Unsteady Aerodynamic Forces Caused by Oscillation of Blades in Multistage Bladerows

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The unsteady lifting surface theory for a model composed of five bladerows is developed to predict unsteady aerodynamic forces on oscillating blades of aerodynamically coupled multistage annular bladerows. The theory is formulated to calculate generalized aerodynamic forces which appear in Lagrange's equations of blade motions, so that it can be used as the aerodynamic tool for the inter-row coupling flutter analysis of multistage bladerows. Numerical studies were also conducted to investigate the mutual aerodynamic influences among bladerows. The simultaneous effects of co-rotating upstream and downstream bladerows located at next to the next position significantly modify the aerodynamic response of oscillating blades. The aerodynamic force induced by the oscillating bladerow is not very small even on the bladerows remoter than the next-to-the-next bladerow.

Key Words: Cascade Flutter, Multistage Bladerows, Rotor-Stator Interaction, Unsteady Aerodynamics, Lifting Surface Theory

Nomenclature

A : a integer number indicating that integral mode parameters lower than $-A$ or higher than A truncated in numerical computation
 $a(r, z)$: displacement amplitude of blade vibration normal to blade surface
 C_{aj} : axial chord length of Row j
 $C_{Lj(\mu)}$: unsteady local lift coefficient of Row j of frequency component $\omega_{j(\mu)}$; Eq. (28)
 F_{nm}^{j-iv} : generalized force on Row j ; Eq. (27)
 G_j : axial coordinate of midchord section of Row j in absolute cylindrical coordinate system; Fig. 1
 h : hub-to-tip ratio
 $\Im[\]$: imaginary part of complex number
 K_q : velocity kernel function; Eq. (16)
 K_{qn} : component of K_q normal to blade surface
 K_{qs} : component of K_q in undisturbed stream-wise direction
 M_a : axial Mach number
 N_{Bj} : number of blades of Row j
 $n_{(\mu)}$: circumferential wave number of flow disturbances; Eq. (9)
 q : disturbance velocity
 q_{nj} : component of q normal to blade surface
 $\Re[\]$: real part of complex number
 r : radial coordinate
 r_T^* : tip radius
 t : time
 W_a^* : mean axial flow velocity
 W_m^j : m -th order normalized natural mode shape of blade vibration of Row j
 z : axial coordinate in frame of reference fixed to duct

z_j : axial coordinate in frame of reference fixed to Row j
 Δp_j : unsteady pressure difference across blade surface (blade loading) of Row j
 $\Delta p_{j(\mu)}$: $j(\mu)$ component of Δp_j ; Eq. (14)
 $\delta_{\alpha,\beta}$: Kronecker's delta
 $\eta_j = \theta_j - \Omega_j z_j$: helical coordinate
 Ω_j : rotational speed of Row j : $\Omega_j = \Omega_j^* r_T^* / W_a^*$
 ω : reduced frequency ($\omega^* r_T^* / W_a^*$) of blade vibration
 $\omega_{(\mu)}$: frequency component of disturbance identified by five integral mode parameters $\mu_1, \mu_2, \dots, \mu_5$ in frame of reference fixed to duct; Eq. (8)
 $\omega_{j(\mu)}$: reduced frequency with integral mode parameters $\mu_1, \dots, \mu_{j-1}, \mu_{j+1}, \dots, \mu_5$ in frame of reference fixed to Row j ; Eq. (10)
 σ : interblade phase angle divided by blade-to-blade angle ($2\pi/N_{Bj}$)
 $\sigma_{j(\mu)}$: number of nodal diameters of duct mode related to component of reduced frequency $\omega_{j(\mu)}$ in frame of reference fixed to Row j ; Eq. (12)
 θ : circumferential angle coordinate in frame of reference fixed to duct
 θ_j : circumferential angle coordinate in frame of reference fixed to Row j
 θ_{j0} : angular position of reference blade of Row j at time $t = 0$ in absolute coordinate system
 ρ_0^* : mean fluid density
Subscripts
 j : row number ($j = 1, 2, \dots, 5$)
 $j(\mu)$: abbreviation for $j(\mu_1, \dots, \mu_{j-1}, \mu_{j+1}, \dots, \mu_5)$, and stands for frequency component identified by four integral mode parameters $\mu_1, \dots, \mu_{j-1}, \mu_{j+1}, \dots, \mu_5$ in frame of reference fixed to Row j

(μ): abbreviation for ($\mu_1, \mu_2, \dots, \mu_5$), and stands for frequency component identified by five integral mode parameters $\mu_1, \mu_2, \dots, \mu_5$ in frame of reference fixed to duct

The signs of the rotational speeds are assumed to be $\Omega_j > 0$ (clockwise looking from downstream) and $\Omega_j < 0$ (counterclockwise).

1. Introduction

From recent studies based on models of multiple bladerows¹⁻⁶ it is now commonly understood that the unsteady aerodynamic response of oscillating blades is significantly influenced by the presence of neighboring bladerows.

