Advanced Guidance Scheme for Lunar Descent and Landing from Orbital Speed Conditions

By Ibrahim Mustafa MEHEDI and Takashi KUBOTA
Department of Electrical Engineering and Information Systems, The University of Tokyo, Tokyo, Japan

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Precise landing technology is one of the most important technologies for future lunar or planetary exploration missions. To achieve a precise landing, an advanced guidance scheme is necessary. This paper outlines a comparison of different solution methods for motion control equations utilized in guidance schemes for lunar descent, and proposes an advanced solution that allows a full depiction of descent vehicle motion from orbital states down to the final landing event. In the conventional solution methods, there exist some poor assumptions such as during descent, constant vertical gravitational acceleration is the only other force acting on the descent vehicle. This inadequate postulation limits the validity of the system solutions within a very low altitude terminal descent area; that is, close to the lunar surface. In this paper, an advanced descent solution is proposed where the centrifugal acceleration term is retained along with the gravitational acceleration term. It allows a complete representation of the descent module motion from orbital speed conditions down to the final landing state. Mathematical derivations of the new scheme are verified in terms of a conventional scheme, and comparative simulation results for a fully integrated solution, conventional schemes and a proposed advanced scheme are demonstrated to test the performance.

Key Words: Guidance, Lunar Descent, Centrifugal Acceleration

Nomenclature

\( u \): lander velocity vector magnitude or speed
\( g_l \): lunar gravitational acceleration
\( N \): ratio of thrust \( F \) and vehicle mass \( m \)
\( y \): real-time altitude of lander from lunar surface
\( x \): horizontal span
\( c \): cross range distance
\( y_l \): lunar radius
\( t \): time
\( \alpha \): velocity vector pitch angle relative to the local vertical
\( \beta \): angle of thrust vector relative to reverse direction of lander velocity
\( \psi \): cross range angle
\( \phi \): thrust roll angle

Subscripts

0: initial
D: descent

1. Introduction

Returning to the moon is a demanding issue. Many scientists and engineers have confirmed considerable interest in the past couple of decades.\(^{1-5,12-16}\) It is essential for a lunar lander to land vertically and softly on the lunar surface.\(^6\) Gravity-turn descent is one of the solutions for this purpose, and this type of descent technique requires that the lander thrust vector be oriented opposed to the velocity vector along the complete flight path of the vehicle.\(^7\) Using the inertial measurement unit, information about the velocity vector can be identified and inserted as an input of attitude controller that can maintain the thrust vector parallel to the velocity vector instantaneously but in the opposite direction, as shown in Fig. 1. The great benefit of using gravity-turn descent is to guarantee an upright landing and optimize fuel consumption.\(^8\)

The lunar guidance scheme takes a horizontally oriented spacecraft from orbital speeds at a point of hundreds of kilometers from the desired landing point to an almost vertical orientation and very low speed. Implemented guidance schemes for lunar landing date back to the Apollo era.\(^5\) Although the Apollo lunar descent guidance schemes worked well to meet the criteria of the 1960s, they can not fulfill the goal of lunar exploration that encompasses the desire to easily and cheaply explore many locations on the moon.

The primary task of the descent scheme is to easily and efficiently solve spacecraft motion equations that help to generate reference trajectories for lunar descent and landing. This paper proposes an advanced solution scheme for lunar descent equations to circumvent complexity. A fully integrated solution for spacecraft motion equations is indeed a time-consuming issue, and such solutions can’t be implemented using an onboard real-time trajectory generation algorithm. The Apollo target trajectory generation scheme is numerically complex and cumbersome. Therefore, it is necessary to find a qualitative solution instead of a numerical one. Apart from the Apollo solution, other researchers have proposed solution schemes for lunar descent equations. But the motion equations for spacecraft descent are that solved in conventional way have some limitations too. This
is because, in a conventional solution, the lunar surface is assumed to be a planar flat surface and the centrifugal acceleration term is ignored.\(^6,7,11\) Ignoring the centrifugal acceleration term during lunar descent, it is assumed that a constant and vertical gravitational acceleration is the only other force acting on the descent vehicle.\(^7\) This confines the vehicle to be landed precisely on the lunar surface. Moreover, since centrifugal forces are unaccounted for, the conventional solution method limits validity to regimes where the descent vehicle velocity is very small relative to the local orbital velocity. Therefore, it can only be used to describe terminal descent, when the vehicle has braked from orbital velocity and is close to the lunar surface. Consequently, the authors demonstrate an advanced descent solution method for a spherical homogeneous lunar surface where centrifugal forces are considered and descent can be initiated from orbital speed conditions. In this paper, some logical values are examined to determine a better approximation for the centrifugal acceleration term without ignoring it, but the gravity is assumed to be constant in magnitude. These assumptions are reasonable with the descent starting from vehicle orbit.\(^3\) Compared to the conventional descent method, the proposed advanced solution allows a full representation of descent module motion from orbiting conditions down to the final vertical landing state. To prove the significant improvement in the new solution, three steps are performed in this study: fully integrated solution, conventional solution and advanced solution.

