}(n) \, dn $$

(21)

Because we assumed the measurement error followed a symmetrical distribution in Section 2, the particle filter algorithm declares a fault to be present if the one-sided tail probability of a residual $pr_f^i$ exceeds a value of $P_{fa}/2$.

### 4.2 Protection level

LAAS users must calculate both fault-free and faulted protection levels ($XPL_0$ and $XPL_1$, respectively). The faulted protection levels indicate the position error protection assuming that a fault measurement exists. LAAS assumes that a fault is induced from the reference receiver errors. However, we assumed that the fault includes all possible errors contained in the measurements that are used for position estimation. The formulas used to calculate the protection levels based on a snapshot method assuming a Gaussian error were derived as follows. The Kalman filter and the particle filter methods can obtain the protection levels using a similar procedure, which will be discussed later.

In GNSSs, the measurement equation can be expressed using a linear function:

$$ z = Hx + u. $$

(22)

The state vector is estimated using the weighted least squares method and the estimation error can be obtained from Eq. (23):

$$ \delta \hat{x} = H_+^T \delta z, \ H_+ = (H^T WH)^{-1} WH^T, $$

(23)

where $W = \text{diag}[\sigma_{a1}^2, \ldots, \sigma_{a2}^2, \ldots, \sigma_{a26}^2]^{-1}$. When a fault $T$ occurs in a measurement $j$, the protection levels are calculated using the following equations:

$$ HPL_0 = k_{fmd} \sigma_h $$

$$ \equiv HPL_{0,n}, $$

$$ XPL_0 = k_{fmd} \sigma_p $$

$$ \equiv XPL_{0,n}, $$

and

$$ VPL_0 = k_{fmd} \sigma_v $$

$$ \equiv VPL_{0,n}, $$

$$ HPL_1 = k_{md} \sigma_h $$

$$ \equiv HPL_{1,n} + HPL_j, $$

$$ XPL_1 = k_{md} \sigma_p $$

$$ \equiv XPL_{1,n} + XPL_j, $$

$$ VPL_1 = k_{md} \sigma_v $$

$$ \equiv VPL_{1,n} + VPL_j, $$

where $\sigma_h^2 = (H^T WH)^{-1}_{1,1} + (H^T WH)^{-1}_{2,2}$ is the variance of the horizontal position error, $\sigma_v^2 = (H^T WH)^{-1}_{3,3}$ is the variance of the vertical position error, $k_{fmd} = 5.810$ and $k_{md} = 2.898$. For simplicity, the symbols $XPL_n$ and $XPL_j$ are used to express the noise and bias components, respectively.

The standard deviation multipliers $k_{fmd}$ and $k_{md}$ are defined in ‘LAAS MASPS’ for three ‘Performance Type 1’ reference receivers. The value of the fault $T$ is equal to the threshold for each FDE algorithm because it is the maximum fault that the system cannot detect. The filtering methods can estimate the protection levels with the above procedure, however, the values of $XPL_n$ and $XPL_j$ are determined differently for each algorithm. The Kalman filter method can obtain the standard deviations, $\sigma_h$ and $\sigma_v$, from the estimated covariance matrix, and $T_j$ is calculated from the residual standard deviation and the individual false alarm rate. The particle filter methods can estimate the protection levels from the estimated posterior density and the required (fault-free) missed detection probability as mentioned in Section 2.2. In the case of the vertical protection level, the following equation is solved

$$ P_{fa} = 2 \int_{x_u}^{\infty} P_{N(0,1)}(n) \, dn = \frac{P_{md}}{2}. $$

(26)

where $x_u$ is the up-direction position.