The salient feature of the unsteady flow field in multistage bladerows is so-called frequency and mode scattering.⁷ Unsteady disturbances originating from a simple harmonic oscillation of blades of one of the bladerows are scattered into multiple frequency components due to aerodynamic coupling between bladerows in mutual rotational motions. Therefore aerodynamic forces on all blades are composed of (an infinite number of) multiple frequency components. Furthermore, we note that the frequency components are not independent, but are dependent on each other, even if small disturbance is assumed and the governing equations are linearized. Consequently, when we deal with flutter problems of multistage bladerows, we encounter the heavy task of computing unsteady flow fields and predicting unsteady aerodynamic forces. In particular, the frequency and mode of blade vibration at a critical flutter condition are unknowns to be determined by aeroelastic analysis, and it is necessary to conduct many aerodynamic computations to find the condition.

Recent progresses in the computer power and the numerical computation technology have displaced classical analytical methods from the leading role in the theoretical studies of fluid mechanics, because the applicability of analytical methods is generally restricted to various assumptions, e.g., small disturbances, simple geometries, etc. Nevertheless, due to the overwhelmingly high computation efficiency compared to CFD techniques, analytical methods are still useful in extensive studies on essential features of fundamental parameters.

Recently, we developed the unsteady lifting surface theory⁸ for a model of three bladerows, and found from a numerical study on a rotor/stator/rotor model that unsteady aerodynamic forces on a rotor bladerow induced by oscillation of the other rotor bladerow in co-rotation is not negligible.

Using the advantage of the lifting surface theory that enables extensive aeroelastic analyses at smaller computation cost, we have conducted⁹ aeroelastic flutter analysis of a model of three bladerows, where the two rotor bladerows are simultaneously vibrating under aerodynamic coupling. The results show that inter-row coupling significantly increases the risk of flutter.

The results from the previous studies indicate the necessity to account for simultaneous coupling with co-rotating bladerows located upstream and downstream for more accurate prediction of flutter in actual multistage compressors and turbines. To this end, this paper describes the unsteady lifting surface theory for a model of five bladerows to calculate unsteady aerodynamic forces on blades caused by blade vibrations.

Moreover, numerical studies are also conducted and the simultaneous effects of upstream and downstream neighboring bladerows on the aerodynamic response of oscillating blades are investigated. Furthermore, properties and magnitudes of unsteady aerodynamic forces induced on nonoscillating blades of neighboring and remote bladerows are also investigated. In particular, even unsteady aerodynamic forces on rotor blades induced by oscillation of rotor blades placed further away than the next rotor bladerow are never negligibly small.

2. Model Description and Mathematical Formulation

2.1. Analytical model and geometries

We consider a model for five bladerows (Fig. 1) in an annular duct with outer radius r_T^* and hub-to-tip ratio h . The fluid is an inviscid nonconducting perfect gas. The undisturbed flow is a subsonic uniform axial flow of velocity W_a^* , density ρ_0^* and axial Mach number M_a . Unstarred symbols denote dimensionless quantities. Lengths, velocities, pressures, and times are normalized by r_T^* , W_a^* , $\rho_0 W_a^{*2}$, and r_T^*/W_a^* respectively. We designate the bladerows as Row 1, Row 2, ..., Row 5, or by indices 1, 2, ..., 5, as shown in Fig. 1.

The mathematical formulations assume that the rows are rotating at individual angular velocities $\Omega_j (= \Omega_j^* r_T^*/W_a^*)$: $j = 1, 2, \dots, 5$, for greater versatility in parameter selection. However, the model is reduced to a stator-rotor-stator-rotor-stator model by specifying $\Omega_1 = \Omega_3 = \Omega_5 = 0$, and to a rotor-stator-rotor-stator-rotor model by specifying $\Omega_2 = \Omega_4 = 0$. We denote cylindrical coordinate systems fixed to Row j : ($j = 1, 2, \dots, 5$), and the duct by (r, θ_j, z_j) , ($j = 1, 2, \dots, 5$), and (r, θ, z) , respectively. The coordinates fixed to the duct and Row j are related to each other by:

Fig. 1. Fan of five bladerows in individual rotation.

$$\theta = \theta_j - \Omega_j t + \theta_{j0}, \quad (1)$$

$$z = z_j + G_j \quad (2)$$

where, G_j denotes the axial position of the center of Row j in the duct-fixed coordinate system.

Further, we define the helical coordinate η_j by:

$$\eta_j = \theta_j - \Omega_j z_j. \quad (3)$$

Note that $\eta_j = \text{constant}$ denotes an undisturbed streamline in the frame of reference fixed to Row j . Transforms from η_j to θ and from η_j to $\eta_{j'}$ are made by:

$$\eta_j = \theta + \Omega_j(t - z_j) - \theta_{j0} \quad (4)$$

$$\eta_j = \eta_{j'} - (\Omega_{j'} - \Omega_j)(t - z_{j'}) + \Omega_j(G_j - G_{j'}) + \theta_{j'0} - \theta_{j0}. \quad (5)$$

We denote the numbers of blades of the bladerows by N_{Bj} , and the axial chord lengths of the blades by C_{aj} . We assume that the blades have constant axial chord length and no axial sweep and no dihedral lean.