2. Fundamentals of Lunar Descent

Fundamental three-dimensional equations of motion to describe the spacecraft proposition concerning a uniform sphere-shaped lunar body\(^\text{10}\) are divided into two parts. One is the equations of spacecraft motions for dynamic states, as follows:

\[
\begin{align*}
\ddot{u}(t) &= g_l \cos \alpha - N \cos \beta \\
\dot{\alpha}(t) &= \frac{1}{u} \left[ \left( \frac{u^2}{y + y_l} - g_l \right) \sin \alpha - N \sin \beta \cos \phi \right] \\
\dot{\psi}(t) &= \frac{1}{u \sin \alpha} \left[ N \sin \beta \cos \phi \right]
\end{align*}
\]

where \(u\) is the spacecraft velocity vector magnitude or spacecraft speed, \(g_l\) is the lunar gravitational acceleration, \(N\) is the ratio of thrust \(F\) and vehicle mass \(m\), \(\alpha\) is the pitch angle of the vehicle velocity vector relative to the local vertical axis, \(\beta\) is the angle of thrust vector relative to the reverse direction of spacecraft velocity, \(y\) is vehicle altitude from the lunar surface, \(y_l\) is the lunar radius, and \(\psi\) is the cross range angle.

The other part is the kinematic states to describe the fundamental equations of spacecraft motion. These are:

\[
\begin{align*}
\dot{y}(t) &= -u \cos \alpha \\
\dot{x}(t) &= u \sin \alpha \cos \psi \frac{y_l}{y + y_l} \\
\dot{c}(t) &= u \sin \alpha \sin \psi \frac{y_l}{y + y_l}
\end{align*}
\]

where \(x\) and \(c\) are the horizontal span and cross range distance, respectively.

2.1. Initial assumptions

The right-hand side of the spacecraft-governing equations are reduced to the function of velocity vector pitch angle, \(\alpha\). For this purpose, some reasonable assumptions are made regarding thrust to mass ratio, thrust vector angle and lunar gravitational acceleration force. To generate an ideal descent trajectory, it is rational to assume a constant value for \(N\) (i.e., \(F/m\)) and \(g_l\), and control input \(\beta\) is set to zero. But in the situation of constant thrust acceleration, \(m\) is not constant, and so \(F/m\) becomes variable. However, this error is removed by the real-time guidance algorithm. Therefore, using initial values for mass and gravity is a straightforward assumption for this solution. To facilitate the simplification of mathematical operation, roll control states are held to zero \((\phi(t) = 0\) and \(\phi(t) = 0\)). To activate the system as a planar motion and constrain motion to two dimensions, the initial states are initialized to zero \((c(0) = 0\), \(\psi(0) = 0\), and \(\psi(0) = 0\)). Consequently, the governing equations are reduced to their two-dimensional forms, where Eqs. (1) and (4) remain the same. The only changes observed are as follows:

\[
\begin{align*}
\ddot{\alpha}(t) &= \frac{1}{u} \left[ \left( \frac{u^2}{y + y_l} - g_l \right) \sin \alpha - N \sin \beta \right] \\
\dot{\alpha}(t) &= u \sin \alpha \frac{y_l}{y + y_l}
\end{align*}
\]

It is reasonable to assume that \(y \ll y_l\) in order that \(y_l/ y + y_l \approx 1\). Then, the equation for horizontal span becomes
\[ \dot{x}(t) = u \sin \alpha \]  

(9)

3. Fully Integrated Solution

To find the fully integrated numerical solutions for speed \( u \), time \( t \), horizontal distance \( x \) and vertical range \( y \) as a function of velocity vector pitch angle \( \alpha \) during powered descending phase, the authors performed the following mathematical derivations for simplification.

