MHD Analysis of Magnetic Diffusion Effect on Magneto Plasma Sail

By Tomoya FUJIMOTO,¹⁾ Hirotaka OTSU,²⁾ Ikkoh FUNAKI^{3),4)} and Yoshiki YAMAGIWA⁵⁾

¹⁾Graduate School of Engineering, Shizuoka University, Hamamatsu, Japan

²⁾Faculty of Science and Technology, Ryukoku University, Otsu, Japan

³⁾Japan Aerospace Exploration Agency, Sagami, Japan

⁴⁾Japan Science and Technology Agency, Kawaguchi, Japan

⁵⁾Faculty of Engineering, Shizuoka University, Hamamatsu, Japan

(Received October 24th, 2008)

To propel a spacecraft away from the Sun, a magneto plasma sail (MPS) spacecraft produces an artificial magnetic cavity to block the hypersonic solar wind. To make a large magnetic cavity sufficient to obtain significant thrust, the MPS spacecraft increases the magnetic cavity size using an onboard coil with assistance from a plasma jet. This process is called magnetic field inflation. In this study, we performed ideal and resistive magnetohydrodynamic (MHD) analyses to investigate the magnetic diffusion effect on the magnetic field inflation process. Our results indicate that a dipole-like magnetic field is drastically deformed by a plasma jet; when the magnetic Reynolds number R_m was 10 or more, the magnetic field lines were nearly identical to the streamlines of the plasma jet. Hence, no magnetic diffusion effect appeared for $R_m > 10$. Meanwhile, when R_m is an order of unity, the magnetic diffusion effect was remarkable in the current sheet formed around equatorial region. For example, when the divergence angle of a plasma jet in the polar direction was 30° , the magnetic field strength at 40 m from the spacecraft (calculated by resistive MHD model) was 19% smaller than the ideal MHD model ($R_m = \infty$).

Key Words: Magneto Plasma Sail, Solar Wind, Propulsion, Magnetohydrodynamics

Nomenclature

$\mathbf{B}$: total magnetic flux density vector, T
 B : total magnetic flux density, T
 $\mathbf{B}_0$: initial magnetic flux density vector, T
 $\mathbf{B}_d$: induced magnetic flux density vector, T
 c_a : Alfvén velocity, m/s
 e : elementary electric charge, C
 E_t : total energy density, J/m³
 $\mathbf{I}$: unit vector
 $\mathbf{j}$: current density vector, A/m²
 L : length, m
 M : Mach number
 m_i : mass of ion, kg
 M_a : Alfvén Mach number
 p : pressure, Pa
 r : distance from the dipole center, m
 r_{Li} : ion Larmor radius, m
 R : specific gas constant, J/(kg·K)
 R_m : magnetic Reynolds number
 t : time, s
 T : temperature, K
 $\mathbf{u}$: velocity vector, m/s
 u : velocity, m/s
 X : coordinate
 Y : coordinate
 β : kinetic beta value
 γ : specific heat ratio
 μ_0 : magnetic permeability of vacuum, H/m

ρ : density, kg/m³

θ : colatitude, degree

σ : electrical conductivity, S/m

Subscript

c: characteristic state

ideal: result by ideal MHD analysis

resistive: result by resistive MHD analysis

1. Introduction

1.1. Background

Various deep-space missions to planets in the outer solar system are now being planned. Such missions require innovative propulsion systems with high specific impulse and high thrust density to drastically cut mission times. In recent years propulsion systems using solar wind have been the focus of interest.

