Trajectory Optimization of Multi-Asteroids Exploration with Low Thrust

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Multi-asteroid tour missions require consideration of the visiting sequences and trajectory optimization for each leg, which is a typical global optimization problem. In this paper, the problem is divided into a multi-level optimization problem. Determination of the visiting sequence plays a key part for a tour mission. In this paper, the energy differences between different orbits and phase differences are used to estimate the energy required for a tour mission. First, this paper discusses the relation between fuel cost for transfer and classical orbit elements difference based on the energy relation of Keplerian orbits and the characteristics of low-thrust spacecraft. Second, the phase difference is combined with the energy difference to achieve the rendezvous energy. In addition, the lower and upper bounds of rendezvous time can be estimated by analyzing the phase difference. Very-low-thrust trajectory optimization problems have always been considered difficult problems due to the large time scales. In this paper, a hybrid algorithm of PSO (particle swarm optimization) and DE (differential evolution) is used to achieve a solution to the energy-optimal tour mission. Based on GTOC-3 (Global Trajectory Optimization Competition), this paper determines the exploration sequence and provides the optimal solution.

Key Words: Low-Thrust, Trajectory Optimization, Hybrid Algorithm

1. Introduction

With the development of space technology and particularly deep-space exploration technology, asteroid explorations have come into focus. NASA has achieved the multi-purpose exploration of Saturn and Titan with the *Cassini* spacecraft launched in 1997. The *Stardust* spacecraft launched in 1999 successfully made a flyby with the *Wild2* comet and brought a comet sample back. In 2004, ESA launched the *Rosetta* spacecraft to discover the secrets of a cometary nucleus and which will take 10 years to rendezvous with the Comet 67 P/Churyumov-Gerasimenko. The *HAYABUSA* mission led by the Japan Aerospace Exploration Agency is designed to explore a small near-Earth asteroid named 25143 Itokawa and to return a sample of material to Earth for further analysis. The *DAWN* mission was successfully launched in 2007 to explore two asteroids with only one spacecraft, which would be a milestone in asteroid explorations.

Based on existing engine technology, the thrust required by the rocket and the fuel cost of the spacecraft would exceed human tolerances if a spacecraft were designed to explore a celestial body directly. The trajectory must include many orbit maneuvers and wide use of a multi-impulse transfer, low-thrust spacecraft has advantages of great efficiency and high precision. A great deal of recent work involves the trajectory optimization using low-thrust techniques. Many methods and theorems have been used to solve the problem. Based on Bellman's principle of optimality, Ross¹⁾ developed an anti-aliasing trajectory optimization method relating low-thrust trajectory optimization problems to the well-known problem of aliasing in informa-

tion theory. Cui²⁾ used the concept of the Lyapunov feedback control law to derive semi-analytical expressions for suboptimal thrust angles, and obtained a near-optimal solution that approaches the optimal solution by using sequential quadratic programming (SQP) to adjust the five weights in the Lyapunov function. Ulybyshev³⁾ used the inner-point algorithm to optimize the Earth elliptic orbit maneuver by processing the discrete legs. Betts⁴⁾ investigated orbit transfer optimization by utilizing the SQP method, reflecting the advantages and features of low thrust. Coverstone-Carroll⁵⁾ used a hybrid optimization algorithm that integrated a multi-objective genetic algorithm with a calculus-of-variations-based low-thrust trajectory optimizer to identify the optimal trajectory of both Earth-Mars and Earth-Mercury missions. Olds⁶⁾ indicated that DE was a promising global method suited to trajectory optimization. However the performance of the method is sensitive to the selection of the routine's tuning parameters. Kluever⁷⁾ applied the "thrust-coast-thrust" flight sequence to design the optimal trajectory for Earth-Moon transfer orbit, simulating the case from near-Earth circular orbit to the lunar polar orbit. Considering the multi-revolution low-thrust trajectory optimization with the "thrust-coast-thrust" flight sequence. Hull⁸⁾ converted optimal control problems into parameter optimization problems and accomplished the method by replacing the control and state histories by control and state parameters and forming the histories by interpolation.

