Adjustable Adaptive Fuzzy Attitude Control 
using Nonlinear SISO Structure of Satellite Dynamics

By Morteza MORADI,1) Reza ESMAELZADEH2) and Ali GHASEMI3)

1)Islamic Azad University, Nowshahr Branch, Nowshahr, Mazandaran, Iran
2)Faculty of Aerospace Engineering, Malek Ashtar University of Technology, Tehran, Iran
3)Department of Electrical and Computer Engineering, Safashahr Branch, Islamic Azad University, Safashahr, Iran

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This paper presents a method for three-dimensional attitude stabilization of a satellite. The pitch loop of the satellite is controlled by a momentum wheel; whereas the roll/yaw loops are stabilized using two magnetic torques along their respective axes. In order to design an efficient controller, the stability conditions are considered based on a nonlinear model of system. An adjustable adaptive fuzzy system is proposed as the method to design the controller. The span of membership functions are tuned using errors of fuzzy inputs with respect to their references. Results show that fuzzy sets cover all variations of fuzzy inputs and optimal fuzzy output is gained. The Lyapunov synthesis method is used to prove the stability of the closed-loop system. The efficiency of the controller in converging of the position error to close to zero is also shown using some numerical simulations.

Key Words: Adjustable Adaptive Fuzzy Attitude Control, Magnetic Torque, Momentum Wheel, Nonlinear Systems, Satellite Attitude Control

1. Introduction

Many valuable controlling methods have been developed since the first satellite was launched in 1957. Generally, all these techniques can be classified into active and passive. For active techniques, some of the well-known examples are micro-thrusters, momentum wheels, control momentum gyros and the Earth’s magnetic field. On the other hand, spin stabilization and aerodynamics are kinds of passive control. A hybrid control system is employed in this paper. On the X axis, momentum wheel is used; on the Y, Z axes magnetic field is used.

Magnetic control systems are relatively lightweight and their low power usage makes them inexpensive.1–3) For bias momentum satellites, union control schemes that use bias momentum wheels and magnetic torque rods have been developed.4–6) Some researchers have developed spin control7) and gravity-gradient control.8) Using the periodic nature of the satellite system to design a periodic controller has also been considered. In order to stabilize this periodic linear time-varying (LTV) system, some tools have been developed to extend the $H_{\infty}$ synthesis problem while some mixed $H_2$ and $H_{\infty}$.9,10)

Sometimes, designers focus on using solar radiation pressure (SRP) for attitude control of satellites. Using SRP for attitude control of high-altitude satellites and interplanetary probes has been proposed.11–14) In some cases, proportional-differential (PD) controllers are used and stability is proven via linear modeling.15–17) Others have used adaptive and optimal control for better application18–20) and fuzzy systems have also been used for satellite attitude control.21–23) In our case, an adaptive fuzzy controller24–26) is employed. Two major problems are considered in the design process.

Firstly, in the real world, when satellite systems are active, some nonlinear properties can cause unforeseen problems such as instability or saturation. In addition, in this paper, the Earth’s magnetic field is used for attitude control where there are high nonlinear properties (see Eq. (A.7). To solve these problems, the nonlinear model of system and nonlinear properties of the magnetic field are considered and the whole model of the system is converted to three canonical nonlinear SISO systems. Then, the controller is designed for this new model. Therefore, unknown and unforeseen variations in the system are considered in design of the controller.

Secondly, designing a fuzzy system is more difficult if there is no information regarding the domain of the variations of the system parameters. Fuzzy sets should cover all variations of fuzzy inputs so that the optimal control signal results in output of the fuzzy system. Sometimes, the domain of the variations is large. Thus, the fuzzy sets with large span are needed. If we use the fixed structure fuzzy system, when the domain of variations becomes small, because this domain is small with respect to the large span of fuzzy sets, at all times, only specific rules and membership functions in each fuzzy sets are employed. Therefore, the output of fuzzy system is approximately constant. A tuning algorithm is proposed to solve this problem. The span of membership functions are tuned using the absolute value of errors.27) To compute the magnetic field, the Earth’s magnetic field $B$ is assumed to be a dipole. The variations in the field are considered without the Earth’s rotation and orbit precession.
2. System Model and Equations

The system model is shown in Fig. 1. The centers of $S$ and the satellite must be the same. $X_s, Y_s, Z_s$ represent the orbital reference frame normal to the orbital plane, local vertical and the third axis of this right-handed frame. $XYZ$ are body-fixed coordinate axes, $\theta$ is the true anomaly (elliptic orbit) or angle (circular orbit) that shows the position of the satellite in orbit. $(\alpha, \gamma, \varphi)$ are Euler angles that are made by rotating about $X_s, Y_s, Z_s$. Euler’s equations of motion can be applied to the satellite as\(^{4,15}\)

$$
\mathbf{H} + \omega_s^2 \mathbf{H} = 3 \left( \frac{\mu}{R^3} \right) \mathbf{C}_e \mathbf{I}_s + \mathbf{T}_c
$$

in which, $h, \omega_s$, $R$ and $I_s$ are the momentum wheel, angular velocity of the satellite, orbital radius and inertia matrix, respectively, and $\mu = GM_E$, where $G$ and $M_E$ are universal gravitational constant and Earth’s mass, respectively.