\[ \dot{u}(\alpha) = \frac{du}{dt} = \frac{u(g_1 \cos \alpha - N)}{(u^2 / \gamma_1 - g_1) \sin \alpha} \]  

(10)

then

\[ \frac{1}{u} \left( \frac{u^2}{\gamma_1} - g_1 \right) du = \frac{g_1 \cos \alpha - N}{\sin \alpha} \, d\alpha \]  

(11)

This can be integrated as:

\[ \int_{u_0}^{u} \frac{1}{u} \left( \frac{u^2}{\gamma_1} - g_1 \right) du = \int_{\alpha_0}^{\alpha} \frac{g_1 \cos \alpha - N}{\sin \alpha} \, d\alpha \]  

(12)

Then following equation can be obtained:

\[ \frac{u^2}{2\gamma_1} - u_0^2 / 2\gamma_1 - g_1 \ln(u/u_0) - g_1 \ln(\sin \alpha) - N \ln \left( 1 + \frac{\cos \alpha}{\sin \alpha} \right) = 0 \]  

(13)

Therefore, the equation for speed:

\[ u(\alpha) = \left[ -g_1 \gamma_1 W \right]^\frac{1}{2} \]  

(14)

where the Lambert \( W \) function is expressed as:

\[ W = -\frac{u_0^2 \sin \alpha - 2}{g_1 \gamma_1 \left( \frac{\sin \alpha}{\sin \alpha} \right)^{\frac{2}{N}}} \]  

(15)

Now the full numerical solution for descent time \( t_D \) as a function of velocity vector pitch angle \( \alpha \) can be obtained by integrating the equations with the help of Eqs. (1) and (10):

\[ i_D(\alpha) = \frac{dT}{d\alpha} = \frac{u}{d\alpha} \frac{du}{dT} = \frac{\dot{u}(\alpha)}{u(t)} \]  

(16)

Again, for vertical range and horizontal flight path distance, it can be written that

\[ \dot{y}(\alpha) = \frac{dy}{d\alpha} = \frac{dy}{dT} \frac{dT}{d\alpha} = \dot{y}(t) t_D(\alpha) \]  

(17)

substituting the values from Eq. (4):

\[ y(\alpha) = -u \cos(\alpha) t_D(\alpha) \]  

(18)

where \( u \) can be replaced from Eq. (14). For the solution of horizontal span as a function of velocity vector pitch angle \( \alpha \), the same procedure can be followed.

\[ \dot{x}(\alpha) = \frac{dx}{d\alpha} = \frac{dx}{dT} \frac{dT}{d\alpha} = \dot{x}(t) t_D(\alpha) \]  

(19)

From Eq. (9):

\[ \dot{x}(\alpha) = u \sin(\alpha) t_D(\alpha) \]  

(20)

4. Conventional Descent Solution

The traditional solution for descent is obtained by assuming that the lunar surface is similar to a plane, so that the lunar radius is \( \gamma_1 \rightarrow \infty \). Therefore, the centrifugal acceleration term is ignored in order to obtain \( u^2 / y + \gamma_1 = 0 \).

With this perimeter, Eq. (7) reduces to:

\[ \dot{\alpha}(t) = -\frac{g_1}{u} \sin \alpha \]  

(21)

This reduced equation can be used to obtain a single, distinguishable differential equation with \( \alpha \) as the self-regulating variable, such that:

\[ \dot{u}(\alpha) = \dot{u}(t) / \dot{\alpha}(t) = -\frac{u(g_1 \cos \alpha - N)}{g_1 \sin \alpha} \]  

(22)

then

\[ \frac{1}{u} \frac{du}{d\alpha} + \frac{\cos \alpha}{\sin \alpha} - N \frac{1}{g_1 \sin \alpha} = 0 \]  

(23)

At this time, Eq. (23) can be integrated to find the descent speed \( u \) as a function of velocity vector pitch angle \( \alpha \):

\[ u(\alpha) = u_0 \left[ \sin \alpha_0 / \sin \alpha \right] \left[ \tan(\alpha_0 / 2) \right]^{\frac{N}{\gamma_1}} \left[ \frac{N}{g_1 \sin(\alpha) - \cot \alpha} \right] \]  

(24)

Differentiating Eq. (24):

\[ u(\alpha) = \frac{du}{d\alpha} = u_0 \left[ \sin \alpha_0 / \sin \alpha \right] \left[ \tan(\alpha_0 / 2) \right]^{\frac{N}{\gamma_1}} \left[ \frac{N}{g_1 \sin(\alpha) - \cot \alpha} \right] \]  