If a spacecraft is designed to explore as many asteroids as possible, it is important to determine the exploration sequence and optimize the flight trajectory so the spacecraft can rendezvous with the asteroids in proper time, which has become the focus of research. Generally, the enumeration method applied by the STOUR⁹⁾ software can search

all the global optimal solutions. However, the computing time and space costs increase as dimensions and problem complexity increase, and cannot be handled by a personal computer.

Based on the Global Trajectory Optimization Competition (GTOC-3)¹⁰⁾ organized by the Dipartimento di Energetica of the Politecnico di Torino from October to December 2007, this paper provides an algorithm for selecting candidate asteroids based on the energy and phase relation and for determining the flight sequence considering constraints on launch window and flight time. Modified equinoctial elements are used in numerical integration using non-dimensional forms. Finally, a heuristic algorithm is used for global optimization, and a pattern search algorithm is used for local optimization.

2. Problem Description

GTOC-3 chose 140 near-Earth asteroids as candidates from the JPL space objects database. A spacecraft launched from Earth must rendezvous with three selected asteroids during its tour and return to Earth. The initial mass of the spacecraft is 2000 kg which can be used as propulsion fuel. The hyperbolic velocity excess of the spacecraft relative to the Earth is 0.5 km/s and the direction can be set at will. The maximum magnitude of the engine is 0.15 N with specified direction and a specific impulse of 3000 s. The launch window is from 2016 to 2025 and the whole flight time is less than 10 years. The stay-time at each of the three asteroids must be at least 60 days. The effect of Earth and the asteroids are neglected in the whole trajectory, and Earth flybys at altitudes exceeding 6871 km can be considered. The objective of the optimization is to maximize the nondimensional quantity:

$$J = \frac{m_f}{m_i} + K \frac{\min_{j=1,3}(\tau_j)}{\tau_{\max}} \quad (1)$$

where m_i and m_f are the initial and final mass of the spacecraft, respectively. τ_j ($j = 1, 2, 3$) represents the stay-time at the j -th asteroid in the rendezvous sequence and $\min_{j=1,3}(\tau_j)$ is the shortest asteroid stay-time. $K = 0.2$, $\tau_{\max} = 10$ years is the journey time.

First, we analyze the objective function. During the long journey, we must try to save the fuel and prolong the stay-time at the rendezvous bodies. Only one long stay-time has no effect on the object function because the shortest stay-time will be utilized. If the stay-time of the spacecraft is longer, the flight time will be shorter, resulting in more fuel cost. Tradeoffs between stay-time, flight time and other mission parameters are a key part of the optimization.

This paper divides the mission into three parts. First, we select three bodies from the 140 candidates based on the energy relation and synodic-period. To increase the integration precision, we divide the whole trajectory into seven parts that include four legs (connecting a celestial body with another celestial body) and three delay times. The optimization algorithm converts the optimal control problems into

parameter optimization problems including the launch date, flight time, stay-time, magnitude and direction of thrust.

3. Dynamic Model

The motion of a body is described by a system of second-order ordinary differential equations:

$$\ddot{\mathbf{r}} + \mu \frac{\mathbf{r}}{r^3} = \mathbf{a}_p \quad (2)$$

where the radius $r = \|\mathbf{r}\|$ represents the magnitude of the inertial position vector. μ is the gravitational constant of the central body. $\mathbf{a}_p$ is defined as perturbation acceleration. For the two-body problem dynamic model, we process the low thrust as a perturbation on the small side. In the inertial coordinate, the parameters change rapidly and the classical orbit elements exhibit singularities for $e = 0$, and $i = 0, 90^\circ$. The dynamic model in Cartesian coordinates reduces the effect of optimization. An appropriate dynamical model not only affects computing time, but also determines the integral accuracy. Kechichian¹¹⁾ analyzed the near-Earth orbit transfer of low-thrust spacecraft where a dynamic model was defined using a modified set of equinoctial coordinates, which could avoid singularities in the classical elements. However, the integration steps must be increased to maintain the integration speed and lower integration precision. We applied the non-dimensional modified equinoctial elements to avoid singularities during the numerical integration. The equations of motion are described as follows:¹²⁾

