Swing-By Window Expansion for WSB Lunar Gravity Capture via Low Continuous Thrust

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(Received January 13th, 2012)

Weak stability boundary (WSB) lunar gravity-capture transfers can save much effort (dV) on arrival at the Moon but there is a very narrow window of only a few days in a month to achieve these transfers. This is because WSB transfers require a specific orientation of the orbit with regard to the Sun, the Earth and the Moon to efficiently use the chaotic dynamics of the multi-body problem. In this paper, we propose controlling the orbit through small continuous acceleration. The results show that the window of the WSB transfer can be expanded up to a full month.

Key Words: WSB Lunar Transfer, Lunar Gravity-Capture, Lunar Swing-By, Low Continuous Thrust

Nomenclature

r : position
 v : velocity
 a : acceleration
 θ : angle
 μ : gravitational constant
 $C3$: energy
 dV : maneuvering

Abbreviations

WSB: weak stability boundary
TOF: time of flight

1. Introduction

Weak stability boundary (WSB) lunar transfers are receiving much attention as an option to save maneuvering. Spacecraft move in the WSB regions so that they can efficiently use the solar perturbation and increase the orbital energy of the transfer trajectory to reduce maneuvering on arrival at the Moon. This kind of transfer has already been adopted and performed by the JAXA's Hiten mission in 1991.¹⁾ Furthermore, when the approaching relative speed on arrival is sufficiently small, spacecraft can be "captured" by the gravity of the Moon without any maneuvering and stay in the vicinity for a while. This mechanism is called lunar gravity-capture and is now attracting attention.

The apogee of this transfer has to be large enough for spacecraft to use the solar perturbation. Two options can be considered to send spacecraft from a low earth orbit (LEO) into the WSB region: (I) Direct transfer to the WSB region with the use of rockets. (II) Indirect transfer via lunar swing-by (Fig. 1). The second option is advantageous in that it requires less maneuvering on departure at

the Earth than the first one. This paper discusses the second transfer scenario from the Earth to the Moon.

The problem we have to consider next is the launch window, in other words, the window for lunar swing-by. It depends on the positional relationship of the orbit with regard to the Sun and the Earth whether the orbital energy can increase as a result of the solar perturbation. In addition, to achieve low relative speed on arrival at the Moon, the position of the Moon is also important. In order to realize the WSB transfer leading to lunar gravity-capture, spacecraft must swing-by the Moon satisfying a certain orientation of the orbit with regard to the planets. That is why the window for this lunar swing-by is very narrow for ballistic flight. There are only a few days in a month that provide a chance and this is not acceptable from a practical view.

It is clearly essential to improve the usability of the WSB lunar transfer by solving the problem of the launch window for application to real missions. One of the options is to use low continuous thrust realized by electronic engines. Many studies have been conducted on electronic engines and the Japanese asteroid explorer Hayabusa had xenon ion engines in it and verified its ability and durability. Such engines have better fuel consumption than chemical engines and are expected to be used more in future missions.

This paper discusses how introducing low continuous thrust expands the window of lunar swing-by for lunar gravity-capture. The results show that sufficiently small acceleration can make the WSB lunar transfer easy to use.

2. Modeling

The calculation conditions are described here. The system consists of one spacecraft and three celestial bodies, the Sun, the Earth and the Moon. Since the gravity of the Earth and the Sun are major forces that cause chaotic dynamics of the WSB orbit, the Sun-Earth-Spacecraft circular restricted three-body problem (CR3BP) is adopted as a model. The

Fig. 1. Lunar swing-by WSB transfer in the Earth-centered inertial frame.

Moon is assumed to have a negligible-mass and moves in a circular orbit around the Earth on the plane defined by the Sun and the Earth. This means the Moon does not affect the motion of the other planets nor spacecraft.

When modeling the system described above, the lunar gravity-capture cannot be numerically calculated. Instead the arrival conditions at the Moon are introduced. These conditions are developed by considering the Moon-Spacecraft two-body problem.

2.1. Sun-Earth-Spacecraft CR3BP

The equation of motion of the Sun-Earth-Spacecraft CR3BP in the Sun-Earth fixed rotating frame is written as

$$\begin{aligned}\ddot{x} - 2\dot{y} &= -U_x \\ y + 2\dot{x} &= -U_y \\ \ddot{z} &= -U_z\end{aligned}\quad (1)$$

$$U(x, y) = -\frac{1}{2}(x^2 + y^2) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}(1-\mu)\mu \quad (2)$$

where μ is the mass parameter between the Earth and the Sun. r_1 and r_2 denote the distance with regard to the Sun and the Earth, respectively.

