Optimal Interplanetary Trajectories for Impulsive Deflection of Potentially Hazardous Asteroids under Velocity Increment Uncertainties*

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This paper investigates the interplanetary trajectories associated with the impulsive deflection of a potentially hazardous asteroid (PHA) considering the uncertainty of the velocity increment that a spacecraft gives to the PHA at the collision. The velocity increment is assumed to have uncertainties of magnitude and direction due to estimation errors of asteroid shape and mass distributions. The uncertainty is modeled using a convex model assuming that magnitude and direction vary independently. The effect of uncertainty is assessed by evaluating the worst (i.e. minimum) value for the closest approach distance between the PHA and Earth. The worst value of the closest approach distance can be determined analytically without searching the whole convex hull. The optimal spacecraft trajectory is designed by maximizing the worst value of the closest approach distance in terms of the Earth departure date and the asteroid arrival date of the spacecraft under C3 (Earth departure energy) constraint. Using a numerical example with a fictitious asteroid, the importance of considering the velocity increment uncertainty is demonstrated by comparing the optimal trajectory with the deterministic optimal trajectory. The uncertainty of the velocity increment direction is shown to have a significant effect on the deflection of the PHA.

Key Words: Spacecraft, Asteroid Deflection, Uncertainty, Potentially Hazardous Asteroids

1. Introduction

Most of the asteroids in our solar system are found between the orbits of Mars and Jupiter called the main-belt, while some asteroids approach and cross Earth’s orbit. They are called potentially hazardous asteroids (PHAs), with a possibility of impacting Earth. Many scientists have studied PHA orbit propagation and quantified the hazards of Earth impact.1)

Several strategies indicate that to prevent an asteroid collision with Earth, deflection by kinetic energy is more effective than fragmentation of the asteroid itself.2) Park and coauthors3–5) investigated the optimal magnitude and direction of velocity increment required to deflect asteroids, but the interplanetary trajectory of the spacecraft has not been discussed. Ivashkin6) showed that a spacecraft with a mass of 8 tonnes at launch could deflect Toutatis, which has a radius of less than 270 m. He also presented some models for the spacecraft impact asteroid with several different values of required momentum for deflecting the asteroid. Other papers7–9) studied asteroid-deflection missions using electric propulsion or solar-sail technology. Izzo10) studied an asteroid deflection mission for 99942 Apophis, which approaches Earth in 2029, stating that asteroid deflection should be performed at least 3 years before the approach to Earth using multiple gravity assist.

Don Quijote mission proposed by ESA will investigate the effect of impulsive asteroid deflection using two spacecraft: one will impact an asteroid and the other will investigate the asteroid to determine the collision point and then observe the change in orbit by rendezvousing with the asteroid.11) The mission achievements will be used for future asteroid-deflection missions.

The asteroid-deflection mission requires the orbital parameters, mass, shape and the surface structure of the target PHA. However, it is difficult to determine exact values due to limitations on measurement accuracy. Therefore, the asteroid-deflection mission should consider uncertainties. Vasile12) studied the optimal trajectory for asteroid deflection, including the uncertainty of orbital elements and mass of the asteroid. However, he did not consider uncertainties of the velocity increment that a spacecraft impacts to the asteroid.

This paper investigates the optimal interplanetary trajectory for deflecting PHAs with uncertainty of the velocity increment. This uncertainty is modeled as a convex model assuming that uncertainties of magnitude and direction of velocity increment vary independently. The optimal trajectory is designed by maximizing the worst (minimum) value of the closest Earth approach distance in the uncertain range of the velocity increment. The effect of velocity increment uncertainty is investigated by numerical examples.

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Section 2 outlines the asteroid deflection mission in the present study. Section 3 defines the orbit design problem considering uncertainty of the velocity increment. Section 4 describes some examples of optimal trajectories with and without velocity increment uncertainty. Comparing these examples shows the importance of considering uncertainty of the velocity increment.