$$\begin{aligned} \dot{\mathbf{h}} &= \mathbf{h} \cdot \mathbf{n} \cdot \mathbf{f}_t \\ \dot{\mathbf{f}} &= \mathbf{h} \cdot \sin L \cdot \mathbf{f}_r + [\mathbf{h} \cdot \cos L + \mathbf{n} \cdot (\cos L + f)] \cdot \mathbf{f}_t \\ &\quad - \mathbf{n} \cdot X \cdot g \cdot \mathbf{f}_n \\ \dot{\mathbf{g}} &= -\mathbf{h} \cdot \cos L \cdot \mathbf{f}_r + [\mathbf{h} \cdot \sin L + \mathbf{n} \cdot (\sin L + g)] \cdot \mathbf{f}_t \\ &\quad + \mathbf{n} \cdot X \cdot f \cdot \mathbf{f}_n \\ \dot{\mathbf{h}} &= \frac{1}{2} \cdot \mathbf{n} \cdot s^2 \cdot \cos L \cdot \mathbf{f}_n \\ \dot{\mathbf{k}} &= \frac{1}{2} \cdot \mathbf{n} \cdot s^2 \cdot \sin L \cdot \mathbf{f}_n \\ \dot{\mathbf{L}} &= \mathbf{n} \cdot X \cdot \mathbf{f}_n + \frac{1}{n^2 \cdot \mathbf{h} \cdot \mu} \end{aligned} \quad (3)$$

where, $\mathbf{n} = \frac{\mathbf{h}}{1 + f \cos L + g \sin L}$, $X = h \sin L - k \cos L$, $s^2 = 1 + h^2 + k^2$, $h = \sqrt{\mu p}$. p is defined by $p = a(1 - e^2)$. f_r , f_t and f_n are defined as the thrust accelerations in a local radial-tangential-normal (RTN) rotating frame. Taking account of the representation of the asteroid orbit element provided by the competition organizer, we can transform the modified equinoctial elements into the classical orbital elements.

$$\begin{aligned} \mathbf{h} &= \sqrt{a(1 - e^2)/\mu}, & f &= e \cos(\omega + \Omega) \\ g &= e \sin(\omega + \Omega), & h &= \tan(i/2) \cos \Omega \\ k &= \tan(i/2) \sin \Omega, & L &= \omega + \Omega + \theta \end{aligned} \quad (4)$$

where six classical orbital elements are defined as follows:

Fig. 1. Relation of semi-axis and inclination.

Fig. 3. Orbit energy.

Fig. 2. Semi-major axis and RAAN.

Fig. 4. Velocity increment from Earth to asteroids.

a : Semi-major axis

e : Eccentricity

i : Inclination

Ω : Right ascension of ascending node

ω : Argument of perigee

θ : True anomaly

4. Energy Relation Analysis

We analyze the inclination and energy relation of the 140 asteroids in Fig. 1 to Fig. 4 (where the bigger circle represents Earth). The minimum semi-major axis of the asteroid is 0.900329 astronomical units (AU) and the maximum is 1.0984144 AU. In the group of asteroids, every asteroid has a number from 1 to 140 according to the incremental semi-axis of orbit from the minimum semi-axis to the maximum. The energies of asteroids is determined by the semi-major axis and is computed by:

$$E = -\frac{m\mu}{2a} \quad (5)$$

where, μ and a are the same as above, and m is the spacecraft mass.

Considering the orbit transfer, the change of angle of orbit

plane will lead to large fuel costs. We remove objects with inclinations bigger than 0.06 radian. For an object with large eccentricity, it is difficult to satisfy the rendezvous precision of position and velocity as additional fuel cost is required. We remove objects with an eccentricity of more than 0.17 too. The candidate 16 asteroids are shown in Table 1. The epoch is expressed as modified Julian date (MJD) in the J2000 heliocentric ecliptic frame. M is the mean anomaly of the asteroid orbit.

In preliminary design of the interplanetary trajectory, we consider low thrust as low impulse and calculate the velocity increment accomplished instantaneously. At the start of velocity increment estimation, we apply the Hohmann orbit transfer with two impulses, and the velocity increment is expressed as follows:

$$\begin{aligned} \Delta V &= \Delta V_1 + \Delta V_2 \\ \Delta V_1 &= \sqrt{\mu} \left(\sqrt{2/r_{p1} - 2/(r_{p1} + r_{a1})} \right. \\ &\quad \left. - \sqrt{2/r_{p1} - 2/(r_{p1} + r_{a2})} \right) \end{aligned}$$

Table 1. Candidate asteroids.