2.2. Disturbance flow field

Assume that the blades of Row j_v are oscillating at an angular reduced frequency ω and an inter-blade phase angle $2\pi\sigma/N_{Bj_v}$, and further the blades of the other rows are not vibrating. Further assume that the blade rows are operating with zero steady blade loading (zero time mean angle of attack, zero camber and zero thickness), and the amplitude of the blade oscillation is very small. Hence the flow field is governed by linearized equations.

This blade motion generates disturbances that interact with all blades of all bladerows. Finally, the disturbance flow field can be described as:

$$D = \sum_{\mu_1=-\infty}^{\infty} \sum_{\mu_2=-\infty}^{\infty} \cdots \sum_{\mu_5=-\infty}^{\infty} D_{(\mu)}(r, z) \times e^{i\omega_{(\mu)}t + i n_{(\mu)}\theta} \quad (6)$$

$$= \sum_{\mu_1=-\infty}^{\infty} \sum_{\mu_2=-\infty}^{\infty} \cdots \sum_{\mu_5=-\infty}^{\infty} D_{(\mu)}(r, z_j + G_j) \times e^{i\omega_{(\mu)}t + i(\mu_j N_{Bj} + \sigma_{j(\mu)})(\theta_j + \theta_{j0})}. \quad (7)$$

Here, the notation (μ) implies the abbreviation for $\mu_1, \mu_2, \dots, \mu_5$, and the notation $j(\mu)$ implies the abbreviation for $j(\mu_1, \dots, \mu_{j-1}, \mu_{j+1}, \dots, \mu_5)$. Further,

$$\omega_{(\mu)} \equiv \omega + \sigma\Omega_{j_v} + \sum_{j'=1}^5 \mu_{j'} N_{Bj'} \Omega_{j'}, \quad (8)$$

$$n_{(\mu)} \equiv \sigma + \sum_{j'=1}^5 \mu_{j'} N_{Bj'}, \quad (9)$$

$$\omega_{j(\mu)} \equiv \omega + \sigma(\Omega_{j_v} - \Omega_j) + \sum_{\substack{j'=1 \\ j' \neq j}}^5 \mu_{j'} N_{Bj'} (\Omega_{j'} - \Omega_j) \quad (10)$$

$$= \omega_{(\mu)} - n_{(\mu)} \Omega_j, \quad (11)$$

$$\sigma_{j(\mu)} \equiv \sigma + \sum_{\substack{j'=1 \\ j' \neq j}}^5 \mu_{j'} N_{Bj'} \quad (12)$$

$$= n_{(\mu)} - \mu_j N_{Bj}. \quad (13)$$

Note that Eq. (6) describes the disturbance flow field in terms of the frame of reference (r, θ, z) fixed to the duct, whereas Eq. (7) describes it in terms of the frame of reference (r, θ_j, z_j) fixed to Row j . Therefore $\omega_{(\mu)}$ denotes the frequencies of the disturbances in the frame of reference fixed to the duct, whereas $\omega_{j(\mu)}$ denotes the frequencies of the disturbances observed in the frame of reference fixed to Row j , and $\omega_{j(\mu)}$ does not depend on the integral mode parameter μ_j .

On the other hand, $n_{(\mu)}$ denotes the circumferential wave numbers of the disturbances. Note also that the interblade phase angle of the disturbances of frequency $\omega_{j(\mu)}$ observed in Row j is given by $2\pi\sigma_{j(\mu)}/N_{Bj}$. Further, note that $\sigma_{j(\mu)}$ is also independent of the integral mode parameter μ_j .

We should note that in the case of a single bladerow in a simple harmonic oscillation of a frequency ω with an inter-blade phase angle with a nodal diameter σ , the disturbances observed in the frame of reference fixed to the bladerow itself are composed of the simple harmonic frequency ω and duct modes with circumferential wave numbers σ plus integral multiples of the number of blades of the bladerow. However, for multiple bladerows, the inter-row aerodynamic coupling causes multiplications of frequencies and duct modes of disturbance waves as shown by Eqs. (8)–(13). In regard to the multiplications, note that in the frame of reference fixed to a bladerow, e.g., Row j , the co-rotating bladerows (Row j' with $\Omega_{j'} = \Omega_j$) do not contribute to multiplication of frequencies $\omega_{j(\mu)}$, because $\omega_{j(\mu)}$ is independent of the integral parameter $\mu_{j'}$ if $\Omega_{j'} = \Omega_j$. But the co-rotating bladerows still contribute to multiplication of duct modes, because $\sigma_{j(\mu)}$ is dependent on the integral parameter $\mu_{j'}$ of $j' \neq j$ irrespective of the rotational velocity $\Omega_{j'}$.

Hereafter, we call the component of $j(0)$, i.e., the component of the frequency $\omega_{j(0)} = \omega + \sigma(\Omega_{j_v} - \Omega_j)$ and the nodal diameter $\sigma_{j(0)} = \sigma$, the fundamental component, and further we call other components derivative components.