2. Outline of Asteroid Deflection Mission

The spacecraft in this mission is assumed to follow a ballistic interplanetary trajectory after leaving Earth and to intercept an asteroid. The spacecraft imparts a velocity increment \( \Delta V \) to the asteroid by the perfectly inelastic impact model, where the spacecraft mass \( m \) is assumed to be much smaller than the mass of the asteroid \( M (m \ll M) \) and negligible. The mission constrains the upper limit to \( C3 \), the square of the escape velocity at Earth departure due to launcher performance. The spacecraft and asteroid orbits are calculated by patched-conic approximation, assuming an unperturbed elliptic orbit around the Sun for the interplanetary trajectory of the spacecraft and asteroid, and hyperbolic orbit around Earth for the asteroid. Figure 1 shows the strategy of asteroid-deflection mission, detailed as follows:

1. The asteroid is assumed to hit Earth dead center.
2. The spacecraft heliocentric orbit is determined from the Earth departure time \( t_{\text{dep}} \) and asteroid arrival time \( t_{\text{arr}} \) using the Lambert method. \( C3 \) can be calculated from the Earth orbital velocity \( V_{\infty} \) and spacecraft Earth departure velocity \( V_{\text{dep}} \) as follows:
   \[
   C3 = |V_{\text{dep}} - V_{\infty}|^2
   \]
   If multiple-revolution trajectories are obtained, the trajectory with the smallest value of \( C3 \) is used.

3. The spacecraft impacts the asteroid with asteroid arrival velocity \( V_{\text{arr}} \). The velocity increment \( \Delta V \) added to the asteroid can be approximated as follows:
   \[
   \Delta V = \frac{m}{M + m} V_{\text{arr}} \approx \frac{m}{M} V_{\text{arr}} \quad (2)
   \]
4. The velocity increment \( \Delta V \) changes the asteroid orbit, so the asteroid orbital period is increased as shown in Fig. 1.
5. The asteroid passes Earth delayed from its original orbit, avoiding collision. Figure 2 shows the hyperbolic trajectory of the asteroid nearing Earth, and the Earth closest approach distance \( d \) is obtained from the following equation.
   \[
   d = \frac{\mu_e}{|V_{\infty}|} \left( \sqrt{1 + \frac{|V_{\infty}|^2 d_{\infty}^2}{\mu_e^2}} - 1 \right) \quad (3)
   \]
   where, \( \mu_e \) is a gravity constant of Earth, \( V_{\infty} \) is the hyperbolic excess velocity of asteroid with respect to Earth, and \( d_{\infty} \) is the distance from Earth center to the asteroid incoming asymptote.

3. Orbit Design Problem under Uncertainties

3.1. Uncertainty model of velocity increment

It is difficult to estimate exactly the velocity increment that the spacecraft impacts to the PHA at collision exactly because shape, surface material, and mass distribution of the PHA are not known exactly. The velocity increment varies from the case of a perfectly inelastic collision, so magnitude and direction have uncertainties. In this research, the quantity of magnitude and direction change of the velocity increment are modeled using independent uncertainty parameters.

The uncertainty parameter of the velocity increment magnitude is denoted as \( \alpha \) representing the ratio between the effective momentum and the momentum for perfectly inelastic collision. The range of the magnitude uncertainty parameter \( \alpha \) is determined from the average and standard deviation of \( \Delta V \) magnitude \( (\mu_\alpha, \sigma_\alpha) \).

The uncertainty parameters of the direction change of the velocity increment are defined as \( d\theta \) and \( d\phi \) describing
changes in the velocity increment directions of in-plane and vertical components, respectively, as shown in Fig. 3. The velocity increment under uncertainty $\Delta V'$ is described as follows:

$$
\Delta V' = \alpha |\Delta V| \left( \cos d\phi \cos(\theta + d\theta) \cos d\phi \sin(\theta + d\theta) \sin d\phi \right)
$$

where $\theta$ is the angle between the $X_c$ axis of the spacecraft-center reference frame and $\Delta V$. Considering the impact of the spacecraft with the asteroid, the uncertainty of parameters for $\Delta V$ direction $d\theta$ and $d\phi$ are interdependent. It is natural to assume the $\Delta V$ spread is conic, so the following relationship is assumed:

$$
d\theta^2 + d\phi^2 \leq \gamma^2
$$

where $\gamma$ is the upper limit of the uncertainty range determined from the standard deviation $\sigma_\gamma$. Therefore, the uncertainty range of the velocity increment $(\alpha, d\theta, d\phi)$ is modeled by a convex hull as shown in Fig. 4. The effect of considering uncertainty of the velocity increment is evaluated by the worst value $d'$ corresponding to the minimum closest approach distance between Earth and the asteroid. The worst value for the prescribed interplanetary trajectory determined from Earth departure time and asteroid arrival time is evaluated by the following optimization problem:

Minimize : $d' = d(\alpha, d\theta, d\phi)$

subject to : $\mu - 3\sigma \leq \alpha \leq \mu + 3\sigma$

$$
d\theta^2 + d\phi^2 \leq \gamma^2
$$

If $d'$ is represented as a convex function in terms of the uncertainty parameters, the minimum value of $d'$ lies along the boundary of the uncertainty convex hull. That is a convex model in which the minimum value is easier to find. In this study, the worst value is determined analytically without searching the convex hull as discussed later.