Asteroid	Epoch	a (AU)	e	i (deg)	ω (deg)	RAAN (deg)	M (deg)
5	57388	0.911399	0.110785	2.62269	7.63927	313.3385	320.8183
11	57388	0.916343	0.112213	2.83037	7.8651	72.72983	84.25695
16	57388	0.923846	0.133888	2.42676	199.6059	346.0455	20.5096
19	57388	0.929445	0.167548	0.89598	131.3047	14.74941	242.9305
30	57388	0.943754	0.164469	1.29373	332.363	233.5381	262.454
37	57388	0.950898	0.121726	0.57414	28.54873	175.1522	203.8559
49	57388	0.977400	0.066971	0.11024	274.9223	192.3114	192.1027
61	57388	0.988346	0.045056	2.7989	330.0373	161.6296	27.92816
64	57388	0.990779	0.12121	2.52245	224.9658	42.1994	243.6173
76	57388	1.011128	0.084271	1.42373	277.0281	45.04477	91.71255
85	57388	1.023656	0.098612	1.40819	95.17529	140.1505	112.67
88	57388	1.026839	0.049186	1.44562	24.50924	73.9738	25.71983
96	57388	1.037806	0.073948	1.27901	111.2593	196.8853	234.988
111	57388	1.062251	0.086315	3.10602	63.83549	184.4865	114.4433
114	57388	1.064616	0.156569	1.17974	90.08023	200.487	235.688
122	57388	1.077682	0.158464	3.03659	96.3364	148.6021	6.596425

Note: (a , e , i , ω , RAAN, M) are defined as classical orbit elements.

Table 2. Candidate exploration sequences.

No.	A1	A2	A3	DV	No.	A1	A2	A3	DV
1	88	76	49	3.6713	16	96	85	88	4.3883
2	49	76	88	3.6754	17	49	37	76	4.3983
3	76	88	49	3.9223	18	49	37	96	4.4003
4	49	88	76	3.9411	19	85	37	49	4.4614
5	96	85	49	4.0266	20	88	96	49	4.4757
6	49	85	96	4.0348	21	49	37	85	4.4923
7	85	96	49	4.2350	22	49	96	88	4.5085
8	88	37	49	4.2599	23	76	85	49	4.5147
9	49	96	85	4.2614	24	88	76	96	4.5225
10	49	37	88	4.2722	25	96	76	88	4.5287
11	88	85	49	4.2748	26	49	85	76	4.5372
12	49	85	88	4.2914	27	85	76	88	4.5487
13	96	37	49	4.3791	28	88	76	85	4.5522
14	88	85	96	4.3800	29	49	76	96	4.5930
15	76	37	49	4.3826	30	96	76	49	4.5951

Note: A1, A2, and A3 indicate the three asteroids and the number in the table represents the serial number of the asteroids in the competition document.

$$\begin{aligned}
\Delta V_2 &= \sqrt{V_i^2 + V_f^2 - 2V_i V_f \cos i_{\text{rel}}} \\
V_i &= \sqrt{\mu} \sqrt{2/r_{a2} - 2/(r_{p1} + r_{a2})} \\
V_f &= \sqrt{\mu} \sqrt{2/r_{a2} - 1/a_2} \\
\cos i_{\text{rel}} &= \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos \Omega_1 \cos \Omega_2 \\
&\quad + \sin i_1 \sin i_2 \sin \Omega_1 \sin \Omega_2
\end{aligned} \tag{6}$$

where, r_{p1} , r_{a1} are defined as the perigee and apogee of the initial orbit, respectively. r_{a2} is the apogee of the orbit of the destination object. i_1 and Ω_1 represent the inclination and RAAN (right ascension of ascending node) of the initial orbit. a_2 is the semi-major axis of the orbit of the destination object, i_2 and Ω_2 represent the inclination and RAAN of the