$$
\begin{bmatrix}
I_{11} & -I_{12} & -I_{13} \\
-I_{12} & I_{22} & -I_{23} \\
-I_{13} & -I_{23} & I_{33}
\end{bmatrix}
$$

where $\omega_s = \left[ \omega_x, \omega_y, \omega_z \right]^T = \left[ \theta + \dot{\alpha}, \sin \gamma \sin \gamma, \cos \gamma \cos \gamma \right]^T$ and $\omega_s^2 = \left[ \omega_x^2, \omega_y^2, \omega_z^2 \right]^T = \left[ 0, -\omega_y^2, -\omega_z^2 \right]^T$.

$$
\mathbf{C}_e = \begin{bmatrix}
\cos \alpha \sin \gamma \cos \gamma + \sin \alpha \sin \gamma \\
\cos \alpha \sin \gamma \cos \gamma - \sin \alpha \sin \gamma \\
\cos \alpha \sin \gamma \cos \gamma - \sin \alpha \sin \gamma
\end{bmatrix}
$$

The orbital radius, rate and acceleration can be calculated by:

$$
R = \frac{a(1 - e^2)}{1 + e \cos \theta}, \quad \dot{\theta} = \frac{\sqrt{\mu a(1 - e^2)}}{R}, \quad \ddot{\theta} = \frac{2 \mu}{R^3} e \sin \theta
$$

where $a$ and $e_s$ are the semimajor axis and orbital eccentricity, respectively. To conveniently compute and simulate our model, $r$ is replaced by $\theta$:

$$
\Phi = [\alpha, \gamma, \varphi]^T
$$

$$
\dot{\Phi} = \frac{\mu}{R^3} (1 + e_s \cos \theta) \Phi'' - (2 e_s \sin \theta) \Phi'
$$

where $(\cdot)'$ and $(\cdot)''$ are first- and second-order derivatives with respect to $\theta$. Replacing Eq. (3) into Eq. (1), the Euler’s

$$
M(\Phi, \theta, \dot{\gamma}, \dot{\varphi}) = N(\Phi, \dot{\Phi}^\prime, \theta, e_s, I_s) = T_c
$$

in which $M$ is a $3 \times 3$ matrix, $N$ and $T_c$ are $3 \times 1$ matrices and $T_c$ is the input torque. Considering Eq. (B.13), Eq. (4) can be rewritten as a state space model by defining the state vector:

$$
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} =
\begin{bmatrix}
\alpha \\
\gamma \\
\alpha' \\
\gamma' \\
\varphi \\
\varphi'
\end{bmatrix}
$$

$$
\begin{bmatrix}
x_1' \\
x_2' \\
x_3' \\
x_4' \\
x_5' \\
x_6'
\end{bmatrix} =
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
$$

$$
\begin{bmatrix}
x_1'' \\
x_2'' \\
x_3'' \\
x_4'' \\
x_5'' \\
x_6''
\end{bmatrix} =
\begin{bmatrix}
x_4 \\
x_5 \\
x_6 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
$$

where

$$
\begin{bmatrix}
x_1'' \\
x_2'' \\
x_3'' \\
x_4'' \\
x_5'' \\
x_6''
\end{bmatrix} =
\begin{bmatrix}
f_4(\Phi, \Phi', \theta, I_s, \mu) - g_4(\Phi, \theta, I_s) h_x' \\
f_5(\Phi, \Phi', \theta, I_s, \mu) - g_5(\Phi, \theta, I_s) m_y \\
f_6(\Phi, \Phi', \theta, I_s, \mu) - g_6(\Phi, \theta, I_s) m_z
\end{bmatrix}
$$

In which $g_4, g_5, g_6 > 0$, as mentioned in Eq. (B.10). Equation (6) can be supposed to include three different SISO systems that work independently with unknown functions $f_4, f_5, f_6, g_4, g_5, g_6$. They can be written as:

$$
\begin{bmatrix}
x_1'' \\
x_2'' \\
x_3'' \\
x_4'' \\
x_5'' \\
x_6''
\end{bmatrix} =
\begin{bmatrix}
x_4 \\
x_5 \\
x_6 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
$$

$\begin{bmatrix}
x_1'' \\
x_2'' \\
x_3'' \\
x_4'' \\
x_5'' \\
x_6''
\end{bmatrix} =
\begin{bmatrix}
x_4 \\
x_5 \\
x_6 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
$$

