Numerical Analysis of Aerodynamic Interference between Two Circular Cylinders Using the Overset Grid Method*

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Flows over two circular cylinders in tandem, side-by-side, and staggered arrangements were analyzed using the overset grid method, which is capable of handling a variety of sizes and arrangements. The Reynolds number was 100 based on the cylinder diameter. The present computation code was validated by comparison with benchmark solutions for flow around a single cylinder. Wind-tunnel experiments were conducted for the side-by-side cylinder flow for comparison with numerical simulations. Calculation showed two critical spacings in the tandem arrangement where the aerodynamic forces and Strouhal number change discontinuously. Three critical spacings and four distinct flow patterns were found numerically in the side-by-side arrangement. Similar critical spacings were found in the staggered arrangement calculation and formed critical lines. Furthermore, a pocket region was found for a staggered arrangement surrounded by the critical line.

Key Words: Aerodynamics, Circular Cylinder, CFD

Nomenclature

- \( C_l \): lift coefficient (time average)
- \( C_d \): drag coefficient (time average)
- \( C_p \): pressure coefficient (time average)
- \( D \): diameter of circular cylinder
- \( Re \): Reynolds number
- \( r \): radial direction
- \( S \): spacing
- \( S_t \): Strouhal number of circular cylinder 1 from lift (= \( St_1 \))
- \( y \): coordinate normal to flow direction
- \( \alpha, \beta \): interpolation constant
- \( \theta \): circumferential direction
- \( b \): rear stagnation
- \( c \): cylinder or critical
- \( p \): pressure
- \( s \): fore stagnation
- \( v \): viscous
- \( 1 \): forward circular cylinder
- \( 2 \): rear circular cylinder
- \( \overline{\cdot} \): time average
- \( ' \): rms value of fluctuation

1. Introduction

Aerodynamic interaction over multiple bodies occurs in many engineering applications and has been the focus of recent attention. Examples of such structures and flow fields include the Space Shuttle Carrier Aircraft (SCA), stage separation of a two-stage-to-orbit (TSTO) vehicle, slipstreams of F1 race cars and around skyscrapers in big cities. Aerodynamic interference associated with these applications is dependent on the flow conditions and sizes, shapes, and arrangements of objects.

As a basic study of an aerodynamic two-body problem, we conducted numerical simulations of air flow over two circular cylinders in tandem, side-by-side, and staggered arrangements to investigate the effect of arrangements on flow characteristics. The purpose is to clarify the relationship between the arrangements of two circular cylinders and mutual aerodynamic interactions for incompressible flow, and then to further understand the underlying mechanisms qualitatively and quantitatively.

First, we tested the validity of the computation code by comparison with benchmark solutions for flow around a single circular cylinder. Physical quantities, such as drag/lift coefficients and Strouhal number, were plotted against the arrangement spacing. In addition, a wind-tunnel experiment was conducted for the side-by-side cylinder flow to examine the results of the numerical simulations.

2. Numerical Method

2.1. SMAC method

Numerical simulations of two-dimensional incompressible flow over two circular cylinders in tandem, side-by-side, and staggered arrangements at \( Re = 100 \) were conducted using the SMAC (Simplified Marker And Cell) method.\(^1\)

At a Reynolds number of 100, the flow is laminar and the wake fluctuates periodically. The governing equations for flow are the continuity and Navier-Stokes equations for two-dimensional incompressible flow. They are integrated...
in time using the first-order explicit Euler method and resolved in space with the second-order central difference scheme for the diffusion terms and the third-order upwind scheme (Kawamura-Kuwahara scheme) for the convection terms.2)