orbit of the destination object. Based on the Hohmann orbit transfer, we chose 30 candidate exploration sequences with minimum velocity increment (Table 2) where A1 indicates the number of the first asteroid in the exploration sequence (excluding Earth) and A2 the second, and A3 the third. In interplanetary missions, prolonging the flight time or using the gravity-assist technique, which can increase or decrease the orbit energy of the spacecraft, reduces the fuel cost. The results in Table 2 where DV indicates the sum of DV1 (from Earth to A1), DV2 (from A1 to A2), DV3 (from A2 to A3), and DV4 (from A3 to Earth) can only be regarded as a reference. Here DV1, DV2, DV3, and DV4 represent the velocity increment of each leg and can be computed according to the Eq. (6).

5. Flight Time and Synodic-Period

For a spacecraft to rendezvous with a celestial body, it must arrive at the position with the same velocity, so launch date has some constraints. If a spacecraft is launched randomly, it consumes more fuel to chase the destination object. We should take account of the synodic-period and the phase difference between the departure and arrival bodies in the preliminary trajectory design. For a low-thrust mission, the range of the orbit transfer is finite. If the spacecraft transfers from one celestial body to another celestial body, the phase difference must be as small as possible to rendezvous in a short time. Otherwise, the spacecraft will spend fuel in implementing orbit transfer, and in chasing the rendezvousing body, which will influence the mission quality and increase the cost.

If a spacecraft with mass m_i is launched from the i -th asteroid at t_0 , and will arrive at the j -th asteroid where the spacecraft mass is m_j , the flight time is $\Delta t = t_1 - t_0$. Using Eq. (5), we can obtain the energy change of the spacecraft for the orbit transfer as:

$$\Delta E_{ij} = \frac{m_i \mu}{2a_i} - \frac{m_j \mu}{2a_j} \quad (7)$$

The work done by the low thrust can be calculated by:

$$W_{i \rightarrow j} = \int_{t_i}^{t_j} \vec{T} \cdot \vec{v} dt \quad (8)$$

Then, the least energy required for the transfer can be written as:

$$\frac{m_j \mu}{2a_j} - \frac{m_i \mu}{2a_i} \approx T \bar{v} \Delta t \quad (9)$$

During the whole process, the mean velocity is approximated as:

$$\bar{v} \approx \left(\sqrt{\mu/a_i} + \sqrt{\mu/a_j} \right) / 2 \quad (10)$$

For preliminary analysis and estimation of the performance, the magnitude of the thrust is assumed to be invariable and the direction accords with the local velocity. Therefore, the mass consumed can be obtained as:

$$\dot{m} = -\frac{T}{I_{sp} g_0} \quad (11)$$

$$m_j = m_i + \dot{m} \Delta t = m_i - \frac{T}{I_{sp} g_0} \Delta t \quad (12)$$

$$\Delta t = \frac{m_i \mu}{T \left(\sqrt{\frac{\mu}{a_i}} + \sqrt{\frac{\mu}{a_j}} - \frac{\mu}{a_j I_{sp} g_0} \right)} \left(\frac{1}{a_i} - \frac{1}{a_j} \right) \quad (13)$$

When the spacecraft transfers from one asteroid to another, to avoid large energy consumption, the phase difference should be small. Assuming that the eccentricity and the inclination of the orbits are small, the phase of spacecraft in ecliptic plane with respect to equinox on a Keplerian orbit can be given approximately as:

$$\psi \approx \Omega + \omega + M \quad (14)$$

where Ω and ω are defined above. ψ indicates the phase of spacecraft and M is the mean anomaly given by:

$$M = M_0 + nt = M_0 + \sqrt{\frac{\mu}{a^3}} t \quad (15)$$

where M_0 is the initial mean anomaly. The angular velocity of the spacecraft can also be approximated as:

$$n = \frac{1}{2} \left(\sqrt{\frac{\mu}{a_i^3}} + \sqrt{\frac{\mu}{a_j^3}} \right) \quad (16)$$