Let $\Delta p_j^{(m)}(r, z_j, t)$ denote the unsteady blade loading (lower surface pressure minus upper surface pressure) of the m -th blade of Row j . Then, it is composed of multiple frequency components that can be expressed by:

$$\Delta p_j^{(m)}(r, z_j, t) = \sum_{\mu_1=-\infty}^{\infty} \sum_{\mu_2=-\infty}^{\infty} \cdots \sum_{\mu_{j-1}=-\infty}^{\infty} \times \sum_{\mu_{j+1}=-\infty}^{\infty} \cdots \sum_{\mu_5=-\infty}^{\infty} \Delta p_{j(\mu)}(r, z_j) \times e^{i\omega_{j(\mu)}t + i2\pi m \sigma_{j(\mu)}/N_{Bj}}. \quad (14)$$

The lifting surface theory is formulated by treating those unsteady blade loadings as disturbance sources, and expressing the disturbance velocity field in the integral form:

$$\begin{aligned}
q(r, \theta, z, t) = & \sum_{\mu_1=-\infty}^{\infty} \sum_{\mu_2=-\infty}^{\infty} \cdots \sum_{\mu_5=-\infty}^{\infty} e^{i\omega_{(\mu)}t + in_{(\mu)}\theta} \\
& \times \sum_{j=1}^5 e^{-in_{(\mu)}(\Omega_j z_j + \theta_{j0})} \\
& \times \int_h^1 \int_{-C_{aj}/2}^{C_{aj}/2} \Delta p_{j(\mu)}(\rho, \zeta) \\
& \times K_q^{(\mu_j)}(r, z_j - \zeta | \rho; N_{Bj}, \Omega_j, \omega_{j(\mu)}, \sigma_{j(\mu)}) \\
& \times d\zeta d\rho \quad (15)
\end{aligned}$$

where $K_q^{(\mu)}$ denotes the Fourier component of the velocity kernel function K_q expressed as:

$$\begin{aligned}
K_q(r, \eta, z | \rho; N, \Omega, \omega, \sigma) = & \sum_{\mu=-\infty}^{\infty} e^{i(\mu N + \sigma)\eta} \\
& \times K_q^{(\mu)}(r, z | \rho; N, \Omega, \omega, \sigma). \quad (16)
\end{aligned}$$

We give the details elsewhere.⁸⁾

3. Integral Equations for Blade Loading Functions

Equation (15) expresses the disturbance flow field as linearized superposition of disturbances generated from five bladerows. Here, the unsteady blade loadings play roles as disturbance sources in appearance. However, in the present problem the real origin of the disturbance is vibration of blades, and hence the unsteady blade loadings should be calculated for specified blade motions. The unsteady blade loadings are determined so that the disturbance velocity satisfies the flow tangency condition on blade surfaces.

In the present case, where the blades of Row j_v are vibrating at a reduced frequency ω and an interblade phase angle $2\pi\sigma/N_{Bj_v}$ with normal displacement $a(r, z_{j_v})e^{i\omega t}$, the flow tangency condition on the blade surfaces is expressed as:

$$\begin{aligned}
[q_{nj}]_{\eta_j=\pm 0} = & \delta_{j,j_v} \left(i\omega a(r, z_j) + \frac{\partial a(r, z_j)}{\partial z_j} \right) e^{i\omega t} \quad (17) \\
& : -C_{aj}/2 \leq z_j \leq C_{aj}/2, \\
& (j = 1, 2, \dots, 5)
\end{aligned}$$

Where q_{nj} denotes the component of the disturbance velocity q normal to blade surfaces of Row j . At a blade surface of Row j , it holds that $\eta_j = 0$ and hence $\theta = -\Omega_j(t - z_j) + \theta_{j0}$.

Then, substituting Eq. (15) into Eq. (17) and requiring each frequency component to satisfy Eq. (17), we obtain a system of integral equations for the unsteady blade loading functions as follows:

$$\begin{aligned}
& \int_h^1 \int_{-C_{aj}/2}^{C_{aj}/2} \Delta p_{j(\mu)}(\rho, \zeta) \\
& \times K_{qn}(r, 0, z_j - \zeta | \rho; N_{Bj}, \Omega_j, \omega_{j(\mu)}, \sigma_{j(\mu)}) d\zeta d\rho \\
& + \sum_{\substack{j=1 \\ j \neq j}}^5 \sum_{\mu_j=-A}^A e^{-in_{(\mu)}\{\Omega_j z_j - \Omega_j z_j + \theta_{j0} - \theta_{j0}\}}
\end{aligned}$$

$$\begin{aligned}
& \times \int_h^1 \int_{-C_{aj}/2}^{C_{aj}/2} \Delta p_{j'(\mu)}(\rho, \zeta) \\
& \times K_{qnj \leftarrow j'}^{(\mu_j)}(r, \bar{z}_j - \zeta | \rho; N_{Bj'}, \Omega_{j'}, \omega_{j'(\mu)}, \sigma_{j'(\mu)}) d\zeta d\rho \\
= & C_{j(\mu)}(r, z_j), \\
& (\mu_1 = 0, \pm 1, \dots, \pm A; \dots; \mu_{j-1} = 0, \pm 1, \dots, \pm A; \\
& \mu_{j+1} = 0, \pm 1, \dots, \pm A; \dots; \mu_5 = 0, \pm 1, \dots, \pm A) \\
& (j = 1, 2, \dots, 5) \quad (18)
\end{aligned}$$