Zubrin's Magnetic Sail¹⁾ is one such sail propulsion system. We call it a 'pure Magnetic Sail' that produces thrust by intercepting the dynamic pressure of the solar wind using a magnetic field around an onboard superconducting coil. Figure 1(a) is a schematic of the flow field around the pure Magnetic Sail. The interaction between the solar wind and magnetic field forms a magnetosphere around the spacecraft acting as a sail. Thrust is produced when the solar wind plasma flow is interrupted by this magnetosphere. Since the pure Magnetic Sail uses the continuous and ultra-fast solar wind plasma flow, it should enable deep-space missions to be accomplished quickly. However, we must create a large magnetosphere to produce significant

Fig. 1. Principle of thrust production process in Magneto Plasma Sail.

thrust because the dynamic pressure of the solar wind is very small even around the Earth. For the pure Magnetic Sail, a large magnetosphere might be formed using a superconducting coil of several tens of kilometer in diameter; but this is impractical at the current technology level. In order to overcome this issue, the idea to inflate the magnetosphere by a plasma jet was proposed instead of employing a huge coil.²⁾ This propulsion system is called mini-magnetospheric plasma propulsion (M2P2) or Magneto Plasma Sail (MPS).

When plasma is injected from a spacecraft, the magnetic field around the spacecraft is frozen to the plasma flow, so the magnetic field is inflated far around the spacecraft (Fig. 1(b)). As a result, a large magnetosphere can be created around the spacecraft by a very small coil. The propulsive performance of the MPS system was investigated based on magnetohydrodynamics (MHD) simulations by Winglee et al.²⁾ However, the MHD approximation is invalid under Winglee's calculation conditions, and it is thought that the propulsive performance has been overestimated. Currently, the MPS system is being re-examined by some researchers.

1.2. Current MPS research

The MPS generates thrust based on two basic phenomena: (1) mutual interference between the solar wind and the magnetic field; and (2) the frozen-in field where the magnetic field lines are transported along the plasma flow. To understand the basic propulsive mechanisms of MPS, these phenomena have been studied previously by numerical approaches based on the ideal MHD model.³⁻⁵⁾

However, the MHD model is valid only when the characteristic length of a plasma flow is much greater than typical ion Larmor radius ($r_{Li}/L_c \ll 1$). For a solar wind to magnetosphere interaction, the typical ion Larmor radius at the edge of the magnetosphere is known to be ~ 70 km at 1 AU. The MHD simulation in this study uses an MPS spacecraft with a larger magnetosphere, i.e. $L_c \gg r_{Li}$. In contrast, an MPS with $r_{Li} \sim L_c$ requires particle simulation including the ion kinetic effect.⁶⁾

In previous studies,³⁻⁶⁾ the electrical conductivity is assumed as infinity to simulate the ideal frozen-in field. However, in reality, injected plasma has finite electrical

conductivity. When the plasma has finite electrical resistivity, the magnetic field may slip across the plasma using the effect called magnetic diffusion. The magnetic diffusion effect in the MPS flow field has not been studied before.

1.3. Objectives

In this study, we performed ideal MHD and resistive MHD analyses of magnetic field inflation for an MPS spacecraft to investigate the magnetic diffusion effect due to finite electrical conductivity in magnetic field inflation.

2. Numerical Model

2.1. Governing equations

This study focuses on macroscopic movement of injected plasma, so resistive MHD equations without viscosity is used as governing equations.⁷⁾ The set of equations is composed of the Euler equation incorporating the MHD effect and the induction equation. When the electrical conductivity can be assumed to be infinity (the third term on the left hand side of Eqs. (3) and (4) can be neglected), this equation system corresponds to the ideal MHD equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot \left(\rho \mathbf{u} \mathbf{u} + \left(p + \frac{\mathbf{B} \cdot \mathbf{B}}{2\mu_0} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{\mu_0} \right) \\ = -\nabla \cdot \mathbf{B} \left(\frac{\mathbf{B}}{\mu_0} \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\sigma \mu_0} \nabla \times (\nabla \times \mathbf{B}) \\ = -\nabla \cdot \mathbf{B} \mathbf{u} \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial E_t}{\partial t} + \nabla \cdot \left(\left(E_t + p + \frac{\mathbf{B} \cdot \mathbf{B}}{2\mu_0} \right) \mathbf{u} - \frac{(\mathbf{u} \cdot \mathbf{B}) \mathbf{B}}{\mu_0} \right) \\ + \frac{1}{\sigma \mu_0} \nabla \cdot (\mathbf{j} \times \mathbf{B}) = -\nabla \cdot \mathbf{B} (\mathbf{u} \cdot \mathbf{B}) \end{aligned} \quad (4)$$