3.2. Optimal trajectory design problem under uncertainty

In the present spacecraft trajectory optimization, the Earth departure time $t_{dep}$ and asteroid arrival time $t_{arr}$ are chosen as design variables to maximize the worst value of the Earth closest approach distance $d'$ under $C3$ constraint, which imposes an upper limit $C3^u$ for mission feasibility. Then, the optimal trajectory problem is formulated as follows:

Maximize : $d'(t_{dep, t_{arr}})$

subject to : $C_3 \leq C_3^u$

$$
t_0 \leq t_{dep} \leq t^u_{dep}
$$

$$
t_0 \leq t_{arr} \leq t^u_{arr}
$$

where, $t^l_{dep}$, $t^u_{dep}$, $t^l_{arr}$ and $t^u_{arr}$ are the lower and upper limits of the Earth departure date and the asteroid arrival date, respectively. Considering problem (6), the optimal trajectory problem with uncertainty is formulated as a max-min problem.

4. Results and Discussion

4.1. Mission conditions

The asteroid is assumed to impact Earth on January 1, 2060 at the ascending node. The asteroid shape is assumed to be a perfect sphere 150 m in diameter and mass of 5.3 Mtonne, with an average density of 3.0 g/cm$^3$. This orbit is determined by referring to 3361 Orpheus which has average orbital parameters for PHAs. The orbital elements of the fictitious asteroid for the present study and Orpheus are listed in Table 1. The upper limit of $C3$ is determined as 50 km$^2$/s$^2$ matching the capability of the average of the HII-A and M-V launchers. The spacecraft mass is assumed to be 1.0 tonne when it impacts the asteroid. The average and standard deviation of the uncertainty parameters are listed in Table 2.

4.2. Optimal trajectories without uncertainty

First, the optimal spacecraft trajectory to maximize the closest approach distance without uncertainty of $\Delta V$ (asteroid velocity increment at spacecraft impact) is obtained.

<table>
<thead>
<tr>
<th>Table 1. Orbital elements of asteroids.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fictitious asteroid</strong></td>
</tr>
<tr>
<td><strong>Orpheus</strong></td>
</tr>
<tr>
<td>$a$ [AU]</td>
</tr>
<tr>
<td>$e$</td>
</tr>
<tr>
<td>$i$ [deg]</td>
</tr>
<tr>
<td>$\Omega$ [deg]</td>
</tr>
<tr>
<td>$\omega$ [deg]</td>
</tr>
<tr>
<td>$v_0$ [deg]</td>
</tr>
</tbody>
</table>

Epoch 1/1/2060 9/13/2000
Table 2. Assumption of fictitious asteroid, spacecraft, and $\Delta V$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth collision date of asteroid</td>
<td>1/1/2060</td>
</tr>
<tr>
<td>Asteroid diameter [m]</td>
<td>150</td>
</tr>
<tr>
<td>Asteroid density [$g/cm^3$]</td>
<td>3.00</td>
</tr>
<tr>
<td>Asteroid mass [tonne]</td>
<td>5.30E+06</td>
</tr>
<tr>
<td>Spacecraft mass [tonne]</td>
<td>1.00</td>
</tr>
<tr>
<td>$C_3$ limit at Earth [km/s$^2$]</td>
<td>50.0</td>
</tr>
<tr>
<td>Average of $\Delta V$ magnitude $\mu_a$</td>
<td>0.60</td>
</tr>
<tr>
<td>Standard deviation of $\Delta V$ magnitude $\sigma_a$</td>
<td>0.10</td>
</tr>
<tr>
<td>Average of $\Delta V$ direction $\mu_r$ [deg]</td>
<td>0.0</td>
</tr>
<tr>
<td>Standard deviation of $\Delta V$ direction $\sigma_r$ [deg]</td>
<td>10.0</td>
</tr>
</tbody>
</table>

> Fig. 5. Distribution of closest approach distance.

> Fig. 6. Optimal trajectory for nominal $\Delta V$ case.

> Fig. 7. Closest approach distance with respect to $\Delta V$ magnitude uncertainty.