where,

$$\begin{aligned}
C_{j(\mu)}(r, z_j) = & \delta_{j,j_v} \delta_{\mu_1,0} \cdots \delta_{\mu_{j-1},0} \delta_{\mu_{j+1},0} \cdots \delta_{\mu_5,0} \\
& \times \left\{ i\omega a(r, z_j) + \frac{\partial}{\partial z_j} a(r, z_j) \right\}, \quad (19)
\end{aligned}$$

$$\bar{z}_j = z_j - (G_{j'} - G_j),$$

$$\begin{aligned}
& K_{qnj \leftarrow j'}^{(\mu)}(r, z | \rho; N, \Omega, \omega, \sigma) \\
= & \frac{1}{\sqrt{1 + \Omega_{j'}^2 r^2} \sqrt{1 + \Omega_j^2 r^2}} \\
& \times [-(\Omega_j - \Omega_{j'}) K_{qs}^{(\mu)}(r, z | \rho; N, \Omega, \omega, \sigma) \\
& + (1 + \Omega_j \Omega_{j'} r^2) K_{qn}^{(\mu)}(r, z | \rho; N, \Omega, \omega, \sigma)]. \quad (20)
\end{aligned}$$

Numerical task to solve the system of integral equations must be carried out by truncating the infinite series to finite ones. In the equations above, truncation is made by cutting off the integral mode parameters lower than $-A$ or higher than A , where A is an appropriate integer number.

The system of the integral equations can be solved by a standard method. As we have shown,⁸⁾ we can express the blade loading functions $\Delta p_{j(\mu)}(r, z_j)$ in terms of a double-mode function series expansion, and transform the system of the integral equations (18) into a system of algebraic equations for the mode function series coefficients. Retaining the double-mode function series up to L terms of radial mode functions and I terms of chordwise mode functions, and requiring the equations to be satisfied at L representative radial points ($r = r_1, r_2, \dots, r_L$) and I representative axial positions ($z_j = z_{j1}, z_{j2}, \dots, z_{jI}$), we deal with a system of $5 \times (2A + 1)^4 \times L \times I$ linearized algebraic equations.

Finally, it is worth emphasizing that the mathematical formulation shown above is quite general, and the extension to a model of multistage bladerows with any number of bladerows is just a matter of substitution.

4. Definitions of Aerodynamic Force Coefficients

4.1. Generalized force

Applying the standard method of the flutter analysis, we express the displacement of a reference blade of Row j_v normal to the blade surface in the form

$$a_{j_v}(r, z_{j_v})e^{i\omega t} = \sum_{m=1}^{\infty} W_m^{j_v}(r, z_{j_v}) w_m^{j_v} e^{i\omega t} \quad (21)$$

where, $W_m^{j_v}(r, z_{j_v})$ denotes the normalized natural mode shape function of the m -th order and $w_m^{j_v}$ denotes the modal

amplitude. Note that $W_m^{j_v}(r, z)$ is a real function, but, in general, the modal amplitude $w_m^{j_v}$ is a complex number. Then, the blade loading function $\Delta p_{j(\mu)}(r, z_j)$ defined in Eq. (14) can be expressed by:

$$\Delta p_{j(\mu)}(r, z_j) = \sum_{m=1}^{\infty} w_m^{j_v} \Delta p_{j(\mu),m}^{j-j_v}(r, z_j) \quad (22)$$

where, $\Delta p_{j(\mu),m}^{j-j_v}(r, z_j)$ denotes the blade loading component of frequency $\omega_{j(\mu)}$ of Row j caused by the blade vibration of Row j_v with the m -th natural vibration mode $W_m^{j_v}(r, z_{j_v})$ of unit amplitude $w_m^{j_v} = 1$.

Note that the linearized summation shown by Eq. (22) is permitted by linearized theory. We have only to compute $\Delta p_{j(\mu),m}^{j-j_v}(r, z_j)$ for each m separately, specifying a single mode oscillation with unit amplitude. The modal amplitudes $w_m^{j_v}$ are determined from aeroelastic analysis.