$\begin{bmatrix}
x_1'' \\
x_2'' \\
x_3'' \\
x_4'' \\
x_5'' \\
x_6''
\end{bmatrix} =
\begin{bmatrix}
x_4 \\
x_5 \\
x_6 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}$

Figure 2 shows the model of the system in axes $l = X, Y, Z$ and the control loop. The state vector of the system is compared with the reference vector and the error vector is gained. Using the error vector and second-order derivative of reference, $v_l$ is produced. Control signal $u$ is used to pro-
duce the input torque. In addition, because of the nondiagonal elements of the Earth magnetic matrix \( C_B \) that is computed in Eq. (A.7), there are undesired torques. To solve this problem, the interaction torques are computed (see Eq. (B.11)) and are used as one of the fuzzy inputs. Using the interaction torques as fuzzy input, the controller has better application to produce the control signal. To design the controller, each axis is considered as a SISO system that is demonstrated in the center of Fig. 2. The new model is then used to design the adjustable adaptive fuzzy controller presented in this paper.

3. Design of the Controller

From Eqs. (7)–(9), we have three SISO systems. At first, the design process is considered for a SISO system. Then, three independent controllers can be employed to control the axes of the satellite. Consider a SISO nonlinear system that is described by the following differential equation:

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x, \Theta) - g(x, \Theta)u \\
\end{aligned}
\]

in which \( y \in \mathfrak{Y} \) is the measured output, \( u \in \mathfrak{U} \) is the control signal, \( x = [x_1, x_2]^T \) is a state vector and \( \Theta \) is an unknown function. \( \mathfrak{Y} \) includes all the variables that vary \( f(x, \Theta), g(x, \Theta) \) and make them unknown functions. The state vector is assumed to be available for measurement.

A zero-order Takagi-Sugeno fuzzy system with point fuzzification method, Mamdani product type inference, and center-average defuzzification approach is used. Let us define \( M(a, b; X) \) as a nonzero membership function in the interval \( X \in (a, b) \) and zero in \( X \notin (a, b) \). Consider the \( i \)-th rule of the proposed fuzzy system as:

\[ R_i^0: \text{if } X_{i1} \text{ is } A_{1i}', \text{ and } X_{i2} \text{ is } A_{2i}', \text{ and } X_{i3} \text{ is } A_{3i}' \text{, and } X_{i4} \text{ is } A_{4i}; \text{ then } y_i = \theta_i \]

in which \( X_i = [X_{i1}, X_{i2}, X_{i3}, X_{i4}] \in U \subset \mathfrak{Y}^4 \) and \( y_i \in V \subset \mathfrak{Y} \) are the crisp input and output. In the simulation section, two kinds of fuzzy systems are considered, one with a fixed structure and the other with an adjustable structure (proposed method). For the fixed structure, on the \( l \) (\( l = X, Y, Z \)) axis we have \( X_{il} = [e_{1l}, e_{2l}, v_l, T_{int,l}] \), where \( e_{1l}, e_{2l} \) are state errors, \( v_l \) is computed in Eq. (14) and \( T_{int,l} \) is gained from Eq. (B.11). For the adjustable structure, state variables (angle and its derivative) on each axis are used instead of \( e_{1l}, e_{2l} \). \( A_j \)s are fuzzy sets with membership function \( \mu_{A_j} (X_j) = M(a_{j1}, a_{j2}; X_j) \) for some \( \alpha_{j1} < \alpha_{j2}, i = 1, 2, \cdots, m \) and \( j = 1, 2, \cdots, n, m \) is the number of rules, \( n \) is number of fuzzy inputs and \( \Theta = [\theta_1, \theta_2, \cdots, \theta_m]^T \) is the output of the rules. On each axis we have \( m = 81 \) and \( n = 4 \). Then, the output of a Takagi-Sugeno fuzzy system is defined as \( \dot{y} := \hat{f}(X_t, \Theta) \) and is obtained as a weighted average of the rule outputs by the following equation:

\[
\dot{y} := \hat{f}(X_t, \Theta) = \sum_{i=1}^{81} \theta_i \mu_i(X_t) = \sum_{i=1}^{81} \frac{\theta_i \xi_i(X_t)}{\sum_{i=1}^{81} \mu_i(X_t)}
\]

where

\[
\xi_i(X_t) = \mu_i(X_t) \prod_{j=1}^{m} \mu_j(X_{kj})
\]

This class of fuzzy logic systems has universal approximation properties, i.e., it is able to approximate any continuous function with an arbitrary accuracy. If \( f(x, \Theta) \), \( g(x, \Theta) \) are supposed to be known functions, for Eq. (10), the ideal input of system can be written as:

\[
u^* = \frac{1}{g(x, \Theta)} \left( f(x, \Theta) - k^T e - r^{(2)} \right)
\]

in which \( r^{(2)} \) is the second-order derivative of the reference signal, \( e = [e, \ddot{e}]^T \), \( k = [k_1, k_2]^T \), \( e = r - y \), \( r = [r, \dot{r}]^T \). Substituting Eq. (12) into Eq. (10) results in:

\[
e^{(2)} = -k^T e = -k_1 e - k_2 \dot{e}
\]