2.2. Definition of lunar gravity-capture

In the vicinity of the Moon, the trajectory is categorized by its orbital energy with regard to the Moon, $C3$. The $C3$ energy with regard to the Moon is defined by

$$\begin{aligned}C3 &= v^2 - \frac{2\mu_{\text{Moon}}}{r} \\ &= -\frac{\mu_{\text{Moon}}}{a}\end{aligned}\quad (3)$$

where r and v are the distance and the velocity with regard to the Moon and a is the semi-major axis of the orbit.

When spacecraft are captured in lunar orbit, the $C3$ energy measured at perilune has a negative value. On the other hand, it should have positive $C3$ to escape from lunar gravity. This means that the energy changes its sign during the course of lunar gravity-capture. In this paper, this sign reversal of the $C3$ energy is assumed to occur when the tra-

Fig. 2. Lunar swing-by/arrival condition.

jectory enters the lunar SOI. Thus the arrival condition that leads to lunar gravity-capture is defined as

$$C3|_{\text{lunar SOI}} = 0 \quad (4)$$

When the radius of lunar SOI is 66,300 km, Eq. (4) can be written as a function of relative speed with regard to the Moon,

$$|v_{\text{S/C w.r.t. Moon}}|_{\text{arrival}} = 0.38 \text{ km/s} \quad (5)$$

2.3. Lunar swing-by condition

Figure 2 shows a sample of the WSB transfer. The post swing-by velocity of spacecraft is a total of the lunar orbital velocity and the swing-by velocity. The swing-by velocity has a constant magnitude of 1.0 km/s with regard to the Moon, assuming the pre-swing-by Earth-Moon transfer is a Hohmann-type trajectory.

$$|v_{\text{S/C w.r.t. Moon}}|_{\text{swing-by}} = 1.0 \text{ km/s} \quad (6)$$

2.4. Classification by time of flight

The WSB transfer can be classified by its time of flight (TOF). Here, TOF means the time from lunar swing-by to lunar gravity-capture. It roughly increases by a factor of the lunar sidereal period, 27.3 days. This paper deals with two groups: Group 1 includes two-to-three lunar periods transfers and Group 2 includes three-to-four lunar periods transfers.

3. Ballistic Flight

This section describes the length of opportunity for the WSB ballistic orbits to achieve lunar gravity-capture when the spacecraft approaches the Moon again.

3.1. Forward targeting algorithm

The WSB ballistic lunar trajectories are calculated using a forward targeting algorithm. The following is a brief description.

We firstly give the initial lunar position with regard to the Earth and then consider the post swing-by flight direction

Fig. 3. TOF of ballistic flight.

(see Fig. 2). The post swing-by velocity of spacecraft is represented as the sum of the relative swing-by velocity with regard to the Moon and the lunar orbital velocity. Since the magnitude of the relative swing-by velocity is assumed to be a constant value of 1.0 km/s, the only variable is the direction of it, the swing-by angle $\theta_{\text{swing-by}}$, which is measured from the lunar orbital velocity vector.

Here, we conduct a one-dimensional search on the distance between spacecraft and the Moon at perilune, $r_{\text{S/C w.r.t. Moon}}$. By varying $\theta_{\text{swing-by}}$ using a bisection process, we can obtain a trajectory that can return to the Moon. The calculation is terminated if the $r_{\text{S/C w.r.t. Moon}}|_{\text{Perilune}}$ becomes sufficiently small.

The initial guess of swing-by angle $\theta_{\text{swing-by}}$ is selected in order that the spacecraft can return to the Earth-Moon system after it passes on the apogee.

3.2. Calculating the swing-by window

The window for ballistic transfer is defined as the timing of swing-by that leads to lunar gravity-capture. The WSB lunar gravity-capture requires two conditions: Firstly, the orbit has to crossover the lunar orbit, and secondly the relative speed on arrival at the Moon must be less than 0.38 km/s (see Eq. (5)). The timing of swing-by is equivalent to the lunar phase when swing-by is performed. It is measured from the anti-Sun direction in the Sun-Earth line fixed rotating frame. As a result of the gravitational symmetry, the motion of the orbit in Quadrants 3 and 4 in the rotating frame is expected to be the same as in Quadrants 1 and 2. Thus, the calculation is conducted for the case $\theta_{\text{Moon}} = 0-180$ deg.

3.3. Calculation results

Figure 3 shows the TOF of the WSB orbit. There clearly exist two groups with different TOF, and the blank in the center of the graph indicates that spacecraft which swing by the Moon at this phase cannot reach the Moon again.