(25)

Now the solution for time, vertical and horizontal ranges can be acquired utilizing the value of speed \( u \) in previously derived mathematical equations to obtain the descent trajectory in terms of the traditional descent solution. Therefore, descent time acquired equations are:

\[ t_D(\alpha) = \frac{dT}{d\alpha} = \frac{du}{d\alpha} \frac{du}{dT} = u_0 \left[ \sin \alpha_0 / \sin \alpha \right] \left[ \tan(\alpha_0 / 2) \right]^{\frac{N}{\gamma_1}} \left[ \frac{N}{g_1 \sin(\alpha) - \cot \alpha} \right] \]  

(26)
vertical range,

\[ y'(\alpha) = \frac{dy}{d\alpha} = \frac{dy}{dt^2} \left( \frac{d\alpha}{dt} \right) \]

and horizontal span,

\[ x'(\alpha) = \frac{dx}{d\alpha} = \frac{dx}{dt^2} \left( \frac{d\alpha}{dt} \right) \]

and horizontal span,

\[ x'(\alpha) = \frac{dx}{d\alpha} = \frac{dx}{dt^2} \left( \frac{d\alpha}{dt} \right) \]

\[ = \frac{u^2}{g_l} \cot \alpha = u_0^2 \left[ \tan(\alpha_0/2) \right]^{2N/\pi} \cot \alpha \]

4.1. Descent specifications

Along with the assumptions described in Section 2.1, other required descent specifications are assumed to integrate the developed equations and to compare the lunar descent scheme simulation results obtained by the fully integrated and conventional solutions. The Moon has no atmosphere and no approximation regarding the centrifugal acceleration term is taken into consideration for conventional descent illumination. Specific descent speculations are shown in Table 1.

4.2. Simulation results

Figure 3 shows a comparison of different trajectory responses for spacecraft descent to the lunar surface while the governing equations are solved using the complete integration method and conventional descent illumination. The computer simulation results for the fully integrated solution is assumed to be the ideal measurement for the lunar descent trajectory, and this ideal situation can not be expected in realistic cases. But the authors would be pleased if responses with minimal divergence are obtained. In these simulation results, it can be seen that the responses of the traditional descent solutions and fully integrated solutions are different for the equations of lunar module motion. It is noticeable that traditional descent solutions have the largest impact on the final altitude variation with respect to the fully integrated solution due to the fact that the centrifugal acceleration term is ignored. This fact proves that the conventional solution method limits validity to regimes where the descent vehicle velocity is very small relative to the local orbital velocity, and therefore, it can only be used to describe the terminal descent portion. Consequently, further analysis using the new advanced scheme for lunar descent and landing needs to be performed so that the entire trajectory, from orbital speed to the final landing event, can be evaluated qualitatively.

5. Advanced Lunar Descent Solution

5.1. Assumptions

To solve the same governing equations for the proposed advanced scheme, it is again necessary that the right-hand side of the equations be kept as a function of velocity vector pitch angle \( \alpha \). The assumption of a centrifugal acceleration term is newly considered for a homogeneous spherical lunar surface. The assumptions for mass and lunar gravity are identical to those in Section 2.1. But for the centrifugal acceleration term, a constant value \( \Gamma \), defined as the ratio between centrifugal acceleration and lunar gravitational acceleration, can be logically chosen. Though this is noticeably a varying value, during the reference trajectory generation phase, it is reasonable to consider it as an assumption at the initial stage because real-time guidance will compensate for the errors between the model and the environment. Therefore:

\[ \Gamma = \frac{u^2}{y + g_l} \]  

(29)

\[ \frac{u^2}{y + g_l} - g_l = -(1 - \Gamma)g_l \]  

(30)

5.2. Mathematical derivations

With these assumptions, and ensuring consistency with traditional lunar descent works, \(^7\), \(^10\), \(^11\) speed can be recognized by following differential equations formulated as a function of the velocity vector pitch angle \( \alpha \):

\[ \dot{u}(\alpha) = \frac{\dot{u}(t)}{\dot{\alpha}(t)} = u \left[ \frac{\frac{g_l \cos \alpha - N}{(1 - \Gamma)g_l \sin \alpha} \right] \]  