Hereafter, we deal with bladerows of 2.5 stages, i.e., a rotor-stator-rotor-stator-rotor system ($\Omega_1 = \Omega_3 = \Omega_5 = \Omega \neq 0$ and $\Omega_2 = \Omega_4 = 0$), or a stator-rotor-stator-rotor-stator system ($\Omega_1 = \Omega_3 = \Omega_5 = 0$ and $\Omega_2 = \Omega_4 = -\Omega \neq 0$). Further, assume that Row 1, Row 3 and Row 5 are simultaneously vibrating at a common frequency ω and a common number of nodal diameters σ , but Row 2 and Row 4 are not vibrating. Then, from Eq. (10), we obtain:

$$\begin{aligned} \omega_{1(\mu)} &= \omega_{3(\mu)} = \omega_{5(\mu)} \\ &= \omega - (\mu_2 N_{B2} + \mu_4 N_{B4})\Omega, \end{aligned} \quad (23)$$

$$\begin{aligned} \omega_{2(\mu)} &= \omega_{4(\mu)} = \omega + \sigma\Omega \\ &+ (\mu_1 N_{B1} + \mu_3 N_{B3} + \mu_5 N_{B5})\Omega. \end{aligned} \quad (24)$$

Note that the fundamental frequency $\omega_{1(0)} = \omega_{3(0)} = \omega_{5(0)}$ of the aerodynamic forces on Row 1, Row 3 and Row 5 is equal to the blade vibration frequency ω , and only the fundamental frequency component of the aerodynamic force exerts effective aerodynamic work on the oscillating blades. Furthermore, note that the fundamental frequency component of the aerodynamic force $\Delta p_{j(0)}(r, z_j)$ is overwhelmingly predominant, as we have shown⁸⁾ previously and also confirmed later.

Then, the aerodynamic work per cycle on a blade of Row j normalized by: $\pi \rho_0^* W_a^{*2} r_T^{*3}$ is given by

$$\begin{aligned} W_j &= \frac{1}{\pi} \oint dt \int_h^1 \sqrt{1 + \Omega_j^2 r^2} dr \\ &\times \int_{-C_{aj}/2}^{C_{aj}/2} \Re[\Delta p_j(r, z_j, t)] \\ &\times \Re \left[\frac{\partial}{\partial t} \{ a_j(r, z_j) e^{i\omega t} \} \right] dz_j \end{aligned} \quad (25)$$

$$= \sum_{j_v=1,3,5} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \Im \left[\overline{w_n^j} w_m^{j_v} F_{nm}^{j-j_v} \right] \quad (26)$$

where, $F_{nm}^{j-j_v}$ denotes the generalized force defined by

$$\begin{aligned} F_{nm}^{j-j_v} &= \int_h^1 \sqrt{1 + \Omega_j^2 r^2} dr \\ &\times \int_{-C_{aj}/2}^{C_{aj}/2} W_n^j(r, z_j) \Delta p_{j(0),m}^{j-j_v}(r, z_j) dz_j. \end{aligned} \quad (27)$$

Table 1. Specified parameters.

Hub/tip ratio h	0.7
Axial Mach number M_a	0.6
Number of rotor blades N_{Bj}	40
Number of stator vanes N_{Bj}	63
Axial chord length (rotor) C_{aj}	0.1
Axial chord length (stator) C_{aj}	0.0794
Rotational speed (rotor) Ω_j	1.0

The generalized force defined above appears in flutter equations describing blade motions.⁹⁾

4.2. Unsteady local lift coefficient

To evaluate frequency components of the aerodynamic forces on oscillating and non-oscillating bladerows, we define the frequency component of the unsteady local lift coefficient by:

$$C_{Lj(\mu)}(r) = \frac{1}{C_{aj}} \int_{-C_{aj}/2}^{C_{aj}/2} \Delta p_{j(\mu)}(r, z_j) dz_j. \quad (28)$$

5. Numerical Results and Discussions

5.1. Specified conditions

Here, we deal with bladerows of two and a half stages composed of rotor-stator-rotor-stator-rotor. Numerical computations were conducted for the conditions in Table 1.

The relative Mach number for the rotors varies from 0.732 at the root to 0.849 at the tip, the stagger angle of the rotors varies from 35° at the root to 45° at the tip, and the pitch/chord ratio of the rotors varies from 0.901 at the root to 1.111 at the tip. On the other hand, the pitch/chord ratio of the stators varies from 0.880 at the root to 1.257 at the tip. Further, this paper deals with the cases where the blades of one of the rotors are vibrating, but the stators are not. The results are shown for the blade vibration of the reduced frequency $\omega C_{aj} = 1.0$.

The distances between two adjacent bladerows are assumed equal, i.e., $G_2 - G_1 = G_3 - G_2 = G_4 - G_3 = G_5 - G_4 = G$, and denoted by, e.g. $G = 1.1 C_a$, where $C_a (= 0.0897)$ is the mean axial chord length of the two adjacent bladerows.

For the numbers of the terms of the double-mode function series expansion, we took $L = 7$ and $I = 7$, which were found to give sufficient accuracy in previous studies.⁸⁾ On the other hand, the harmonic numbers of frequencies $\mu_1, \mu_2, \dots, \mu_5$ are truncated by specifying $A = 1$ in Eq. (18), because the previous study on three bladerows⁸⁾ indicated that $A = 1$ gives sufficient accuracy for evaluation of aerodynamic forces of the fundamental frequency component $\omega_j(0)$ while the relative flow velocities for all bladerows are subsonic.