For Eq. (10), the fuzzy inputs for adjustable structure are chosen as:

\[
X_l = [x, v, T_{int-g}] \quad X_l \in \mathcal{X}_l, \quad v = k^T e + r^{(2)}
\]

in which \( T_{int-g} \) is generally defined as the interaction torque that affects Eq. (10). Consider \( f(x, \Theta), g(x, \Theta) \) not to be known functions. A fuzzy logic controller in the form of Eq. (11) is applied to approximate \( u^* \). The input can be defined as:
\[ u = \hat{u}(X_t, \theta) = \sum_{i=1}^{81} \beta_i \xi_i(X_t) \]  
(15)

Substituting Eq. (15) in Eq. (10) and subtracting Eq. (12):

\[ e^{(2)} = -k^T e + g(x, 3)u^* - g(x, 3)\hat{u}(X_t, \theta) \]  
(16)

An arbitrary \( \varepsilon^* \) can be chosen which results in \( \xi(X_t) = [\xi_1(X_t), \xi_2(X_t), \ldots, \xi_{81}(X_t)]^T \), and the ideal parameter vector \( \theta^* = [\theta_1^*, \theta_2^*, \ldots, \theta_{81}^*]^T \) so that:

\[ g(x, 3)u^* - g(x, 3)\hat{u}(X_t, \theta) = \sum_{j=1}^{81} c^j (\theta_j^* - \theta_j)\xi_j(X_t) + \varepsilon \]  
(17)

in which \( |\varepsilon| \leq \varepsilon^* \) and \( c^j \) are positive constants. Considering \( \theta = (\theta^* - \theta) \), \( C = diag([c^1, c^2, \ldots, c^8]) \) from Eq. (16) results in:

\[ e^{(2)} = -k^T e + \theta^T C\varepsilon(X_t) + \varepsilon \] 
\[ \Rightarrow \dot{e} = A_r e + b_r (\theta^T C\varepsilon(X_t) + \varepsilon) \]  
(18)

in which \( A_r, b_r \) are defined as:

\[ A_r = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}, \quad b_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  
(19)

\( k \) can be chosen so that \( A_r \) becomes stable. Therefore, a positive definite symmetric \( 2 \times 2 \) matrix \( p \) can be found to satisfy the Lyapunov equation as:

\[ A_r^T p + pA_r = -Q \]  
(20)

in which \( Q \) is an arbitrary \( 2 \times 2 \) positive definite matrix and is chosen so that \( \lambda_{\min}(Q) > 1 \).

**Lemma.** Tuning \( \theta_i \) by following the \( \sigma \)-modification robust adaptive law bounds the error:

\[ \dot{\theta}_i = -\gamma e^T p_b \xi_i(X_t) - \sigma \theta_i \]  
(21)

**Proof.** Consider the Lyapunov function candidate

\[ V = \frac{1}{2} e^T p e + \frac{1}{2\gamma} \theta^T C \theta \]  
(22)

\[ V = \frac{1}{2} e^T p e + \frac{1}{2} e^T p e + \frac{1}{2\gamma} \theta^T C \theta + \frac{1}{2\gamma} \theta^T C \theta \]  
(23)

Substituting Eq. (18) into Eq. (23), \( \dot{V} \) is obtained as:

\[ \dot{V} = \frac{1}{2} e^T A_r^T p e + e^T p A_r e + \frac{1}{\gamma} \theta^T C \theta \] 
\[ + e^T p b \theta^T C \xi(X_t) + e^T p b \varepsilon \]  
(24)

Applying adaptive law of Eq. (21) changes \( \dot{V} \) to:

\[ \dot{V} = -\frac{1}{2} e^T Q e - \theta^T C e \theta + \theta^T C \theta \] 
\[ + \theta^T C e \theta + \theta^T C \varepsilon \theta + \theta^T p b \varepsilon \]  
(25)

Using the following equation:

\[ \theta^T C \theta = \frac{1}{2} \theta^T C \theta - \frac{1}{2} \theta^T C \theta \]  
\[ e^T p b \varepsilon \leq |k_e p_b e^*|, \quad \lambda_{\min}(Q) \| e \|^2 < e^T Q e \]  
(26)

in which \( k_e, \varepsilon^* \) are arbitrary positive constants that satisfy \( k_e > |e|, \forall t, \varepsilon^* > |\varepsilon| \), the following inequality is gained:

\[ \dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) \| e \|^2 - \frac{\sigma}{2} \theta^T C \theta + w \]  
(27)

\[ w = \frac{\sigma}{2} \theta^T C \theta + |k_e p_b e^*| \]