$$E_t = \frac{p}{\gamma - 1} + \rho \frac{\mathbf{u} \cdot \mathbf{u}}{2} + \frac{\mathbf{B} \cdot \mathbf{B}}{2} \quad (5)$$

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B} \quad (6)$$

Although physically true that $\nabla \cdot \mathbf{B} = 0$, it is not conserved numerically. The numerical error of $\nabla \cdot \mathbf{B}$ exerts a nonphysical force on the plasma flow. To reduce the error of $\nabla \cdot \mathbf{B}$, we utilize the 8-wave formulation⁸⁾ where the terms proportional to $\nabla \cdot \mathbf{B}$ are left on the right hand sides of Eqs. (2)–(4). At actual simulation, to accurately calculate the strong magnetic field around the spacecraft, the total magnetic field ($\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_d$) is separated into the initial (intrinsic) magnetic field $\mathbf{B}_0$ and the induced (deviation) magnetic field $\mathbf{B}_d$, and only the induced magnetic field was time-integrated.

2.2. Magnetic Reynolds number

The Magnetic Reynolds number R_m is a dimensionless parameter indicating how well a magnetic field is frozen into a plasma flow. It is the ratio of the convection term to the diffusion term in the induction equation (Eq. (3)) and is defined as follows:

$$R_m = \frac{|\nabla \times (\mathbf{u} \times \mathbf{B})|}{|\nabla \times (\nabla \times \mathbf{B})/\sigma\mu_0|} \approx \sigma\mu_0 u_c L_c \quad (7)$$

If R_m is much greater than unity, because the injected plasma cannot diffuse across the magnetic field lines, the magnetic field lines are transported along with the injected plasma flow. In contrast, if R_m is less than unity, the magnetic diffusion effect is dominant. When discussing the interaction between solar wind and inflated magnetic field in space, R_m is assumed to be infinity, because the characteristic length of the solar wind is extremely large and R_m is in the order of 10^8 .⁹⁾ On the other hand, for magnetic field inflation around the spacecraft, R_m should be in the order of 10 – 10^2 ,⁴⁾ suggesting that the ideal frozen-in field theory is invalid and we need to consider magnetic diffusion in magnetic field inflation.

2.3. Numerical methods

This study adopted the MUSCL-type high-ordered TVD Lax-Friedrich scheme¹⁰⁾ with the MINMOD limiter.¹¹⁾ In this scheme, a stable solution is obtained because adequate amounts of numeric viscosity are automatically introduced. The Euler explicit method was used for time integration.

2.4. Simulation model and computational conditions

Figure 2(a) shows the computational domain and boundary conditions. An axis-symmetric flow field was assumed. A spherical spacecraft (1 m in radius) was placed at the origin and a dipole magnetic field was assumed as the initial magnetic field. Plasma was injected from the polar region of the spacecraft. The angle inclined to the polar axis is defined as colatitude θ ($0^\circ < \theta < 90^\circ$). The plasma was injected symmetrically toward the north and the south polar directions from the surface of the spacecraft within the range of $\theta < 30^\circ$. The computational domain was arranged for $r < 50$ m, in which r is the distance from the dipole center (spacecraft center).

Next, the β value for plasma flow was introduced. The β value is the ratio of the dynamic pressure of the injected plasma to the magnetic pressure:

Table 1. Parameters of plasma jet and magnetic field at plasma injection point.