The arrival excess speed at the Moon appears in Fig. 4. It is distributed around $|v_{\text{S/C w.r.t. Moon}}| = 0.2-2.0$ km/s. The excess speed in most case, increases compared with the relative speed on swing-by. The upper limit for lunar gravity-capture is also defined in this graph and is 0.38 km/s, calcu-

Fig. 4. Arrival excess velocity with regard to the Moon.

lated in Eq. (5). To achieve lunar gravity-capture, the arrival excess speed has to be lower than this limit.

Considering the condition mentioned above, the window for Group 1 is $\theta_{\text{Moon}} = 72-73, 115-122$ deg and the window for Group 2 is $\theta_{\text{Moon}} = 70, 115-128$ deg. It is estimated that there are only 1.7 days for Group 1 and 2.5 days for Group 2 per month.

4. Controlled Flight

The previous section indicates that the ballistic WSB lunar transfer has a very narrow swing-by window, in other words, little opportunity for launch. According to the result, if one misses the first swing-by, the next occasion comes approximately a quarter of a month later. It is inefficient and impractical for spacecraft to wait in orbit for such a long time. The following section proposes the way to solve this problem, that is, the use of continuous low thrust.

4.1. Optimization method

The optimized trajectory is obtained using the SCGRA method. This uses a variational approach to get an optimal solution. A precise algorithm is shown in the Ref. 2).

4.2. Boundary condition

The initial swing-by condition is the same as the ballistic flight, that is, the relative speed with regard to the Moon is 1.0 km/s. Furthermore, the arrival condition at the Moon is added; the perilune distance is 0 and the relative speed is 0.38 km/s as shown in Eq. (5).

4.3. Constraints

The mass of the spacecraft is assumed to be constant during the flight and does not change by the consumption of its fuel. The maximum thrust acceleration is defined as 1.0×10^{-7} km/s² corresponding the ability of spacecraft which have a mass of 1,000 kg and a propulsion system of 100 mN.

$$\sqrt{a_x^2 + a_y^2 + a_z^2} \leq 1.0 \times 10^{-7} \text{ km/s}^2 \quad (7)$$

4.4. Cost function

The amount of the fuel which spacecraft has to load is proportional to the sum of the maneuvering. It should be

Fig. 5. Group 1 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 50 \text{ deg}$). Fig. 6. Group 2 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 50 \text{ deg}$).

as small as possible to get more payload. Thus, the cost function is defined by the sum of the control written as

$$J = \int dV$$

$$= \int \sqrt{a_x^2 + a_y^2 + a_z^2} dt \quad (8)$$

4.5. Constraint error and optimality condition

The SCGRA iteration continues until the constraint error P and the optimality condition error Q satisfy the pre-selected conditions. The error P is related to the sum of the norm squared of the error of each constraint and the error Q is introduced in the process of the variation method. This paper requires these errors to satisfy the following inequality,

$$P \leq \varepsilon_P = 1.0 \times 10^{-6}$$

$$Q \leq \varepsilon_Q = 1.0 \times 10^{-4} \quad (9)$$

and the number of steps is 1,000.

4.6. Calculation results

The calculation is conducted for all the cases of lunar swing-by except the phases that can achieve ballistic lunar capture.

Figures 5 and 6 show examples of the calculation results. The arrows in the graphs indicate the acceleration by the low thrust and we see that the spacecraft can return to the Moon satisfying both the initial and final conditions.

Figure 7 shows how much maneuvering is needed to meet the lunar arrival condition of Eq. (5). The necessary dV is correlated to the approaching relative speed with regard to the Moon of the ballistic flight. The figure indicates that the orbit is modified by maneuvering up to 0.46 km/s for Group 1 and 0.73 km/s for Group 2. This means that the WSB lunar capture of Group 1 can be achieved regardless of the timing of lunar swing-by when the spacecraft has a continuous propulsion system which allows acceleration of $1.0 \times 10^{-7} \text{ km/s}^2$ and total dV of 0.5 km/s (this value is equivalent to 58 days of continuous maneuvering).

Table 1 summarizes the results of Fig. 6 and shows the length of the lunar swing-by window per month for Group

Fig. 7. Total maneuvering (dV).

Table 1. Window width per month [days].

	Maneuvering (dV) [km/s]								
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
G1	1.7	5.8	15	18	23	30	—	—	—
G2	2.5	7.5	16	18	21	23	27	29	30

1 and Group 2 under the condition that total maneuvering is between 0 and 0.8 km/s. For example, 0.2 km/s maneuvering in both groups enables the window to expand to approximately half a month and the window is 8.8 times wider for Group 1 (1.7→15 days) and 6.4 times for Group 2 (2.5→16 days) compared to the corresponding ballistic orbit.