It is easy to find a \( \eta(\eta > 0) \) so that it satisfies the following inequalities:

\[ \lambda_{\min}(Q) > \lambda_{\max}(Q), \quad \sigma > \frac{\eta}{\sqrt{v}} \]  
(28)

\( V \) can be calculated as:

\[ \dot{V} \leq -\frac{1}{2} \lambda_{\max}(p) \| e \|^2 - \frac{\eta}{2\gamma} \theta^T C \theta + w \leq -\eta V + w \]  
(29)

\[ V(t) \leq e^{-\eta t} \left( V(0) - \frac{w}{\eta} \right) + \frac{w}{\eta} \]  
(30)

\[ \Rightarrow V(t) \leq \max \left( V(0), \frac{w}{\eta} \right) \forall t > 0 \]

Substituting Eq. (30) into Eq. (22):

\[ \| e \| \leq \sqrt{2 \max \left( V(0), \frac{w}{\eta} \right)} \forall t > 0 \]  
(31)

and using

\[ e^T p b \varepsilon \leq \frac{1}{2} \| e \|^2 + \frac{1}{2} \| p b \| \| e^* \|^2 \]  
result in:

\[ \dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) \| e \|^2 - \frac{1}{2} \| e \|^2 + \rho \] 
\[ \rho = \frac{\sigma}{2} \theta^T C \theta^* + \frac{1}{2} \| p b \| \| e^* \|^2 \]  
(32)

Since \( \lambda_{\min}(Q) > 1 \), Eq. (32) implies that \( \dot{V} \) is negative when

\[ \frac{1}{2} (\lambda_{\min}(Q) - 1) \| e \|^2 > \rho \]

This makes the system ultimately upper bound. Therefore, \( e(t) \) converges to compact set \( \Omega_e \) in a finite time:

\[ \Omega_e = \left\{ e(t) \mid \| e(t) \| \leq \sqrt{\frac{2\rho}{\lambda_{\min}(Q) - 1}} \right\} \]  
(33)

4. Fuzzy System Tuning

The main problem in designing of a fuzzy system is how the left, center and right points of membership functions are
obtained. The new constant parameter is tuned automatically and a good tracking possibility is
error value, the span of introduced membership functions
/\!C!12
\textit{described with a triple above-mentioned problem is solved.}

span of a membership function is presented so that the
used. In this section, the proposed method for tuning the
proposed method for tuning the fuzzy inputs. This process needs information about the var-

designed till the span of fuzzy set covers all variations of fuzzy inputs. This process needs information about the var-
ables. Therefore, other methods and devices should be

Consider the triangular membership function that is
described with a triple \(\{\beta_1 \Delta, \beta_2 \Delta, \beta_3 \Delta\}\) for \(i = 1, 2, 3\)
(i is number of membership functions for each fuzzy set; in this paper three membership functions are used for each fuzzy set) which specifies the lower, center and upper bounds of each triangular membership function, respectively, as shown in Fig. 3. The span of the membership functions can be improved by adjusting \(\Delta\). In each axis, we employ four fuzzy sets including three membership functions for each one. For convenience in notation, the left, center and right points of the triangular membership functions in each axis can be described in a compact form as:

\[
zd = \begin{bmatrix}
\beta_{1d} \Delta, & \beta_{12d} \Delta, & \beta_{13d} \Delta \\
\beta_{21d} \Delta, & \beta_{22d} \Delta, & \beta_{23d} \Delta \\
\beta_{31d} \Delta, & \beta_{32d} \Delta, & \beta_{33d} \Delta
\end{bmatrix}, \quad d = 1, 2, 3, 4
\]  \hspace{1cm} (34)

where \(d\) is number of fuzzy sets. Each row of \(zd\) shows the left, center and right points of a membership function. There are reference signals \(r_1, r_2, r_3\) and \(r_4\) for fuzzy inputs. We define \(e_{1i} = r_1 - x_i, \quad e_{2i} = r_2 - x_i, \quad e_{3i} = r_3 - v, \quad e_{4i} = r_4 - T_{int - \phi}\). In this paper, \(r_1 = r_2 = r_3 = r_4 = 0\). Considering parameters \(1 \leq L_1, 0 < L_2 \leq 1, 1 \leq b_1,\) and \(0 < b_2 \leq 1\), the proposed method to tune \(\Delta\) is:

\[
\Delta_{\text{new}} = \begin{cases}
L_1 \Delta_{\text{old}} & \text{if } |e_i| > b_1 \Delta_{\text{old}} \\
L_2 \Delta_{\text{old}} & \text{if } |e_i| > b_2 \Delta_{\text{old}} \text{ and } \Delta_{\text{old}} > \Delta_{\text{min}} \\
\Delta_{\text{old}} & \text{else}
\end{cases}
\]  \hspace{1cm} (35)

It is obvious that if \(\Delta\) is selected as a factor of the absolute error value, the span of introduced membership functions is tuned automatically and a good tracking possibility is obtained. The new constant parameter \(\Delta_{\text{min}}\) is defined to prevent \(\Delta\) from becoming very small. Therefore, for very small error \(\Delta\) will be constant all the time. The main advantage of using this method is that it guarantees that span of fuzzy sets cover any unforeseen variations in fuzzy inputs. Furthermore, the initial value of \(\Delta\) is chosen arbitrarily and doesn’t need any information to choose right, center and left points of membership functions.