2.2. Overset grid

An overset grid method was used considering applicability to different cylinder arrangements and spacings.1) Figure 1 shows the general view of an overset grid used in the present study, consisting of one Cartesian global grid \( x \times y = 45D \times 30D \) for the whole flow field, and two O-type local grids of diameter \( 2D \) around the circular cylinders. The leading circular cylinder is located at \( x = 15D \) from the inlet of the computational domain. The global grid points are \( x \times y = 200 \times 160 \) and the local grid points are \( r \times \theta = 30 \times 120 \). Grid clustering is used in both grid systems. Data exchange by interpolation between the global and local grids takes place at every single time interval. Physical quantities \( q_p \) at a particular grid point \( p \) were interpolated linearly from four neighboring grid points around \( p \) as shown in Fig. 2. The linear-interpolation is given by

\[
q_p = (1 - \alpha)(1 - \beta)q_{i,j} + \alpha(1 - \beta)q_{i+1,j} + (1 - \alpha)\beta q_{i,j+1} + \alpha\beta q_{i+1,j+1} \tag{1}
\]

Each grid is updated with the interpolated physical quantities of Eq. (1), and the Navier-Stokes equations are integrated each time interval on the global and the local grids alternatively.3–5)

3. Computational Results

The present code with overset grid method was compared with benchmark solutions for flow around a single circular cylinder. Table 1 shows the present solutions compared to three numerical simulations in the published data. Table 1 shows that the code in the present study is satisfactory.

All simulations were conducted at Reynolds number of 100, based on the diameter of a single cylinder. Flows over two circular cylinders are also considered laminar at \( Re = 100 \) unless they are very close to each other. The arrangements of the circular cylinders are shown in Fig. 3 as follows: (a) tandem arrangement parallel to uniform flow of circular cylinder 2 at \( (x_{c2}, y_{c2}) = (s, 0) \); (b) side-by-side arrangement perpendicular to uniform flow of circular cylinder 2 at \( (x_{c2}, y_{c2}) = (0, s) \); (c) staggered arrangement oblique to uniform flow of circular cylinder 2 at \( (x_{c2}, y_{c2}) = (s \cos \theta, s \sin \theta) \).

3.1. Tandem arrangement

In the present study, 33 numerical simulations were conducted ranging from \( s = 1.3 \) to \( s = 10.0 \). Figures 4 and 5 show the Strouhal number and average drag coefficient, respectively, for different circular cylinder spacing \( s \). These figures include the numerical data of Sharman et al.6) Two critical spacings were found as shown below:

![Fig. 1. Overset grid.](image1)

![Fig. 2. Linear interpolation.](image2)

![Fig. 3. Cylinder arrangement.](image3)

<table>
<thead>
<tr>
<th></th>
<th>Park</th>
<th>Posdziech</th>
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<td>36</td>
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<td>36</td>
</tr>
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</table>

Table 1. Code validation for circular cylinder at \( Re = 100.6) \n
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where, $s_{c1}$ refers to the first critical spacing, and $s_{c2}$ refers to the second. At the critical spacing, the Strouhal number and aerodynamic forces change discontinuously. Although Sharman et al.\textsuperscript{6)} and earlier studies mentioned the existence of critical spacing, $s = s_{c1}$, the second critical spacing, $s = s_{c2}$, found here has not been mentioned before.

Figure 6 shows vorticity contours and streamlines at $s = 2.0$, $s = 4.0$, and $s = 6.0$ in the tandem arrangement.

At $s = 2.0$, the wake of cylinder 1 forms a negative pressure region and is stable between the two cylinders. This is called slipstream. As shown in Fig. 5, the average drag of circular cylinder 2 is negative at a small spacing, meaning the cylinder is pushed forward. The flow field around two circular cylinders together with the boundary layer forms a 2-D airfoil-like section.

**3.2. Side-by-side arrangement**

A total of 39 numerical simulations were calculated for the side-by-side arrangement ranging from $s = 1.3$ to $s = 10.0$. Figures 7 and 8 show the Strouhal number and the average drag coefficient, respectively. Three critical spacings were found as shown below:

\[
\begin{align*}
    s_{c1} &\approx 1.3 \sim 1.4 \\
    s_{c2} &\approx 2.2 \sim 2.3 \\
    s_{c3} &\approx 3.15 \sim 3.2
\end{align*}
\]
Figure 9 shows the vorticity contours and streamlines for $s = 1.3$, $s = 2.0$, $s = 3.0$, and $s = 4.0$. These correspond respectively, to the following four flow patterns: deflected, flip-flopping, in-phase-synchronized, and anti-phase-synchronized.