Ion Species	Hydrogen
Specific heat ratio: γ	5/3
Velocity: u [km/s]	35.9 (Mach Number = 1.0)
Temperature [eV]	4.0
Magnetic Field	Dipole Magnetic Field (Dipole moment = 0.02 Tm^3 : 0.04 T at pole, 0.02 T at equator)
β value at Pole: β_0	2.5×10^{-4}
r_{Li}/L_c	9.37×10^{-3}
Mag. Reynolds number, R_m	1, 3, 5, 10, 30, 50, 70, 100, ∞

$$\begin{aligned} \beta &= \frac{\rho |\mathbf{u}|^2 / 2}{|\mathbf{B}|^2 / 2\mu_0} \\ &= \left(\frac{|\mathbf{u}|}{|\mathbf{B}| / \sqrt{\mu_0 \rho}} \right)^2 \\ &= \left(\frac{|\mathbf{u}|}{c_a} \right)^2 \\ &= M_a^2 \end{aligned} \quad (8)$$

In this study, the β values at the pole surfaces of spacecraft ($r = 1$ m, and $\theta = 0^\circ$) are defined as β_0 . Equation (8) shows that the β value is proportional to the injected plasma density. Because it is unrealistic to inject high- β plasma for an MPS system from the viewpoint of propellant consumption, magnetic field inflation by a plasma jet with low Alfvén Mach number or β smaller than unity at the inner boundary is investigated. In this study, we assumed β_0 is 2.5×10^{-4} .

Table 1 shows the injected plasma parameters and the magnetic field strength at the surface of the spacecraft sphere. Inputting the values shown in Table 1 to Eq. (8), the plasma jet density is $2.48 \times 10^{-10} \text{ kg/m}^3$.

Finally, the validity of MHD approximation is discussed. When the ion Larmor radius is comparable to the characteristic length ($r_{Li}/L_c \geq 1$), the MHD model cannot describe a plasma flow. The ion Larmor radius is expressed as

$$r_{Li} = \frac{m_i u}{eB}. \quad (9)$$

Using the values in Table 1, r_{Li}/L at the injection point is calculated as 9.37×10^{-3} ($\ll 1$). Here, the distance from the magnetic dipole center is selected as the characteristic length ($L_c = 1$ m). Therefore, the ion kinetic effect is not significant in the present simulation.

2.5. Boundary condition^{5,12)}

When the boundary conditions at the inner and the outer boundaries are defined, attention on the theory of characteristics is required. In this study, because the Alfvén Mach number at the inner boundary is smaller than unity, two characteristic velocities in radial direction become negative, and information from the computational domain propagates to the inner boundary (upstream direction). Under this condition, all physical values at the inner boundary cannot be specified and at least two physical values

Fig. 2. Computational grid and simulation model.

are implemented based on the information from the computational domain.

The magnified view around the inner boundary is shown in Fig. 2(b). At the injection inner boundary, i.e. at the surface of the spacecraft within the range of $0^\circ < \theta < 30^\circ$, the density and pressure are fixed to the values of the injected plasma. In addition, we selected the 0th-order extrapolation to obtain the tangential component of the induced magnetic field. The velocity is then calculated so the velocity and total magnetic field are parallel at the surface of the spacecraft.

At the non-injection inner boundary, i.e. the surface of the spacecraft within the range of $30^\circ < \theta < 90^\circ$, we selected the 0th-order extrapolation to obtain the density and pressure. For velocity, the reflecting condition was implemented. For the induced magnetic field, the normal component was fixed to zero and the tangential component was given by the 0th-order extrapolation.

At the outer boundary, the information from the outer boundary only propagates downstream because the flow is super-Alfvénic. Hence, we selected the 0th-order extrapolation for all physical values as the boundary condition at the outer boundary.

The polar axis is assumed to be an axis-symmetric boundary, and an axis-symmetric boundary condition was implemented.