(31)

\[ \frac{da}{u} = \left[ \frac{\frac{g_l \cos \alpha - N}{(1 - \Gamma)g_l \sin \alpha} \right] d\alpha \]  

(32)

This Eq. (32) can now be directly integrated to obtain the descent velocity \( u \) as a function of the velocity vector pitch angle \( \alpha \):

\[ u(\alpha) = u_0\int_{\alpha_0}^{\alpha} \left[ \frac{\frac{g_l \cos \alpha - N}{(1 - \Gamma)g_l \sin \alpha} \right] d\alpha \]  

(33)

If

\[ \int_{\alpha_0}^{\alpha} \left[ \frac{g_l \cos \alpha - N}{(1 - \Gamma)g_l \sin \alpha} \right] d\alpha = \ln \left( \frac{\sin \alpha}{\sin \alpha_0} \right) \quad \frac{1}{\Gamma - 1} \]  

(34)

\[ + \ln \left( \frac{\tan \alpha}{\tan \alpha_0} \right) \quad \frac{1}{\Gamma - 1} \]  

Table 1. Lunar descent specifications.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lunar gravitational acceleration ( g_l )</td>
<td>1.623 m/sec(^2 )</td>
</tr>
<tr>
<td>Thrust to mass ratio ( N )</td>
<td>4 N/kg</td>
</tr>
<tr>
<td>Initial lander speed ( u_0 )</td>
<td>1688 m/sec</td>
</tr>
<tr>
<td>Initial velocity vector pitch angle ( \alpha_0 )</td>
<td>90 deg</td>
</tr>
</tbody>
</table>
then

\[ u(\alpha) = u_0 \left( \frac{\sin \alpha}{\sin \alpha_0} \right)^{(1-\tau)/(1-\tau_0)} \left( \frac{\tan \frac{\alpha_0}{2}}{\tan \frac{\alpha}{2}} \right)^{-\frac{\tau_0}{\tau}} \]  

so

\[ \tan(\alpha/2) = \frac{1 - \cos \alpha}{\sin \alpha} \]

Assume

\[ \tau = \frac{1}{1 - \Gamma} \quad \text{and} \quad \rho = N/g_0 \]

where \( \rho > 0 \), so that

\[ u(\alpha) = u_0 \left( \frac{\sin \alpha}{\sin \alpha_0} \right)^{-\tau} \left( \frac{1 - \cos \alpha_0}{\sin \alpha} \right)^{-\tau \rho} \]  

where \( \tau = 1/(1 - \Gamma) \) is a measure of the centrifugal acceleration term. Then, the solution for current speed obtains the shape:

\[ u(\alpha) = u_0 \left( \frac{\sin \alpha}{\sin \alpha_0} \right)^{-\tau(1+\rho)} \left( \frac{1 - \cos \alpha_0}{1 - \cos \alpha} \right)^{-\tau \rho} \]  

Fig. 3. Comparison of fully integrated solution to conventional solution: speed, time, vertical range and horizontal span.
Next, the time to go, horizontal span \( x(\alpha) \) and vertical range \( y(\alpha) \) are resolved in a manner identical to the conventional lunar descent solution, as follows:

\[
t_0(\alpha) = \frac{1}{\dot{\alpha}}(t) \\
= -\frac{\tau u_0}{g_1 \sin \alpha} (1 - \cos \alpha_0)^{-\tau_p} (\sin \alpha)^{-(\tau(1+\rho)-1)} G_0 \\
= G_{t_0} \left( \frac{\sin \alpha}{1 - \cos \alpha} \right)^{-(\tau(1+\rho)-1)} (1 - \cos \alpha)^{\tau_p}, \tag{38}
\]

where

\[
G_{t_0} = -\frac{\tau u_0}{g_1} (1 - \cos \alpha_0)^{-\tau_p} \left( \frac{\sin \alpha}{1 - \cos \alpha} \right)^{-(\tau(1+\rho)-1)} .
\tag{39}
\]

For horizontal span:

\[
\dot{x}(\alpha) = \dot{x}(t)/\dot{a}(t) \\
= -\frac{\tau u_0^2}{g_1 \sin \alpha} (1 - \cos \alpha_0)^{-2\tau_p} (\sin \alpha)^{-(2(1+\rho))} G_x \\
= G_x \left( \frac{\sin \alpha}{1 - \cos \alpha} \right)^{-2(1+\rho)} (1 - \cos \alpha)^{-2\tau_p} , \tag{40}
\]

where

\[
G_x = -\frac{\tau u_0}{g_1} (1 - \cos \alpha_0)^{-2\tau_p} .
\tag{41}
\]