Figures 8 to 13 show the controlled trajectories from lunar swing-by to capture when the lunar phase of swing-by changes from 0 to 180 deg. All the trajectories are expressed in the geo-centric inertial frame and satisfy the lunar arrival condition of the Eq. (4). To make figures clearer, only the trajectories are shown and the arrows indicating acceleration are abbreviated.

Fig. 8. Group 1 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 0-60 \text{ deg}$). Fig. 9. Group 1 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 80-110 \text{ deg}$). Fig. 10. Group 1 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 130-180 \text{ deg}$). Fig. 11. Group 2 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 0-80 \text{ deg}$). Fig. 12. Group 2 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 90-110 \text{ deg}$). Fig. 13. Group 2 ($\theta_{\text{Moon}}|_{\text{swing-by}} = 130-180 \text{ deg}$).

5. Conclusion

The result reveals that only slight control can modify the trajectories and expand the window of lunar swing-by. The following is the reasoning for these results.

The necessary maneuvering has a strong relationship with the approaching excess speed with regard to the Moon of ballistic flight and maneuvering up to 0.46 km/s for Group 1 and 0.73 km/s for Group 2 can guide spacecraft into lunar gravity-capture (Figs. 8–13). When considering direct lunar capture from Hohmann transfer, the required speed reduction is 0.62 km/s (Note that Hohmann transfer has excess speed of 1.0 km/s with regard to the Moon and the acceptable excess speed of lunar gravity-capture is 0.38 km/s). Then, it can also be said that the window expansion with control of less than 0.62 km/s is meaningful in the practical phase.

Based on Fig. 7, we calculate the length, for spacecraft with constant capacity of dV , of the lunar swing-by window (Table 1). The results tell us that only 0.3 km/s maneuvering can expand the window to more than half a month in both Group 1 and Group 2. This window width is 11 times wider than that for ballistic flight in Group 1 (1.7→18 days) and 7.2 times wider in Group 2 (2.5→18 days).

Note that Fig. 5 and Table 1 only deal with the case when the maximum acceleration is with $1.0 \times 10^{-7} \text{ km/s}^2$. However, the characteristic remains that the excessive approaching speed of ballistic flight and the necessary control to

make lunar gravity-capture have a relationship and that the width of the expanded window depends on the total amount of control, dV , regardless of the magnitude of acceleration. Of course, there exists a minimum magnitude of acceleration that enables spacecraft to transfer, but a specific value of it is not discussed here. Thus, the results we obtained indicate that the low continuous acceleration can expand the chance of lunar gravity-capture.

Several studies have been conducted on the WSB lunar transfer. Spurrmann³⁾ introduced the WSB transfer as one of the transfers to the Moon with the least dV . He also pointed out the error sensibility in that a small maneuvering error of the initial injection can cause a big change of the following orbit. Belbruno and Carrico⁴⁾ proposed a practical way of calculating the WSB ballistic trajectories. In addition, the WSB transfer has been already achieved in a real mission by Japanese Hiten.¹⁾ However, these studies are based on ballistic flight or the use of impulse acceleration by chemical engines. It is expected that the low continuous acceleration will be the major power of spacecraft in future missions because of its fuel consumption efficiency. Thus, the design of a continuous-thrust-trajectory will become more and more demanding.

The result that the swing-by window can be expanded with small control can contribute to the improvement of mission feasibility in future lunar missions. The idea that spacecraft can aim for lunar gravity-capture via lunar swing-by can be also used for back-up trajectories. If space-

craft fail to enter the lunar orbit and choose lunar swing-by, they can re-approach the Moon controlling its path with a low continuous thruster.

The problems we have to consider next involve the following three factors. First, the model should be improved. We adopt in this study the Sun-Earth-Spacecraft CR3BP, neglect the influence of the lunar gravity and introduce the lunar capture condition, namely, $C3|_{\text{lunar SOI}} = 0$. To make more accurate calculations, the Sun-Earth-Moon-Spacecraft 4BP should be adopted. Second, our study focus on the trajectory out of lunar SOI and does not analyze the orbit after entering it. This is also due to the absence of lunar gravity. It is important to know how the orbit should be controlled to stay around the Moon. Third, there should be limitations of the direction of the thruster gimbal. We set limits on the amount of acceleration but not its direction. In actual cases, the direction of acceleration depends on the gimbal of the thruster. Calculation of trajectory considering attitude of the spacecraft is necessary.

Acknowledgments

This work was supported by the Japan Society of the Promotion of Science (JSPS).

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