A deflected wake pattern is observed at $s = 1.3$. As shown in Fig. 9, the flow between the two circular cylinder surfaces (gap flow) is deflected constantly to one side, and the circular cylinder on the side experiences higher drag than the other. The flow field as a whole fluctuates periodically with the gap flow remaining deflected to one of the two circular cylinders.

At $s = 2.0$ in Fig. 9, a randomly fluctuating flow without clear periodicity is observed in the calculation. This irregular flow seems to flip-flop randomly between the two flow patterns either deflected upwards or downwards (see $s = 1.3$ of Fig. 9. Consequently, the flow structure is called a flip-flopping wake pattern.\cite{7,8} The Strouhal numbers are shown with error bars in Fig. 7 because no clear peak is observed due to flow characteristics.

At $s = 3.0$, the two wakes become in-phase synchronized, as shown in Fig. 9, after experiencing the initial transitory duration of anti-phase-synchronized pattern due to the numerical impulsive acceleration of the flow. Moreover, the lift coefficients for both cylinders are in-phase while the drag coefficients are out-of-phase. The flow is completely periodic.
Conversely at $s = 4.0$, the two wakes remain in constant anti-phase synchronization. Vortex shedding at both cylinders occurs in the opposite phase and the lift coefficients are in anti-phase (180° phase difference) while the drag coefficients are in the same phase. In other words, the flow field at any instant is symmetric with respect to the centerline between the two cylinders, and the center streamline is straight far downstream as shown in Fig. 9. The flow is completely periodic.

The phase of the lift and drag is governed by the symmetric and non-symmetric nature of the flow field for $s = 4.0$ and $s = 3.0$, respectively.

Figure 10 shows the average lift coefficient against the spacing $s$ in the side-by-side arrangement. As clearly shown, the average lift for both cylinders is a repulsive force; the closer the two cylinders get, the larger the force is. The average lift may be attributed partly to the two types of forces: one is a repulsive force, resulting from the viscous effect in the gap similar to bearing lubrication, and the other is an attractive force due to the potential nature of the flow, i.e., the Bernoulli effect in the gap. Although the lift changes its sign alternately due to the vortex shedding, the time average lift is a repulsive force under the present computational condition at $Re = 100$, possibly due to the viscous effect. Moreover, the average lift coefficient varies as nearly $1/s^2$. The inverse square law reminds us of gravity although it is attractive.

3.3. Staggered arrangement

In the staggered arrangement, 40 cases were simulated numerically from $s = 2.0$, $\theta = 10^\circ$ to $s = 6.0$, $\theta = 80^\circ$. Figures 11, 12 and 13 show the 2D plots of $St_1$, $C_{l_2}$, and $C_{d_1}$ in the $x$-$y$ plane, respectively. Circular cylinder 1 is located at the origin (0,0) and each plot shows the physical quantity when circular cylinder 2 is located at $(x_{c2}, y_{c2})$. Some regions are interpolated coarsely, but are sufficient to display the trends. These three figures have a common critical region around $s = 4.0$, $\theta = 10^\circ[(x, y) = (s \cos \theta, s \sin \theta)]$.

To consider the mechanism in the critical region, the vorticity contours and streamlines at $s = 4.0$, $\theta = 10^\circ$ and $s = 4.0$, $\theta = 20^\circ$ are shown in Fig. 14. At $s = 4.0$, $\theta = 10^\circ$, the vortex from the top side of circular cylinder 1 is held by circular cylinder 2. The flow is deflected downward by circular cylinder 2 and then the vortex shed from the lower side of circular cylinder 1 is also trapped by circular cylinder 2. Consequently, the wake of circular cylinder 1 is closed and forms a negative pressure region. In other
words, the twin vortices remain stable behind circular cylinder 1. As a result, a strong vortex is shed only from the top side of circular cylinder 2 and the flow field remains relatively stable.