For vertical range:

\[
\dot{y}(\alpha) = \dot{y}(t)/\dot{a}(t) \\
= -\frac{\tau u_0^2}{g_1 \sin \alpha} (1 - \cos \alpha_0)^{-2\tau_p} (\sin \alpha)^{-(2(1+\rho))} \cos \alpha G_y \\
= G_y \left( \frac{\sin \alpha}{1 - \cos \alpha} \right)^{-2(1+\rho)} \cos \alpha (1 - \cos \alpha)^{-2\tau_p} \sin \alpha , \tag{42}
\]

where

\[
G_y = \frac{\tau u_0}{g_1} (1 - \cos \alpha_0)^{-2\tau_p} .
\tag{43}
\]

6. Performance Test for Advanced Scheme

To integrate the above equations in a qualitative manner, the value for \( \tau \) must be an integer. This entails \( \tau = 1, 2, 3, 4, \ldots \) Instead of this solution, the ratio \( \Gamma \), mentioned in Section 5.1 can be chosen as some fractional value to make \( \tau \) an integer. But the authors found better results by directly taking logical integer values to obtain qualitative integration of the equations. Choosing a logical value directly for \( \tau \) proves to be more precise in approximation as well. To visualize the simulated results for the advanced lunar descent solution, different values \( 1, 2, 3, 4, \ldots \) for \( \tau \) are utilized in Eqs. (37), (38), (40) and (42), and these equations are numerically integrated with constant approximate values for \( g_1 \) and \( N \), whereas \( g_1 = 1.623 \text{ m/sec}^2 \) and \( N = 4 \text{ N/kg} \). Initial and final values for the velocity vector pitch angle \( \alpha \) are 90 [deg] and 0 [deg], respectively, while the initial speed \( u_0 \) is assumed to be the approximate orbital speed, 1688 m/sec. The influence of differing the constant \( \tau \) is demonstrated in Figs. 4(a), 4(b), 4(c), 4(d) and 4(e) for speed, time, vertical range and horizontal span, respectively. In contrast to the advanced solution, the fully integrated solution to Eqs. (1), (2), (4) and (5), and the traditional lunar descent solutions to Eqs. (25), (26), (27) and (28) are performed for comparison using the same approximations for \( \beta, g_1, N, \alpha \) and \( u_0 \) as used for Eqs. (37), (38), (40) and (42); while no estimations are made regarding the centrifugal acceleration term.

A comprehensive evaluation of this advanced solution using a traditional descent scheme, and a fully integrated solution for the motion equations of a lunar landing mission is conducted in this investigation. It can be noted that varying \( \tau \) has a reasonable impact on different responses for speed, time, vertical range and horizontal span. In particular, it directly influences the vertical range of the descent trajectory. It has already been mentioned that the trajectory responses for the fully integrated solution are models for any other solution method. The issue is to minimize the divergence gap between other methods and the fully integrated solution. From the assessment of the various values for \( \tau \), the authors feel that the value of \( \tau = 2 \) emerges to be a realistic number and improves on the different responses of the advanced solution for speed, time, vertical range and horizontal span when compared to conventional solutions. Additionally, accurate verification of the mathematical calculations conducted for the proposed advanced scheme was obtained by investigating the responses, as shown in Fig. 4. In the figures, it produces the exact same simulation results are obtained for the conventional descent solution and the solution choosing a value of \( \tau = 1 \). Furthermore, the same assumption for conventional descent solutions is reproduced with a value of \( \tau = 1 \) in Eq. (30), in which the centrifugal acceleration term is ignored.

7. Conclusion

The conventional lunar descent and landing problem has been advanced to allow the accurate representation of lunar descent from orbital conditions. Finding a reasonable assumption for the lunar surface and centrifugal acceleration, it significantly advances the sphere of validity of the traditional gravity-turn solution from low-velocity terminal descent to a complete descent from orbital conditions. The accessibility of the descent speed, time, vertical range and horizontal span as a function of the velocity vector pitch angle can be utilized to lessen the computational burden on real-time lunar descent guidance schemes for future landing missions.
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References


Fig. 4. Comparison of advanced solution to fully integrated solution and conventional solution: speed, time, vertical range and horizontal span.