5.2. Cut-on condition of duct modes

First, recall that the aerodynamic interaction among bladerows occurs via acoustic waves propagating both upstream and downstream, and also via vortical waves shed from each blade and convected downstream only. The

Fig. 2. Frequency components of unsteady local lift coefficient $|C_{Lj(\mu)}|$ (absolute value) at midspan $r = 0.8$. $\sigma = -10$. Rotor 3 is vibrating ($j_v = 3$), $G = 1.1C_a$.

acoustic waves are composed of duct modes. Each duct mode is characterized by the frequency $\omega_{(\mu)}$ given by Eq. (8), the circumferential wave number $n_{(\mu)}$ given by Eq. (9) and the radial order l . It is important to note that the cut-on condition of a duct mode is given by

$$(k_l^{n_{(\mu)}})^2 - \omega_{(\mu)}^2 M_a^2 / (1 - M_a^2) < 0 \quad (29)$$

where, $k_l^{(n)}$ denotes the radial eigenvalue of the circumferential wave number n and the radial order l .⁽⁸⁾

From previous studies^(5,8) on two or three bladerows, we found that the fundamental duct mode with $\omega_{(0)} = \omega + \sigma\Omega$, $n_{(0)} = \sigma$ and $l = 0$ is overwhelmingly dominant, and the aerodynamic force is heavily dependent on the state of the fundamental duct mode, i.e., whether it is cut-on or cut-off.

Here, note that the fundamental duct mode is cut-on in the range of $-4 < \sigma \leq 20$ at the blade vibration frequency $\omega C_{aj} = 1.0$ specified in the numerical examples shown here.

5.3. Frequency components of aerodynamic forces

To investigate the magnitudes of frequency components of the aerodynamic forces on the blades and vanes, Fig. 2 shows an example computation of the frequency components of absolute values of the unsteady local lift coefficient $|C_{Lj(\mu)}(r)|$ defined by Eq. (28) at midspan $r = 0.8$ for the case of vibrating Rotor 3 at the middle position. The number of the nodal diameters of $\sigma = -10$ is specified in the example computation. The fundamental duct mode is cut-off under this condition.

As shown by Eq. (14), the aerodynamic force on each bladerow is dependent on four harmonic integral parameters. The results are obtained by taking the integral mode parameters $\mu_1, \mu_2, \dots, \mu_5$ from -1 to $+1$, i.e., computing with $A = 1$ in Eq. (18).

In the case of three bladerows studied previously,⁽⁸⁾ taking $A = 1$ generates about nine mode components for the aerodynamic forces, but in the present case, the number of mode components becomes 81. To identify the mode components in the figure, Mode Index for Row j is defined by numbering as follows:

Mode Index

$$= \sum_{k=1}^{j-1} (2A+1)^{4-k} (\mu_k + A) + \sum_{k=j+1}^5 (2A+1)^{5-k} (\mu_k + A) + 1. \quad (30)$$

Using this numbering, the fundamental component $j(0)$ with all mode integral parameters equal to zero is positioned at Mode Index 41. For comparison, the values obtained when $A = 0$ are shown at the far left side of the Mode Index axis.

First, in general, the fundamental frequency component $j(0)$ is overwhelmingly dominant for each bladerow, and the derivative components are very small.

Second, the aerodynamic force on the oscillating bladerow is larger than that on non-oscillating neighboring bladerows, but the latter is not negligible.

Third, the fundamental components computed when $A = 1$ are very close to those computed when $A = 0$. This suggests that the fundamental components of the aerodynamic forces are only slightly influenced by the derivative duct modes.

All the features noted above are the essential ones already found in a previous study on the model of three bladerows.⁽⁸⁾

Note that the generalized force is defined in Eq. (27) with the fundamental component. The fact that fairly good estimations of generalized forces can be obtained using $A = 0$ is beneficial from the practical standpoint. In the case of five bladerows, the computation cost to obtain the generalized force increases drastically by changing the value of A from 0 to 1. In fact, it takes just a few seconds on a conventional 32-bit PC to compute $F_{n,m}^{j-j'}$ for all combinations of $j = 1, 3, 5$; $j' = 1, 3, 5$; $n = 1, 2, 3, 4$; $m = 1, 2, 3, 4$ for a specified ω and a specified σ with $A = 0$. However, several hours are needed even on a 64-bit supercomputer to do the same work when $A = 1$.

5.4. Possibility of interbladerow coupling flutter

Figure 2 shows the aerodynamic force components other than the fundamental ones are essentially ineffective for aeroelastic inter-bladerow coupling. As Eq. (10) shows, the fundamental frequency $\omega_{j(0)}$ of the aerodynamic force induced on the co-rotating bladerows is same as the blade vibration frequency ω . Therefore, where inter-bladerow coupling flutter is occurring, the blades of co-rotating bladerows are vibrating at a common frequency ω .

On the other hand, the fundamental frequency $\omega_{j(0)}$ of the aerodynamic force induced on the bladerows in a mutual rotational motion $\Omega_j - \Omega_{j_v}$ is different from the blade vibration frequency ω , but equal to $\omega + \sigma(\Omega_{j_v} - \Omega_j)$. Therefore, if aeroelastic inter-bladerow coupling flutter occurs between a rotor of rotational speed Ω and a stator, the vibration frequency of the stator vanes should be $\omega + \sigma\Omega$, i.e., the fundamental frequency of the aerodynamic force induced by the rotor blade vibration, and is different from the vibration frequency ω of the rotor blades. In addition, note that the interaction between the rotor and stator is mutual, because vibration of stator vanes at a frequency $\omega + \sigma\Omega$ also

Fig. 3. Absolute value of generalized force $F_{(1,1)}^{j-j}$ on vibrating rotors. $G = 1.1C_a$.

induces aerodynamic forces on rotor blades, and the fundamental frequency of the induced aerodynamic force is equal to the rotor blade vibration frequency ω .