5. Simulation

In order to study the performance of the proposed controller, the closed-loop system is numerically simulated using Eqs. (7)–(9) and (15). At first, we simulate the closed-loop system with the fixed-structure fuzzy system. Then, we show that the adjustable structure has better application with respect to the fixed structure.

5.1. Fixed-structure fuzzy system

In this part, the fixed-structure fuzzy system is considered. The parameters of controllers and membership functions are defined as:

Parameters for two simulations
\[
\begin{align*}
\alpha &= 51^\circ, & \gamma &= 45^\circ, & \varphi &= 45^\circ, & \alpha' &= 0.01 \\
\gamma' &= 0.015, & \varphi' &= 0.005, & \gamma_z &= 300, & \gamma_y &= 700 \\
I &= [250, 2.5, -2; 2.5, 315, -3; -2, -3, 318] \\
\gamma_z &= 700, & R_u &= 1000, & e_r &= 0.3, & h_n &= 1 \\
-100 < m_y, & m_z < 100, & p_1 &= [50, 7, 7, 3] \\
p_2 &= [90, 25, 25, 10], & p_3 &= [50, 15; 15, 10] \\
b_i &= [0; 1]
\end{align*}
\]

Regarding the matrix \(zd\) in section 4, the left, center and right points of membership functions of the fixed-structure fuzzy sets for all axes are defined as (each row of matrices has left, center and right points of membership function):

\[
\begin{align*}
m_{f_{11}} &= [-2; -0.5; -0.5; -1; 0; 1; -0.5; 0.5; 1] \\
m_{f_{12}} &= [-5; -2; 1; -1; 0; 1; -1; 2; 5] \\
m_{f_{13}} &= [-5; -2; 1; -1; 0; 1; -1; 2; 5] \\
m_{f_{1n}} &= [-50; -20; 10; -10; 0; 10; -10; 20; 50]
\end{align*}
\]

The attitude response is shown in Fig. 4. The domain of response in the first axis after 0.5 orbits is 0.1 deg. The responses of the \(Y, Z\) axes vary with large domain that is not good. After one orbit, the domain of \(\varphi\) is 1.2 deg and \(\gamma\) is 0.9 deg. These variations should be omitted. In Fig. 5, the velocity responses of axes are shown. On the \(X\) axis, the domain of variations after 0.5 orbits became 0.001. However, on the \(Y, Z\) axes, the domain of variations is large. On the \(Y\) axis, after one orbit it is 0.02 and on the \(Z\) axis it is
0.03. The magnetic control signals are shown in Fig. 6. The variations of $m_y$ are inside the constraint range. The domain of $m_z$ at initial times of simulation is out of bounds. Both control signals swing because there exist variations in system outputs. The variations of the momentum wheel are shown in Fig. 7. The variations after 0.4 orbits become almost constant.

These figures show that the fixed-structure fuzzy system cannot produce a suitable control signal. As a result, after one orbit, there still exist variations in outputs. To solve this problem and produce suitable responses in output, the proposed method is employed in the next simulation.

5.2. Adjustable-structure fuzzy system

The tuning coefficients of the fuzzy membership functions are chosen as:

$b_1 = 1.5, \quad b_2 = 0.3, \quad L_1 = 1.1, \quad L_2 = 0.57, \quad \Delta_{\min} = 0.3$

The $z_d$ matrices for the three axes are defined as:

$z_d - x_{(i)} = \begin{bmatrix} -2\Delta & -0.2\Delta & 0.5\Delta \\ -\Delta & 0 & \Delta \\ -0.5\Delta & 0.2\Delta & 2\Delta \end{bmatrix}$

$z_d - T_{\text{ax}, i} = \begin{bmatrix} -2\Delta & -\Delta & 0 \\ -\Delta & 0 & \Delta \\ 0 & \Delta & 2\Delta \end{bmatrix}$. \quad l = X, Y, Z

Using this tuning algorithm for fuzzy membership functions, it is not necessary to have knowledge about the left, center and right points of membership functions because the span of membership functions are tuned by the error domain. Results of the simulation for attitude responses are shown in Fig. 8. Employing the adjustable fuzzy system, the attitude responses are smoother than for the fixed structure. Additionally, the settling time is 0.75 orbits. After 0.8 orbits the attitude responses are 0.1 deg for the $Y, Z$ axes and 0.02 deg for the $X$ axis.