3. Numerical Results and Discussion

3.1. The ideal frozen-in field

In this section, the magnetic field inflation process by the plasma jet with β_0 value is 2.5×10^{-4} and R_m is infinity (corresponding to the ideal MHD model) is discussed. Figures 3 and 4 show the distributions of the magnetic field strength and the magnetic field lines. Figure 3 shows the original dipole magnetic field as the initial magnetic field without a plasma jet, and Fig. 4 shows the one inflated by the plasma jet. These figures confirm that a strong magnetic field near the spacecraft is widely inflated in all directions by a plasma jet except in the equatorial region (along $X = 0$ m line). Figures 4 and 5 show that the magnetic field lines are approximately identical to the streamlines of the plasma jet.

The degree of the magnetic field inflation is evaluated from the above results. For example, along the polar axis ($Y = 0$ m line), the contour ($B = 1.26 \times 10^{-5}$ T) at $X = 15$ m in the initial state is inflated to $X = 21.5$ m by plasma injection. Additionally, the variation from the initial state is more remarkable far from the spacecraft. However, in the equatorial region ($X = 0$ m and $Y > 10$ m in Fig. 4), the magnetic field inflation is not significant and a low magnetic field region appears.

Figure 6 shows the distribution of $|\mathbf{B}_d|/|\mathbf{B}_0|$ (the ratio of induced magnetic field $\mathbf{B}_d$ to initial magnetic field $\mathbf{B}_0$). In this figure, the contour where $\beta = 1$ is also plotted as dashed line. Because a sub-Alfvénic plasma is released from the spacecraft, a region of $\beta < 1$ surrounds the spacecraft. Meanwhile, because the plasma flow is accelerated in the downstream region where the magnetic field is decaying, the plasma jet becomes a super-Alfvénic flow and a $\beta > 1$ region appears. In the $\beta < 1$ region, the induced magnetic field $\mathbf{B}_d$ is much smaller than the initial magnetic field $\mathbf{B}_0$ ($|\mathbf{B}_d|/|\mathbf{B}_0| \ll 1$). However, around the $\beta = 1$ line, $\mathbf{B}_d$ becomes comparable to $\mathbf{B}_0$ ($|\mathbf{B}_d|/|\mathbf{B}_0| \approx 1$), and in the $\beta > 1$ region, $\mathbf{B}_d$ becomes much greater than $\mathbf{B}_0$ ($|\mathbf{B}_d|/|\mathbf{B}_0| \gg 1$). This means that the induced magnetic field inflates the initial magnetic field in the region of $\beta > 1$ and magnetic field inflation becomes remarkable there.

As shown in Fig. 7, a strong induced current appears in the equatorial region, and this current also plays an important role in the magnetic field inflation. The induced current flows in the direction of $\mathbf{u} \times \mathbf{B}$ and the magnetic field is induced in the direction that strengthens the initial magnetic field in the region $\beta > 1$.

3.2. Results of resistive MHD analyses

Here, we used the resistive MHD model, and conducted a parametric survey of some R_m numbers to investigate the magnetic diffusion effect due to finite electrical conductivity in magnetic field inflation. However, we assumed R_m is constant in the whole computational domain to identify the magnitude of R_m that significantly affects the plasma flow of the magnetic field inflation. This assumption is valid because R_m is a slowly varying number in the domain. Here, R_m number is from 1 to 10^2 .

Fig. 3. Initial magnetic flux density and magnetic field lines, T.

Fig. 5. Velocity contour and streamlines, m/s ($R_m \rightarrow \infty$, Ideal MHD model). Fig. 4. Inflated magnetic flux density and magnetic field lines, T ($R_m \rightarrow \infty$, Ideal MHD model). Fig. 6. Distribution of $|B_d|/|B_0|$ (ratio of induced magnetic field to initial magnetic field) and local β value ($R_m \rightarrow \infty$, Ideal MHD model).