The threshold can be estimated from the residual density as follows:

$$ x_v = f_x(x_v-1, v_{v-1}) = F x_{v-1} + v_{v-1}. $$

(28)

Normally, the GNSS observation function is expressed as a linear function as follows:
\[ z_k = h_k(x_k, n_k) = H_k x_k + n_k, \] (29)

where
\[ x_k = \begin{bmatrix} x_e \ x_n \ x_u \ x_t \end{bmatrix}^T, \quad F = I_{4\times4}. \]

The measurement error \((n_k)\) is derived from the assumed distribution shown in Eq. (1). The standard deviation for each pseudorange was determined using the LAAS GAD-B and AAD-A model, which is a function of the elevation angle of each satellite.\(^{13}\)

\[ \sigma_i^2 = \sigma_{\text{PR, gmjd}}^2 + \sigma_{\text{SIS}}^2 + \sigma_{\text{AIR}}^2. \] (30)

‘GAD’ and ‘AAD’ mean accuracy designators that indicate the accuracy performance level of the GPS ground reference receivers and airborne receivers, respectively. ‘-B’ and ‘-A’ stand for the performance category.

The simulator intentionally injected biases \((b_k)\) into the pseudorange of a satellite to assess the FDE performances of the algorithms as follows:

\[ b_k = \begin{cases} 5.0 \text{ m}, & k = 20 \sim 40 \text{ sec} \\ 4.3 \text{ m}, & k = 60 \sim 80 \text{ sec.} \\ 0.0 \text{ m}, & \text{otherwise} \end{cases} \] (31)

The biases simulate sudden jumps that can be included in pseudorange measurements due to a GPS clock failure or a correction message failure. In case of carrier phase measurements, cycle slips will cause the measurement jumps. A significant GPS clock failure would cause a large range drift. For example, the PRN 23 clock anomaly on January 20, 2004, caused a range rate difference of over 70 m/s which can be expressed by a sudden 70 m jump in the pseudorange measurement generated at the 1 Hz rate. The bias values 5 and 4.3 meters are set to investigate the marginal performance of the algorithms, and the two values can show the performance differences of the implemented algorithms.

Four algorithms described in Section 4 were used to detect and exclude the fault and to estimate the protection levels.

5.2. Measurement domain result

Figure 4 shows the decision variables (WSSE) and thresholds of the SSX2 and KFX2 algorithms. When a decision variable exceeded the threshold, the system declared a fault. The SSX2 method could not detect any faults due to the high noise level. The KFX2 method detected the 5 m bias initially, but it failed to detect anything thereafter. When a filtering method fails to detect and exclude a bias, the remaining bias propagates through the filter and becomes more difficult to detect afterwards.

The individual residual methods (KFIR and PFIR) had as many decision variables as the number of measurements at an epoch. The left-hand side of Fig. 5 shows the normalized residual of the fault measurement and the threshold value for the KFIR method, and the right-hand side of Fig. 5 depicts those of the PFIR method. The KFIR method detected the 5 m bias perfectly. However, it did not detect the 4.3 m bias after 65 s. The undetected bias remained in the filtering process, and led to a large position error that propagated in the subsequent process, which led the monitoring system not detecting the fault after the missed detection and the other nominal residuals became noisy. The PFIR method detected the faults at all epochs where the system had a fault measurement.

5.3. Position domain result

Figure 6 shows the vertical position errors (left-hand side) and vertical protection levels (right-hand side) for each algorithm after the FDE procedure. Since the SSX2 method and KFX2 method did not detect faults well, they exhibited large position errors induced by the remaining faults. The KFIR method excluded all of the 5 m biases and provided
more accurate solutions from 20 to 40 s. However, the missed detection of the 4.3 m bias caused a large error after the epoch of 65 s. The PFIR method detected and excluded all of the biases and provided the most accurate position solutions. Table 1 shows the 95% level errors for each method. As for the protection levels, the XPL₀ values are not shown here because they were always smaller than the XPL₁ values in our simulation. The data show that the SSX2 method provided the highest protection level, and the other filtering methods had lower protection levels. The KFX2 method provided a slightly higher protection level than the KFIR method, which means that the KFIR method could detect smaller biases. The PFIR protection level was the lowest, and this shows that the algorithm outperformed the other algorithms in terms of its integrity monitoring ability.

Table 1. The 95% level position errors for each algorithm.

<table>
<thead>
<tr>
<th></th>
<th>SSX2</th>
<th>KFX2</th>
<th>KFIR</th>
<th>PFIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Error (m, 95%)</td>
<td>2.55</td>
<td>2.14</td>
<td>2.08</td>
<td>0.34</td>
</tr>
<tr>
<td>Vertical Error (m, 95%)</td>
<td>3.82</td>
<td>2.55</td>
<td>2.27</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 2. Protection levels under fault-free conditions.