5.5. Effects of neighboring blade rows on generalized forces

Figure 3 shows the absolute value of the generalized force $F_{(1,1)}^{3-3}$ on the oscillating middle rotor ($j = 3$) in the five bladerows (Rows 1-2-3-4-5) as a function of the interblade phase parameter σ . For comparison, the three bladerows (Rows 2(stator)-3(rotor)-4(stator)) and the isolated bladerow (Row 3(rotor) only) are also plotted. Here, note that the fundamental acoustic duct mode of the circumferential wave number $n = \sigma$ and the radial order $l = 0$ is cut-on over the range $-4 < \sigma < 20$. In this example, the row-to-row distance is small, i.e., $G = 1.1C_a$.

A substantial difference is observed between the generalized force on the middle rotor $F_{(1,1)}^{3-3}$ in the five bladerows (1-2-3-4-5) and in the three bladerows (2-3-4). This suggests that the effects of the reflecting disturbances from co-rotating rotors on both sides are important.

5.6. Induced aerodynamic forces on neighboring blade rows

Figure 4 shows the absolute values of the generalized forces $F_{(1,1)}^{3-1}$ and $F_{(1,1)}^{5-1}$ induced by oscillating Row 1 upstream. For comparison, $F_{(1,1)}^{3-1}$ in the three bladerows 1(rotor)-2(stator)-3(rotor) is also plotted.

The generalized force $F_{(1,1)}^{3-1}$ in the five bladerows is somewhat smaller than in the three bladerows in the cut-off range ($-20 \leq \sigma < -4$) of the fundamental duct mode, but they are very close in the cut-on range ($-4 < \sigma \leq 20$).

The generalized force $F_{(1,1)}^{5-1}$ induced on Row 5 at the most remote downstream station is not smaller but rather larger than $F_{(1,1)}^{3-1}$ on Row 3 situated at a closer position in the cut-off range of the fundamental duct mode. However, $F_{(1,1)}^{5-1}$ is generally smaller than $F_{(1,1)}^{3-1}$ in the cut-on range of the fundamental duct mode. Note that the generalized forces induced on the bladerows remoter than the next to the next row are not negligible.

Fig. 4. Absolute value of generalized force $F_{(1,1)}^{j-1}$ induced by vibrating rotor Row 1 on rotors Row 3 and Row 5 at downstream stations. $G = 1.1C_a$.

Fig. 5. Dependence on bladerow gap of absolute value of generalized force $F_{(1,1)}^{3-3}$ induced by vibrating middle rotor Row 3.

The numerical examples above specify, a very small row-to-row distance $G = 1.1C_a$. Figure 5 shows how the generalized forces induced by oscillating middle Row 3 are dependent on the bladerow gap G .

In the cut-on range ($-4 < \sigma \leq 20$) of the fundamental duct mode, the self-induced generalized force $F_{(1,1)}^{3-3}$ is heavily dependent on the bladerow gap. The effect of the bladerow gap is also substantial on $F_{(1,1)}^{1-3}$ and $F_{(1,1)}^{5-3}$, which do not necessarily become small as the bladerow gap increases.

On the other hand, in the cut-off range ($-20 \leq \sigma < -4$) of the fundamental duct mode, the self-induced generalized force $F_{(1,1)}^{3-3}$ is only weakly dependent on the bladerow gap, whereas the generalized force $F_{(1,1)}^{1-3}$ on Row 1 at the upstream end becomes small as the bladerow gap increases. However the generalized force $F_{(1,1)}^{5-3}$ on Row 5 at the downstream end is not decreased by increasing bladerow gap. This is because the interaction between bladerows

via acoustic disturbances propagating upstream and downstream decreases rapidly with increased bladerow gap in the cut-off range of the fundamental duct mode, but the interaction via the vortical disturbances convected only downstream does not decrease with increased bladerow gap.

6. Conclusions

The unsteady lifting surface theory for a model of five bladerows with any blade oscillating, is developed in a general mathematical form applicable to a model for any number of bladerows.

The possibility of rotor-stator inter-row coupling flutter with different vibration frequencies is suggested.

Numerical studies have been conducted and the essential features of the bladerow coupling effects on the unsteady aerodynamic forces found from previous study on a model of three bladerows⁸⁾ are confirmed. In addition the following has been clarified:

1. The simultaneous effects of co-rotating upstream and downstream bladerows located at the next-to-the-next position significantly modify the aerodynamic response of oscillating blades.

2. The aerodynamic force induced by the oscillating bladerow is not negligible even on the bladerows remoter than the next-to-the-next bladerow.

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