Fluid Dynamic Forces Acting on Rectangular Plate with Hole in Stokes Flow

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The effect of a hole on aerodynamic forces acting on a rectangular plate in Stokes flow is shown. The effect does not depend strongly on the area or location of the hole, because the hydrodynamic forces near the hole are increased and because the effect of the hole on the hydrodynamic forces acting on the plate is decreased. This result can be used to decrease the mass of a cantilever in a micro-flow sensor without affecting its characteristics. When this micro-flow sensor is attached to a flapping wing, the decrease in sensor mass is important because inertia acting on the sensor causes deformation of the flapping wing. This result is important in understanding the flight of insects with bristled wings.

Key Words: Viscous Flows, Stokes Flow, Micro Flow Sensor

Micro-flow sensors composed of a cantilever have been developed.^{1,2)} Theoretical analysis to estimate hydrodynamic forces acting on a cantilever with thickness, (a rectangular solid) have clarified³⁾ the characteristics of such micro-flow sensors. At low Reynolds numbers ($Re \approx 1$), the difference between aerodynamic forces acting on a bristled wing and on a membranous wing with the same outline is small.⁴⁾ If aerodynamic forces acting on a cantilever in a micro-flow sensor are independent of hole size, we can decrease the mass of a micro-flow sensor without affecting its characteristics. When this sensor is attached to a flapping wing,²⁾ the decrease in sensor mass is important because inertia acting on the sensor causes deformation of the flapping wing. This paper shows how a hole in a plate without thickness affects the aerodynamic forces acting on the plate in Stokes flow. Moreover, the results clarify our understanding of the flight of an insect with bristled wings.^{5,6)}

1. Numerical Method

As shown in Fig. 1, the rectangular plate $L_X \times L_Y$ has a hole $\alpha L_X \times \alpha L_Y$. The centers of the plate and hole are located at $(0,0)$ and (X_h, Y_h) , respectively. The plate is considered in uniform flows U_X , U_Y , or U_Z . Note that the hydrodynamic forces in the X , Y , and Z directions acting on an object in Stokes flow are determined by the uniform flow velocity in each direction. The Reynolds number is defined as $Re_* = \rho U_* L_Y / \mu$ (ρ : density, μ : viscosity).

The Stokes flow around a rectangular plate with a hole in uniform flow was investigated by boundary-element analysis. The details of the analytical method are explained elsewhere.^{7,8)} Here, the method is explained simply. The velocity $u_i(x)$ at point x on the plate is satisfied by the following equation:

$$C_{ij}(x)u_i(x) = - \int H_{ij}(x, y)u_i(y) d\Gamma(y) + \int G_{ij}(x, y)t_i(y) d\Gamma(y), \quad (1)$$

where,

$$t_i = \left(-\frac{p\delta_{ij}}{\mu} + \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j,$$

$$G_{ij}(x, y) = \frac{1}{8\pi R} \left(\delta_{ij} + \frac{r_i r_j}{R^2} \right),$$

$$H_{ij}(x, y) = -\frac{3}{4\pi R^5} r_i r_j (r_i n_i),$$

$$r = x - y, \quad R = |r|, \quad C_{ij}(x) = \frac{1}{2} \delta_{ij}.$$

y , Γ , p , δ_{ij} , and n are a point on the plate, the plate surface area, the pressure, the Kronecker delta, and a unit outward normal on the plate surface, respectively. From the boundary condition, when the total flow velocity is 0 on the plate surface, $u_i(x)$ is given by U_X , U_Y , and U_Z . The plate surface is divided into many rectangular elements, and solving Eq. (1) gives the values of p and $\frac{\partial u_i}{\partial x_3}$ ($i = 1, 2$) for each element. Moreover, the aerodynamic forces acting on the plate are calculated from the values of p and $\frac{\partial u_i}{\partial x_3}$ ($i = 1, 2$) on each element.

For a rectangular plate without a hole, the force coefficients in the X , Y , and Z directions, C_X , C_Y , and C_Z , are defined by:

$$C_* = F_*/0.5\rho U_*^2 L_X L_Y, \quad (* = X, Y, Z). \quad (2)$$

Furthermore, the force coefficient in a Stokes flow can be expressed by:

$$C_* = k_*/Re_*. \quad (3)$$

Fig. 1. Rectangular plate with hole.

Fig. 2. Force coefficients k_* of rectangular plate without hole.

The * symbol indicates the direction of uniform flow and force acting on the plate.

2. Results

Figure 2 shows the values of k_* for a rectangular plate without a hole. These values agree with those obtained by extrapolating the curves explained elsewhere.³⁾ Figure 3 shows the ratio of the aerodynamic forces between rectangular plates with/without a hole. In this figure, the value of L_X/L_Y is 1, and the center of the hole agrees with the center of the plate: $(X_h, Y_h) = (0, 0)$. This figure shows that the ratios exceed 0.92 when α is smaller than 0.75. The effect of the hole on the aerodynamic forces on the plate is very small.

Calculations with $\alpha = 0.5$ when the hole's center is not at $(0, 0)$ were made for $L_X/L_Y = 1$. The X and Y coordinates of the hole's center are satisfied by $(-8/36)L_X \leq X_h \leq (8/36)L_X$ and $(-8/36)L_Y \leq Y_h \leq (8/36)L_Y$, respectively. Varying the position of the hole's center confirmed that the variation in aerodynamic forces acting on the plate is less than 2%. The effect of the location of the hole's center on aerodynamic forces on plates is very small.

Figure 4 compares $\Delta p/0.5\rho U_Z^2$ (Δp : pressure difference between upper and lower surfaces on $Z = 0$) between squares without a hole ($\alpha = 0$) and with a hole of $(X_h, Y_h) = (0, 0)$ and $\alpha = 0.5$. Figures 4(a) and (b) show the variation in $\Delta p/0.5\rho U_Z^2$ along $Y = \pm 0.375$ and $Y = \pm 0.0139$, respectively. The pressure on the region close to the hole of the square with the hole is larger than that on the corresponding region of the square without a hole.

Fig. 3. Ratios of aerodynamic forces acting on square plate without hole and on square plate with hole.

The location of the hole's center matches that of the square plate. The * symbol indicates the direction of uniform flow and force acting on the plate.

Fig. 4. Comparisons of $\Delta p/0.5\rho U_Z^2$ between square ($L_X = L_Y = 1$) without hole and square with hole for $(X_h, Y_h) = (0, 0)$ and $\alpha = 0, 0.5$. (a): variations along $Y = \pm 0.375$, (b): variations along $Y = \pm 0.0139$. Vertical lines at $X = \pm 0.25$ show the plate exists at $X \leq -0.25$, $X \geq 0.25$ along $Y = \pm 0.0139$ when $\alpha = 0.5$.

3. Conclusion

The effect of a hole on the aerodynamic forces acting on a rectangular plate in Stokes flow is shown. The effect does not depend strongly on the area or the location of the hole.

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