Figures 8 and 9 show the distributions of the inflated magnetic field strength and magnetic field lines when $R_m = 10$ and 1 respectively. Comparing Fig. 4 with Fig. 8, the result for $R_m = 10$ is almost identical to the result for the ideal MHD model, and any effect of finite electrical conductivity is not seen. On the other hand, comparing Fig. 4 with Fig. 9, we see some changes occur in the inflated magnetic field for $R_m = 1$ especially in the equatorial region ($X = 0$ m and $Y > 10$ m). Due to the magnetic diffusion effect, the magnetic flux density becomes gentle for $R_m = 1$ (resistive MHD model). However, this is not a positive result from the viewpoint of magnetic field inflation. Comparing Fig. 4 and Fig. 9, we see that the magnetic field strength inflated by the plasma jet with $R_m = 1$ is obviously smaller than that for the ideal MHD model. For example, along the polar axis ($Y = 0$ m line), while the contour where $B = 1.26 \times 10^{-5}$ T is inflated to $X = 21.5$ m in the case of ideal MHD model, the contour is located at $X = 20$ m when $R_m = 1$.

To quantify the magnetic diffusion effect in magnetic field inflation, the distributions of magnetic field strength along a line 40 m from the dipole center in the θ -direction

Fig. 7. Induced current density in direction of $\mathbf{u} \times \mathbf{B}$, A/m² ($R_m \rightarrow \infty$, Ideal MHD model).

Fig. 8. Inflated magnetic flux density and magnetic field lines, T ($R_m = 10$, Resistive MHD model).

Fig. 9. Inflated magnetic flux density and magnetic field lines, T ($R_m = 1$, Resistive MHD model).

are shown in Fig. 10 for $R_m = 1, 3, 10, 30, 100$ (resistive MHD model) and ∞ (ideal MHD model). These distributions have similar features: (1) the magnetic field is inflated radially for $\theta < 75^\circ$; and (2) weak magnetic flux density regions are seen in the equatorial region. The distributions of the magnetic field strength for $R_m = 10, 30$ and 100 are almost identical to the result of the ideal MHD model. It can be said that the magnetic diffusion effect due to finite electrical conductivity is negligible when the magnetic Reynolds number is about 10 or more, meaning that the ideal MHD approximation is valid for $R_m > 10$.

Meanwhile, the distributions of the magnetic flux density when $R_m = 3$ and 1 are quite different from the other cases shown in Fig. 10. We confirmed that the magnetic field strength when $R_m = 3$ and 1 is small compared to the ideal MHD model. This trend is explained as follows: when R_m is small, the magnetic diffusion term works efficiently in the strong-current region. Near the equatorial region ($X = 0$ m and $Y > 10$ m), as shown in Fig. 7, a strong current is induced for the ideal MHD model. Such a strong and localized current region is called a current sheet. Thus, the

Fig. 10. Magnetic field strength distribution along line 40 m from dipole center.

Fig. 11. Magnetic field strength normalized by result of ideal MHD model at $r = 40$ m on polar axis (at $X = -40$ m and $Y = 0$ m).

magnetic diffusion effect cannot be neglected around the current sheet and slip of the magnetic field lines across the plasma flows occurs when R_m is unity.

Figure 11 shows the ‘efficiency’ of the magnetic field inflation process when R_m is from 1 to 10^2 . Here, efficiency is defined as the normalized magnetic field strength (the magnetic field of the resistive MHD model divided by that of the ideal MHD model) along the polar axis at $r = 40$ m ($X = -40$ m, $Y = 0$ m). From our results, the magnetic diffusion effect is remarkable when R_m is about unity, hence magnetic field inflation based on the resistive MHD model is suppressed by about 19% when $R_m = 1$.

3.3. Design of plasma source for MPS

The above sections explain the detailed structure of the inflated magnetic field. Because different R_m values are used for each simulation, here is a question about which simulation case is best for MPS operation. This topic is discussed here.