The velocity responses for the three axes are shown in Fig. 9. At initial times of simulation, the domains of veloc-
ity responses are large. However, after 0.4 orbits, the domains become less than 0.02. Comparisons between Figs. 9 and 5, and 4 and 8 show that the proposed method could eliminate the variations in outputs of the system. The control signals for the Y, Z axes are shown in Fig. 10. Unlike control signals in Fig. 6, the variations of the control signals in Fig. 10 are eliminated and after 0.3 orbits became less than 5 Am\(^2\). The variations of the momentum wheel are shown in Fig. 11. Regarding Figs. 8–11 with respect to Figs. 4–7, the figures show the ability of the proposed method to make the three axes stable.

6. Conclusion

In this paper, a hybrid magnetic-momentum wheel attitude control problem of a three-axis stabilized satellite was considered, and an adjustable direct adaptive fuzzy logic controller was employed. The Lyapunov synthesis equation was used to prove the stability of the closed-loop system. Simulations were considered for fixed and adjustable fuzzy systems. The numerical simulations showed the ability of the proposed method to eliminate the undesired torques that were caused by nondiagonal elements of the magnetic matrix and also the ability to control the system in the presence of eccentricity.

References

the orbital reference frame is denoted by $\vec{B}_0$ and can be calculated as:

$$\vec{B}_0 = \frac{\mu_f}{R^2} \left[ \frac{(\cos i_0)j_0 - (2 \sin \theta \sin i_0) i_0 + (\cos \theta \sin i_0) k_0}{(2 \sin \theta \sin i_0) i_0 + (\cos \theta \sin i_0) k_0} \right]$$

(A.1)

where $i_0$, $j_0$, $k_0$ are unit vectors of the orbital reference frame, $i_m$ is the satellite orbital inclination with respect to the magnetic equator and $\mu_f$ is the Earth’s magnetic dipole strength. The satellite is considered in the equatorial orbit. In this paper, it is assumed that $i_m = 0$ and therefore the magnetic field in the orbital reference frame can be rewritten as:

$$\vec{B}_0 = \vec{B}_0, \quad B = \frac{\mu_f}{R^2}$$

(A.2)

Using a direct cosine matrix $C$, $\vec{B}_b$ in the body fixed reference frame can be obtained as:

$$\vec{B}_b = C\vec{B}_0$$

$$\Rightarrow \vec{B}_b = \begin{bmatrix} B_{bx} \\ B_{by} \\ B_{bz} \end{bmatrix} = \begin{bmatrix} \cos \phi \cos \gamma & 0 & -\sin \gamma \\ -\sin \phi & 1 & 0 \\ \cos \phi \sin \gamma & 0 & \cos \gamma \end{bmatrix} \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$$

(A.3)

and the magnetic torque can be obtained as:

$$T_m = \vec{M} \times \vec{B}_0$$

where $T_m$ is magnetic torque, $\vec{M}$ is the magnetic momentum of the magnetic rod and is calculated as:

$$\vec{M} = \frac{\vec{m} \times \vec{B}_0}{|\vec{B}_0|}$$

(A.5)

where $\vec{m} = [m_x, m_y, m_z]^T$ is the mapped magnetic dipole moment of the magnetic rod. Substituting Eq. (A.5) in Eq. (A.4), the magnetic torque can be rewritten as:

$$T_m = \begin{bmatrix} T_{mx} \\ T_{my} \\ T_{mz} \end{bmatrix} = C_B \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

(A.6)

$$C_B = \begin{bmatrix} -\left(B_{by}^2 + B_{bz}^2\right) & B_{bx}B_{by} & B_{bx}B_{bz} \\ B_{bx}B_{by} & -\left(B_{bx}^2 + B_{bz}^2\right) & B_{by}B_{bz} \\ B_{bx}B_{bz} & B_{by}B_{bz} & -\left(B_{bx}^2 + B_{by}^2\right) \end{bmatrix}$$

(A.7)

If $\gamma, \varphi \to 0$, then:

$$B_{by}, B_{bz} \to 0 \Rightarrow \left( B_{by}^2 + B_{bz}^2 \right) \approx 0$$

(A.8)

Considering Eq. (A.8) with $m_x = 0$, we replace it with the momentum wheel to obtain:

$$T_b = \begin{bmatrix} -\frac{\dot{h}_t}{T} \\ 0 \\ 0 \end{bmatrix} \Rightarrow T_c = T_m + T_b$$