Then, we obtain:

$$\begin{aligned} \Omega_i + \omega_i + M_i + \frac{1}{2} \left(\sqrt{\frac{\mu}{a_i^3}} + \sqrt{\frac{\mu}{a_j^3}} \right) \Delta t \\ = \Omega_j + \omega_j + M_j + \sqrt{\frac{\mu}{a_j^3}} \Delta t + 2k\pi \quad k = 0, 1, \dots \end{aligned} \quad (17)$$

$$\Delta t = \frac{2(2k\pi + \Omega_j + \omega_j + M_j - \Omega_i - \omega_i - M_i)}{\left(\sqrt{\frac{\mu}{a_i^3}} - \sqrt{\frac{\mu}{a_j^3}} \right)} \quad (18)$$

where the subscript j denotes the variables related to the arrival asteroid and the subscript i denotes the variables related to the leaving asteroid. T indicates the magnitude of thrust. g_0 is standard acceleration due to gravity at Earth's surface and I_{sp} represents the specific impulse of the engine. $\bar{v}$ is defined as the mean velocity of the transfer orbit.

The spacecraft finally rendezvous with Earth. The phase difference of the asteroid and Earth shall be zero during the 10 years from launch. So a sequence with large phase difference and large synodic-period is excluded from consideration because it is difficult for the spacecraft to return to Earth. Consequently, the candidate asteroids must take account of the synodic-period and initial phase different within 10 years. The sequences are shown in Table 3.

The data in the lower-left area of Table 3 indicate the phase difference at the epoch of MJD57388 (2015-12-31) that shows the degree of the asteroids in the row in the front of asteroids in the column. The data in the upper-right area of Table 3 indicate the difference of the angular velocity with unit of degree per Julian year. The constraint of the launch window is 10 years and then the first asteroid with large synodic-period with Earth will be excluded. Here, the first synodic-period is less than 15 years. Considering the constraint of flight time, we must estimate the synodic-period between the first asteroid and the last asteroid. In addition, the synodic-period of the three asteroids with Earth should increase along the order of rendezvous. Otherwise the spacecraft will reduce the orbit energy, requiring more fuel to rendezvous with the next asteroid that has rendezvoused with Earth before this rendezvous. The possible sequences are shown in Table 4 where the numbers of the sequences are the same as in Table 2. Here D1, D2, D3, and D4 indicate the pursuing time computed through the phase difference and synodic-period. In a sequence, D1 is the time of Earth pursuing the first asteroid and D2

Table 3. Initial phase difference and angular velocity difference.
(unit of lower-left: degree, unit of upper-right: degree per Julian year)

No.	37	49	70	72	76	79	85	88	94	96	Earth
37	0	15.683	31.0	31.77	34.17	36.72	40.65	42.26	45.81	47.73	17.41
49	251.78	0	15.31	16.08	18.48	21.04	24.96	26.58	30.13	32.05	12.55
70	354.84	103.06	0	0.77	3.17	5.73	9.65	11.27	14.82	16.74	-2.76
72	297.37	45.59	302.52	0	2.40	4.96	8.88	10.50	14.05	15.96	-3.53
76	6.23	114.45	11.39	68.86	0	2.56	6.48	8.09	11.65	13.56	-5.93
79	338.18	86.40	343.34	40.81	331.95	0	3.92	5.54	9.09	11.01	-8.49
85	300.44	48.66	305.6	3.07	294.21	322.26	0	1.61	5.17	7.08	-12.4
88	76.65	184.87	81.80	139.28	70.42	98.47	136.21	0	3.55	5.47	-14.03
94	102.75	210.97	107.91	165.38	96.52	124.57	162.31	26.10	0	1.92	-17.58
96	135.58	243.8	140.73	198.21	129.35	157.4	195.14	58.93	32.83	0	-19.50
Earth	109.43	160.59	57.53	115.01	46.14	74.19	111.93	335.72	309.62	276.8	0

Table 4. Exploration sequences according to synodic-period with Earth.