4.3. Optimal trajectories under uncertainty

The minimum value of the Earth closest approach distance $d$ under the uncertainty of velocity increment $\Delta V$ can be obtained without searching the whole convex hull. The uncertainty parameters that give the worst closest approach distance are denoted as $(\alpha_{\text{worst}}, t_{\text{dep, worst}}, t_{\text{arr, worst}})$, which are called the “worst $\Delta V$” case.

4.3.1. Worst value evaluation method with uncertainty

The closest approach distance with respect to the uncertainty parameter $\alpha$ in the case of $(t_{\text{dep}}, t_{\text{arr}}) = (7/16/2010, 12/6/2011)$ is shown in Fig. 7. The closest approach distance is found to be almost linear in terms of the magnitude of the velocity increment parameter $\alpha$ when the closest approach distance is larger than the Earth radius. This result shows that the smaller impulse gives a smaller momentum to the asteroid. The distribution of the closest approach distance at the lower limit of the magnitude of velocity increment, $\alpha = 0.3$, is shown as contour curves in Fig. 8 under the same assumptions as in Section 4.2. The axes and contours in Fig. 8 show the same as in Fig. 5. The magnitude of the closest approach distance is changed by the magnitude of the velocity increment $\alpha$. Therefore, the worst closest approach distance with respect to $\alpha$ can be obtained at the lower limit of $\alpha$. Furthermore, the distribution of the closest approach distance with respect to the Earth departure date and asteroid arrival date is almost identical to that of the deterministic case shown in Fig. 5.
Then, the effect of the $\Delta V$ direction on the closest approach distance is investigated. The closest approach distance with respect to the uncertainty parameters of $\Delta V$ direction ($d\theta$, $d\phi$) in the range of $-40 \leq d\theta \leq 40$ deg and $-40 \leq d\phi \leq 40$ deg is shown in Fig. 9 for ($t_{\text{dep}}, t_{\text{arr}}$) = (11/21/2008, 5/31/2010). The cross points (sign “+”) indicate the asteroid velocity direction giving the maximum value of the closest approach distance. Therefore, the optimal direction of $\Delta V$ is identical to that of the asteroid velocity at the spacecraft impact.

Figure 10 shows the change in the semi-major axis $\Delta a$ in terms of $d\theta$ and $d\phi$. $\Delta a$ is obtained by calculating the difference in the semi-major axis caused by the spacecraft. The $\Delta a$ distribution is very similar to the closest approach distance distribution in comparison with Fig. 9, suggesting that the closest approach distance is also strongly influenced by the change in the semi-major axis.

Park and Ross$^3$ also showed that the optimal direction of $\Delta V$ is identical to the direction of the asteroid velocity, and the closest approach distance is proportional to the change in the semi-major axis $\Delta a$.

Here, the asteroid and Earth orbit plane are different, but the component of $\Delta V$ normal to the asteroid orbital plane is insignificant to $\Delta a$. In addition, $\Delta a$ calculated from Eq. (8) is equal to the value shown in Fig. 10. Therefore, the optimal direction of $\Delta V$ is identical to the direction of asteroid velocity, and the distribution of the closest approach distance with respect to the $\Delta V$ direction is regarded as a convex function where the maximum value is taken at the asteroid velocity direction and the minimum value lies on the boundary of an uncertain range circle of radius $\gamma$.

The range of the uncertainty parameter for the worst $\Delta V$ case is shown in Fig. 11, where ($d\theta_{\text{worst}}$, $d\phi_{\text{worst}}$) indicates the worst value of the closest approach distance and the cross point (sign “+”) shows the asteroid velocity direction ($d\theta_{\text{ast}}$, $d\phi_{\text{ast}}$), and the bold circle indicates the boundary of

$$\Delta a = \frac{2\Delta V_{\text{ast}}[-e \sin \nu \cos \zeta + (1 + e \cos \nu) \sin \zeta]}{n\sqrt{1 - e^2}}$$  \hspace{1cm} (8)
Since the distribution of the closest approach distance in terms of the \( V \) direction is a convex function with the peak at the asteroid velocity direction, \((d_{\text{worst}}, d_{\text{worst}})\) lies on the intersection of the uncertainty circle and the line connecting the origin and the \((d_{\text{ast}}, d_{\text{ast}})\). Therefore, the worst value of the closest approach distance with uncertainty can be obtained simply without searching the whole convex hull.