Conversely, at $s = 4.0, \theta = 20^\circ$, vortices shed from the top side of circular cylinder 1 and the bottom side of circular cylinder 2 strengthen each other due to synchronized vortex shedding at this relative position of the two circular cylinders. Consequently, the wake as a whole fluctuates stronger and wider than that of $s = 4.0, \theta = 10^\circ$. This mechanism could result in the critical pocket around $s = 4.0, \theta = 10^\circ$ in Figs. 11, 12, and 13.

4. Supplemental Flow Visualization for Side-by-side Arrangement

A flow visualization experiment was conducted for the side-by-side arrangement to further examine the computed flow pattern, i.e., in-phase- and anti-phase-synchronized flows. A low-speed blow-down wind tunnel in the Department of Aeronautics and Astronautics, Tokyo University, was used. The exit aperture is $250 \text{ mm} \times 250 \text{ mm}$. The smoke-wire method was used to visualize the flow. Circular cylinders with a diameter of $5.0 \text{ mm}$ were used. The Reynolds number was approx. $10^7$ for $D = 5 \text{ mm}$, and the flow was effectively laminar.

The in-phase-synchronized flow at $s = 3.0$ and the anti-phase-synchronized flow at $s = 4.0$ visualized in the experiment are shown in Figs. 15 and 16, respectively. These figures agree qualitatively with the results of the numerical simulations (see Figs. 9 and 10) except for the buoyancy effect on the streak lines. However, the experimental flow patterns were not always the same in the given condition. Table 2 shows occurrences where each flow pattern was observed in the experiment.

At $s = 2.5$, the in-phase-synchronized flow pattern was observed most frequently, but the flip-flop pattern was observed 6 times and the anti-phase-synchronized pattern was observed twice. Beyond $s = 3.0$, the anti-phase pattern became dominant although the in-phase pattern could be observed even at $s = 4.0$. The flow pattern may be bifurcated in some flow conditions, such as the initial condition, and the 3D effect different from the 2D numerical conditions. However, compared to Eq. (3), the trends agreed with the results of the numerical simulations.

The Strouhal number can be approximated by $D/a$, where $a$ is the distance between the two periodic vortices. Therefore, it is possible in theory to estimate the Strouhal number from a photograph like Fig. 15. One might estimate $D : a \approx
from Fig. 15 and $St \approx 1/6 \approx 0.17$. Although it is close to the computational value of 0.172 (see Fig. 7), this is an order of magnitude analysis due to the ambiguity of the vortex position.

5. Conclusions

Flows over two circular cylinders in tandem, side-by-side, and staggered arrangements were investigated numerically at $Re = 100$. There are several critical spacings and critical regions where flow pattern changed abruptly.

In the tandem arrangement, two critical spacings $s = s_c (3.7\sim3.75, 5.0\sim5.1)$ were found for $1.3 \leq s \leq 10$. The second critical spacing has not been mentioned in previous studies.

In the side-by-side arrangement, three critical spacings $s = s_c (1.3\sim1.4, 2.2\sim2.3, 3.15\sim3.2)$ and four distinct wake patterns: deflected, flip-flop, in-phase-synchronized, and anti-phase-synchronized were found for $1.3 \leq s \leq 10$.

In the staggered arrangement, the numerical simulations covered $(x, y) = (s \cos \theta, s \sin \theta)$ from $s = 2.0$, $\theta = 10^\circ$ to $s = 6.0$, $\theta = 80^\circ$. A critical pocket around $s = 4.0$, $\theta = 10^\circ$ was found in the $x$-$y$ plane surrounded by critical lines.

In addition, a supplementary flow-visualization experiment was done in the side-by-side arrangement using the smoke-wire method. Although randomness was observed in the experiment, the visualized flow exhibited similar qualitative trends to the numerical simulations.

References