No.	D1	D2	D3	D4	No.	D1	D2	D3	D4
1	1.73	8.7	6.19	12.8	16	4.27	27.56	84.6	1.73
2	12.8	6.19	8.7	1.73	17	12.8	16.06	10.72	52.93
3	52.93	8.7	6.96	12.8	18	12.8	16.06	10.38	4.27
4	12.8	6.96	8.7	52.93	19	19.99	7.39	16.06	12.8
5	4.27	27.56	16.37	12.8	20	1.73	10.77	7.61	12.8
6	12.8	1.95	27.56	4.27	21	12.8	16.06	16.25	19.99
7	19.99	27.56	7.61	12.8	22	12.8	7.61	10.77	1.73
8	1.73	10.33	16.06	12.8	23	52.93	45.4	1.95	12.8
9	12.8	7.61	27.56	7.61	24	1.73	8.7	9.54	4.27
10	12.8	16.06	10.33	1.73	25	4.27	9.54	8.7	1.73
11	1.73	84.6	1.95	12.8	26	12.8	1.95	45.4	52.93
12	12.8	1.95	84.6	1.73	27	19.99	45.4	8.7	1.73
13	4.27	27.56	84.6	1.73	28	1.73	8.7	45.4	19.99
14	1.73	84.6	27.56	4.27	29	12.8	6.19	9.54	4.27
15	52.93	10.72	16.06	12.8	30	4.27	9.54	6.19	12.8

is the time of the first asteroid pursuing the second asteroid. D3 and D4 are the analogies. The stay-time of the spacecraft must be longer than 60 days at each rendezvous. The spacecraft cannot spend too long time during a single leg because the total flight time is less than 10 years which is also a maximum constraint on the difference of D1 and D4. Adding the launch window of 10 years, the maximum pursuing time in the sequence must be less than 20 years. Because there is no sequence satisfying all the constraints, we selected candidate sequences 1, 21, 24, and 30 sequences from Table 4. Sequence 21 has the best time sequence that increases gradually, but the first time is beyond 10 years. Sequences 1, 24, and 30 have no incremental synodic-period with Earth, but the first time is less than 10 years.

6. Numerical Simulation

Here, we select the asteroid whose synodic-period with Earth is within 10 years as the first object to be explored and the asteroid whose synodic-period with Earth is from 10 to 20 years, as the last object. Based on the above energy relation in Table 2 and the constraints of launch window and

flight time, we apply exploration sequence 1 listed in Table 2 and 4 in this paper shown below:

Earth-A88(1991 VG)-A76(2006 JY26)-
A49(2000 SG344)-Earth

Trajectory optimization belongs to a global optimization, consisting of finding the global optimum of a given performance index in a large domain, typically characterized by the presence of a large number of local optima. In this paper, we use a hybrid algorithm of PSO (particle swarm optimization) and DE (differential evolution) that were both developed in the 1990s and are now applied widely in various fields. PSO¹³⁾ was developed for optimizing continuous non-linear functions by Kennedy and Eberhart based on the action of foraging birds. It has been studied widely in the recent years and many modified PSO algorithms have been used. In general, these modified algorithms are all about how to adjust the parameters and enhance the diversity so as to search the whole solution domain. DE¹⁴⁾ was introduced by Storn and Price and resembles the structure of an EA (Evolutionary Algorithm), but differs from traditional EAs in generation of new candidate solutions and use of a

Table 5. Orbit elements in heliocentric inertial coordinate of MJD57388.

Object	a (AU)	e	i (deg)	ω (deg)	RAAN (deg)	M (deg)
(A88)	1.0268385	0.04918621	1.44562	24.50924	73.97380	25.719831728
(A76)	1.0111280	0.08427146	1.42373	277.02812	45.04477	91.712548835
(A49)	0.9774002	0.06697124	0.11024	274.92230	192.31139	192.10270824
Earth	0.9999880	0.01671681	0.0008854	287.6157	175.406	356.9054

Note: (a , e , i , ω , RAAN, M) are defined as classical orbit elements.

Table 6. Parameters of hybrid algorithm.