### 4.3.2. Optimal trajectories with velocity increment uncertainty

Optimal interplanetary trajectories were designed considering the velocity increment uncertainties (worst \( V \)), and the effects of the uncertainties of the velocity increment on the closest approach distance were investigated. From the deterministic results in Section 4.2, the Earth departure date and the asteroid arrival date were selected as listed in Table 3 with Cases 1 and 2, where the large closest approach distance is obtained in Section 4.2.

Figures 12 and 13 show the distribution of closest approach distances in Case 1 with and without uncertainties. Likewise, Figs. 14 and 15 show the distribution of closest approach distances in Case 2 with and without uncertainties. The cross points (sign “+”) in the figures indicate the optimal Earth departure date and asteroid arrival date of the spacecraft. Figures 16 and 17 illustrate the optimal trajectories corresponding to the cross points with and without uncertainties. These figures indicate that the distribution of the closest approach distance with uncertainty is different from that without uncertainty. The effect on the optimal trajectories due to the uncertainty of \( V \) magnitude can be ignored as described in Section 4.3.1. That is, the optimal trajectories are shown to depend only on the uncertainty of \( V \) direction.

The effect of uncertainty of the \( V \) direction is discussed. The components of the asteroid velocity direction on the

<table>
<thead>
<tr>
<th>Case 1</th>
<th>( t_{\text{dep}} )</th>
<th>1/1/2008–3/30/2008</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( t_{\text{arr}} )</td>
<td>6/1/2010–8/29/2010</td>
</tr>
<tr>
<td>Case 2</td>
<td>( t_{\text{dep}} )</td>
<td>12/1/2010–2/28/2011</td>
</tr>
<tr>
<td></td>
<td>( t_{\text{arr}} )</td>
<td>11/1/2011–1/29/2012</td>
</tr>
</tbody>
</table>

Table 3. Design range of Earth departure and asteroid arrival date.
spacecraft impact with and without uncertainties are described as follows:

\[ \Delta V_{ast} = |\Delta V| \cos \psi \]
\[ \Delta V_{ast}' = |\Delta V| \cos(\psi + 3\sigma_y) \]

where, \( \Delta V_{ast} \) is the component of the asteroid velocity direction for the nominal \( \Delta V \) (deterministic) case, \( \Delta V_{ast}' \) is that for the worst \( \Delta V \) (uncertain) case, and \( \psi \) is an angle between the spacecraft velocity and asteroid velocity vectors. If the \( |\Delta V| \) takes a constant value, the ratio between \( \Delta V_{ast} \) and \( \Delta V_{ast}' \) indicates the uncertainty effect of \( \Delta V \) direction. Hence, the effects of uncertainty due to \( \Delta V \) and the closest approach distance can be described in \( \psi \).

To investigate the effect of the optimal trajectories due to the standard deviation of \( \Delta V \) direction uncertainty on optimal trajectories, the optimal departure date and arrival date are calculated in the range of \( \sigma_y = [0, 10] \) deg. The obtained optimal dates in Case 1 are shown in Fig. 18, which also shows the distribution of the angle between the spacecraft and asteroid velocities, \( \psi \). The optimal departure and arrival dates shift to the smaller angle of \( \psi \) as the standard deviation of the velocity increment direction becomes larger. Figure 19 shows the ratio of the closest approach distance with and without uncertainties. This distribution is similar to that of \( \psi \) in comparison to Fig. 18. Therefore, the effect of \( \Delta V \) uncertainty decreases as the angle between the spacecraft velocity and asteroid velocity vector \( \psi \) decreases. That is, the optimal trajectory under \( \Delta V \) uncertainty is selected because it gives a small angle between the asteroid and spacecraft velocities at collision.

5. Conclusions

This paper describes the optimal interplanetary trajectories for deflecting asteroids, taking the uncertainty of velocity increment of the asteroid due to spacecraft impact into account. The uncertainty of velocity increment is modeled using a convex model, where magnitude and direction are varied independently. The worst (minimum) value of the closest approach distance under the uncertainty is shown to be obtained without searching the whole convex hull (uncertain range), using the relationship of the change in semi-major axis and closest approach distance. Comparing optimal trajectories between those with and without the velocity increment uncertainties demonstrates the importance of including uncertainty in the asteroid-deflection mission design. The optimal trajectories of the spacecraft are sensitive to uncertainty of the velocity increment direction, but insensitive to uncertainty of the velocity increment mag-
The effect of velocity increment direction uncertainty depends on the angle between the velocity vector between the spacecraft and the asteroid at collision.

References