<table>
<thead>
<tr>
<th></th>
<th>SSX2</th>
<th>KFX2</th>
<th>KFIR</th>
<th>PFIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPL₁ (m)</td>
<td>5.49</td>
<td>3.40</td>
<td>2.97</td>
<td>2.09</td>
</tr>
<tr>
<td>(HPL₁/HPL₁,KFIR, %)</td>
<td>184.85</td>
<td>114.48</td>
<td>100</td>
<td>70.37</td>
</tr>
<tr>
<td>VPL₁ (m)</td>
<td>9.88</td>
<td>5.49</td>
<td>4.67</td>
<td>3.44</td>
</tr>
<tr>
<td>(VPL₁/VPL₁,KFIR, %)</td>
<td>211.56</td>
<td>117.56</td>
<td>100</td>
<td>73.66</td>
</tr>
</tbody>
</table>

5.4. Result analysis

The reason that the PFIR method outperformed other methods can be explained as follows.

The SSX2 data showed the worst performance due to the high noise level, which is straightforward to understand. The KFX2 and KFIR methods used the same filtering procedures, but had different decision variables: WSSE and individual residuals, respectively. For a fixed false alarm rate, the \( \chi^2 \) method has a larger threshold because it needs to consider the noise of the other fault-free measurements contained within the WSSE calculations. In contrast, since the individual methods only considered one residual at a time, they had smaller thresholds. This difference is responsible

Fig. 5. Residual and threshold of KFIR and PFIR (fault measurement).

Fig. 6. The vertical position errors and protection levels after exclusion.
for the better detection performance and availability of the KFIR method.

The KFIR and PFIR methods used the same FDE algorithm, but they used the overbounded Gaussian and non-
Gaussian error model, respectively. The impact of the different error models can be described as follows. In Eqs. (24) and (25), the important factors are the detection threshold, \( T \), and the position error noise, \( \text{VPL}_n \). When \( T \) is small, the system can detect smaller biases, and smaller values of \( T \) and \( \text{VPL}_n \) lead to lower protection levels that result in higher availability. Figure 7 shows the one-sided tail probability of the residuals of the KFIR method (dotted) and PFIR method (solid). The fault detection threshold is the residual at which the tail probability was equal to \( P_{\text{fa}}/2 \), as defined previously. In Fig. 7, the thresholds for the KFIR and PFIR methods had a difference equal to \( D_1 \) due to the overbounding effect, and the difference of the residual values at which the tail probabilities were equal to the missed detection rate caused the noise part to have a discrepancy. A projection of the difference, \( D_2 \), to the position domain is shown as the protection level difference:

\[
T_{\text{KFR}} = T_{\text{PFIR}} + D_1 \tag{32}
\]

\[
\text{VPL}_{\text{KFR}} = \text{VPL}_{\text{PFIR}} + \sum_{i=1}^{N_{\text{oa}}} H_{\text{W}}^{(3, i)} \cdot D_2(i) \tag{33}
\]

Equations (32) and (33) describe that the PFIR method has a smaller minimum detectable bias and that both equations indicate the PFIR method has smaller protection levels.

6. Conclusion

In this study, we developed an integrity monitoring algorithm that tests measurement residuals individually using particle filters. The simulation results show that our proposed algorithm detected smaller measurement biases and generated smaller protection levels by about 30%, which implies that systems with the algorithm can provide better integrity monitoring performance than other methods that assume an overbounded Gaussian measurement. The data show that the conservative sigma overbounding methods degrade integrity monitoring performance. Inversely, using a true error model or more accurate overbounding methods, we can provide a more available navigation system without losing any integrity or continuity.

Although our proposed algorithm requires a high computational capacity due to the large particle set used, it can be used in more sophisticated systems or for development and performance assessment of sigma overbounding methods in conventional systems.

In the future, more computationally efficient algorithms need to be developed to enable wider usage of our method. In addition, because our algorithm tests the residuals individually, it can be used to detect simultaneous multiple faults, which is another interesting topic for future research.

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