	F	CR	$c1$	$c2$	NP	$Iteration$
PSO + DE	0.8	0.618	0.5	0.5	1300	3000

“greedy” selection scheme. It has great potential in many numerical benchmark problems and real world applications. In many test suites, the DE algorithm outperforms other methods and has the advantage of fast convergence rate and low computational consumption of function evaluations. Because the above global optimization algorithms are sensitive to selection of tuning parameters, this paper combines these two algorithms to achieve stable performance. In the former 30 iterations of every 50 iterations, PSO is first applied to extend the search ability. In the latter 20 iterations of every 50 iterations, DE is applied to converge rapidly. In the process of parameter optimization, the approximate departure date and arrival date are fixed based on the above phase analysis. But two parameters date errors are set to be optimized to obtain the accurate departure date and arrival date. In the paper, a continuous low-thrust is assumed as the fixed impulse in a small interval and every leg is divided into 13 intervals considering the tradeoff of precision and computation time. Here, the magnitude and direction of the low thrust are to be optimized in the constraint range. We transfer the control optimization problem into a parameters optimization problem and continuously integrate the dynamics equation along the trajectory to avoid matching adjoint variables. Nevertheless, the final rendezvous precision cannot be controlled. Finally, the pattern search algorithm is used to help the above preliminary solution satisfy the precision requirements. The algorithm is a local optimization algorithm for unconstrained optimization using positive spanning directions. It directs the search for a minimum through a pattern containing at least $n + 1$ points per iteration, where the vectors representing the direction and distance of each point relative to the current iterate form a positive basis in R^n . We finally apply the pattern search tool in the Matlab[®] software to get the optimal solution.

The performance of the algorithm is sensitive to the selection of the routine’s tuning parameters. This paper provides the optimal solution and also provides the tuning parameters in Table 6 where NP is the population size, CR is the cross-over probability, F is the mutation scaling factor and $c1$, $c2$ are two weighting factors. The optimal parameters during every leg are shown in Tables 7 to 10.

Table 7. From Earth to A88.

Launch date (MJD and calendar):	58090.8510 (2017-12-2)
Hyperbolic velocity excess (km/s):	[−0.3378, 0.05498, 0.3645]
Arrival date (MJD and calendar):	58479.1488 (2018-12-26)
Initial mass (kg):	2000.0000
Arrival mass (kg):	1960.6172

Table 8. From A88 to A76.

Launch date (MJD and calendar):	58704.1343 (2019-8-8)
Stay-time at A88 (JD):	224.9855
Arrival date (MJD and calendar):	59371.8310 (2021-6-5)
Departure mass (kg):	1960.6172
Arrival mass (kg):	1807.5461

Table 9. From A76 to A49.

Launch date (MJD and calendar):	59806.8411 (2022-8-14)
Stay-time at A76 (JD):	435.0101
Arrival date (MJD and calendar):	60470.0672 (2024-6-8)
Departure mass (kg):	1807.5461
Arrival mass (kg):	1624.7850

Table 10. From A49 to Earth.

Launch date (MJD and calendar):	61059.0684 (2026-1-18)
Stay-time at A49 (JD):	589.0012
Arrival date (MJD and calendar):	61641.9721 (2027-8-23)
Departure mass (kg):	1624.7850
Arrival mass (kg):	1564.6000

Based on the optimal solution, we show the interplanetary trajectory in Figs. 5 to 8.

Finally, the object function can be computed as:

$$J = \frac{m_f}{m_i} + K \frac{\min_{j=1,3}(\Gamma_j)}{\Gamma_{\max}} = \frac{1564.60}{2000} + 0.2 \frac{224.9855}{3652.5} = 0.7946$$

7. Conclusion

Based on the GTOC-3, this paper analyzes the energy relation of orbit transfer and determines the exploration sequence according to the phase difference and the synodic-period with Earth. The method can be used in the preliminary design of the interplanetary trajectory. Trajectory design is a global optimization problem that

Fig. 5. Trajectory from Earth to A88.

Fig. 7. Trajectory from A76 to A49.

Fig. 6. Trajectory from A88 to A76.

Fig. 8. Trajectory from A49 to Earth.

generally can obtain the optimal solution by meta-heuristic search algorithms, such as GA (genetic algorithm), PSO (particle swarm optimization), DE (differential evolution), SA (simulated annealing), EA (evolutional algorithm), etc. This paper uses a hybrid algorithm of PSO and DE to obtain the optimal solution with low thrust. Last, the local optimization algorithm (pattern search) is applied to obtain a more accurate solution to satisfy the required precision of 1000 km in position and 1 m/s in velocity. A combination of the global optimization algorithm and classical local optimization provides ideal method for trajectory design. If Earth flybys are considered during the flight, the solution space is extended exponentially and a more efficient algorithm is required in future.

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