As defined in Eq. (7), R_m is proportional to the electrical conductivity, σ , and to the velocity of the plasma jet, u_c . For a fully ionized plasma, conductivity is expressed as $\sigma = 1.53 \times 10^{-2} T^{3/2} / \ln \Lambda$,¹³⁾ hence R_m is related to the characteristics of the plasma jet (T and u_c) used for magnetic field inflation. To characterize the plasma jet, we used three non-dimensional parameters: β , M , and R_m . Using Eqs. (7) and (8), and $M = u_c / (\gamma RT)^{1/2}$, it is possible to transform these non-dimensional parameters into physical values (ρ , T and u_c). In the present hydrogen plasma simulation ($\beta = 2.5 \times 10^{-4}$, $M = 1.0$ and $L_c = 1$ m), $R_m = 1$ corresponds to $\rho = 9.0 \times 10^{-9}$ kg/m³ ($n = 5.4 \times 10^{18}$ /m³), $u_c = 6.0$ km/s, and $T = 2600$ K if the Coulomb logarithm is approximated as $\ln \Lambda = 15$. In this case, a high-density, low-velocity, and low-temperature plasma jet is required at the spacecraft surface. However, it is usually difficult to produce a fully ionized high-density plasma at such a low temperature. In contrast, a lower density ($\rho = 2.8 \times 10^{-9}$ kg/m³, ($n = 1.7 \times 10^{18}$ /m³) plasma jet with a moderate velocity ($u_c = 10.6$ km/s) and a moderate temperature ($T = 8200$ K) is possible for $R_m = 10$. In other words, the simulation with $R_m = 10$ is appropriate from the viewpoint of plasma production. From this fact and the efficient magnetic field inflation for a large R_m value, we think that $R_m \geq 10$ is suitable for MPS operation.

Note that the MHD analyses in this paper are restricted to a very near-field region around the MPS spacecraft. Using the results from these near-field simulations, it is possible to estimate the magnetospheric size for an MPS spacecraft. The concept of MPS is to achieve large thrust by increasing the magnetospheric size. However, some global simulations of MPS¹⁴⁾ show that large magnetospheric size does not necessarily mean large thrust. In this sense, global simulation including both the solar wind to magnetosphere interaction and the magnetic field inflation is critical to obtain the thrust characteristics. Since the ideal MHD model was found to be good enough to describe the MPS flow for $R_m > 10$, resistive MHD analysis requiring large computational time can be circumvented. Optimization of the thrust performance for an MHD-scale MPS using the ideal MHD is our next step.

4. Conclusions

We performed ideal MHD and resistive MHD analyses of magnetic field inflation process for an MPS to investigate the magnetic diffusion effect of finite electrical conductivity in magnetic field inflation. The following results were obtained:

(1) For the ideal MHD model, the magnetic field lines approximately coincide with the streamlines of the injected plasma flow and the magnetic field near the spacecraft is inflated almost radially. However, in the equatorial region, the magnetic field inflation is not significant and the equatorial plane becomes a current sheet as a result of the transformation of the magnetic field lines.

- (2) When R_m is 10 or more, the magnetic diffusion effect due to finite electrical conductivity of plasma is negligible, i.e. the ideal MHD approximation is valid. $R_m \geq 10$ is suitable for MPS operation because very efficient magnetic field inflation is possible.
- (3) When R_m is almost unity, the magnetic diffusion effect cannot be neglected. In this case, the magnetic field was not inflated effectively compared to the ideal MHD case.

Acknowledgments

I would like to thank to the member of the Magnet Plasma Sail (MPS) working group (in particular Prof. Hiroshi Yamakawa of Kyoto University, Dr. Hiroyuki Nishida of Tokyo University of Agriculture and Technology, and Dr. Kenichi Kubota of JAXA). This work is supported by the Grant-in-Aid for Scientific Research (B) (No. 18360411) and (A) (No. 21246126) of the Japan Society for Promotion of Science, and by the Japan Science and Technology Agency/CREST. The activity of the MPS working group is supported by the engineering committee of the institute of space and astronautical science in JAXA.

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