(A.9)

where $T_b$ is the momentum wheel torque.
Appendix B

Euler’s equation can be rewritten as:

$$I_s \dot{\Theta}C \Phi'' + I_s \dot{\Theta}L + I_s \dot{\Theta}\omega_{\theta \omega} + \dot{\Theta}^2 \omega_{\theta \omega}^2 I_s \omega_{\theta \omega} + \dot{\Theta} \omega_{\theta \omega}^2 h = \frac{3}{R^3} C_s^x I_s C_s + T_c$$  \hspace{1cm} (B.1)

in which $\omega_{\theta \omega}$ is the angular velocity with respect to $\theta$. $L$ is obtained as:

$$L = \begin{bmatrix} -(1 + \alpha') \phi' \sin \phi \cos \gamma - (1 + \alpha') \psi \cos \phi' \sin \gamma - \gamma' \phi' \cos \gamma \\ -(1 + \alpha') \psi \cos \phi' \\ -(1 + \alpha') \phi' \sin \phi \sin \gamma + (1 + \alpha') \psi \cos \phi' \cos \phi - \gamma' \phi' \sin \gamma \end{bmatrix}$$  \hspace{1cm} (B.2)

Solving Eq. (B.1), $\Phi''$ is obtained as:

$$\Phi'' = f + (I_s \dot{\Theta}^2 C)^{-1} T_c$$  \hspace{1cm} (B.3)

$$f = (I_s \dot{\Theta}^2 C)^{-1} \left( -I_s \dot{\Theta}^2 L - I_s \dot{\Theta} \omega_{\theta \omega} - \dot{\Theta}^2 \omega_{\theta \omega}^2 I_s \omega_{\theta \omega} - \dot{\Theta} \omega_{\theta \omega}^2 h + \frac{3}{R^3} C_s^x I_s C_s \right)$$  \hspace{1cm} (B.4)

$$(I_s \dot{\Theta}^2 C)^{-1} T_c = \frac{1}{B} (I_s \dot{\Theta}^2 C)^{-1} p \begin{bmatrix} h_s' \\ m_s \\ m_z \end{bmatrix} = G_T \begin{bmatrix} h_s' \\ m_s \\ m_z \end{bmatrix}$$  \hspace{1cm} (B.5)

$$C^{-1} = \frac{1}{\cos \phi} \begin{bmatrix} \cos \gamma & 0 & \sin \gamma \\ \sin \phi \cos \gamma & \cos \phi & \sin \gamma \sin \phi \\ -\cos \phi \sin \gamma & \cos \phi \cos \gamma \end{bmatrix} \xrightarrow{\gamma, \phi \rightarrow 0} C^{-1} = \frac{1}{\cos \phi} \begin{bmatrix} \cos \gamma & 0 & 0 \\ 0 & \cos \phi & 0 \\ 0 & 0 & \cos \phi \cos \gamma \end{bmatrix}$$  \hspace{1cm} (B.6)

$I_s$ is supposed to be a diagonal matrix

$$I_s = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$  \hspace{1cm} (B.7)

$p$ can be written as:

$$p = w_1 + w_2, \quad w_1 = \begin{bmatrix} -B \dot{\Theta} & 0 & 0 \\ 0 & -(B_{\theta x}^2 + B_{\theta z}^2) & 0 \\ 0 & 0 & -B_{\theta x}^2 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 0 & B_{\theta y} & B_{\theta z}B_{by} \\ 0 & 0 & B_{\theta z} \\ 0 & B_{\theta z}B_{by} & 0 \end{bmatrix}$$  \hspace{1cm} (B.8)

From Eq. (B.8), $G_T$ is obtained as:

$$G_T = G + G_{int} = \frac{1}{B \dot{\Theta}^2} C^{-1} I_s^{-1} w_1 + \frac{1}{B \dot{\Theta}^2} C^{-1} I_s^{-1} w_2$$  \hspace{1cm} (B.9)

$g_4, g_5, g_6$ are taken as:

$$G = \frac{1}{B \dot{\Theta}^2} C^{-1} I_s^{-1} w_1 = \begin{bmatrix} g_4 & 0 & 0 \\ 0 & g_5 & 0 \\ 0 & 0 & g_6 \end{bmatrix} = -G_{op}$$  \hspace{1cm} (B.10)

As $-90 < \gamma, \phi < 90$, it is clear that $g_4, g_5, g_6$ are nonzero and positive ($g_{4,5,6} > 0$).

$$T_{int} = G_{int} \begin{bmatrix} h_s' \\ m_s \\ m_z \end{bmatrix} = \frac{1}{B \dot{\Theta}^2} C^{-1} I_s^{-1} w_2 \begin{bmatrix} h_s' \\ m_s \\ m_z \end{bmatrix}$$  \hspace{1cm} (B.11)

where $T_{int}$ is the interaction torque that is produced by the magnetic field.

$$F = [f_4, f_5, f_6]^T = f + T_{int}$$  \hspace{1cm} (B.12)

$$\Phi'' = f - G_{op} \begin{bmatrix} h_s' \\ m_s \\ m_z \end{bmatrix}$$  \hspace{1cm} (B.13)