A Robust Flight Control System Design for a Small-Scale UAV Helicopter

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The aim of this paper is to design a robust automatic flight control system for a small-scale UAV helicopter. A non-linear search based on differential evolution (DE) algorithm is conducted for a six degrees-of-freedom linear state-space model that matches the frequency-response data set. The accuracy of the identified model is verified by comparing the model-predicted responses with the responses collected during flight experiments. Based on the identified model, the $H_\infty$ loop shaping method is used to design the inner-loop of the unmanned helicopter in order to satisfy the flight performance requirements specified in the military standard ADS-33E. The greatest common right divisor method is used to solve the difficulties in choosing a proper weighting matrix in the $H_\infty$ loop-shaping procedure, compared with the traditional method, the system using the new method have a larger robust stability margin, the decoupling and the bandwidth of the system are also greatly improved. The simulation results prove high standard of the control performance of the unmanned helicopter system in accordance with ADS-33E.

Key Words: Helicopter, Robust Control, System Identification, $H_\infty$ Loop-Shaping Method, Flight Performance, Wind Disturbance

1. Introduction

An automatic flight control system is essential for a UAV helicopter system. To improve flight control performance, many researchers have devoted themselves to the study of implementing more advanced control techniques the small scale UAV helicopter. The key elements in flight control system design are the accurate model and proper control method. The small-scale helicopter is a complicated multi-input/multi-output (MIMO) system. The modeling of the helicopter is especially difficult because of the high level of measurement noise, generally unstable vehicle dynamics, high degree of inter-axis coupling, and the aerodynamic interaction between the rotor and the fuselage. Conventional time-domain techniques are often not well suited to these difficult aspects of helicopters as discussed by Tischler and Gauffman and Fu and Kaletka. In contrast, the frequency domain methods are well suited to the rotorcraft problem, as the frequency identification method can 1) extract a complete set of non-parametric frequency responses that fully characterize the coupled behavior of the helicopter without uncorrelated noise and 2) conduct a nonlinear search for a state-space model that matches the frequency response only in the region of highest accuracy.

For flight control problems with mostly qualitative closed-loop specifications, $H_\infty$ control can be used to quickly establish the achievable performance. This is because $H_\infty$ control can systematically handle the multivariable and uncertain nature of the aircraft. The $H_\infty$ loop-shaping method proposed by McFarlane and Glover is a very sensible and appealing procedure for designing robust controllers. It is a combination of loop-shaping and robust stabilization and is the method used in this paper.

This paper attempts to provide a solution by presenting a modeling and control framework that minimizes the time, cost, and both human and physical resources necessary to design high-performance flight controllers. The outline of this paper is as follows. Firstly, we present a linear model for the UAV helicopter system for the hover and near hover flight conditions using frequency identification. Then the flight performance requirements is stated based on the military standard ADS-33E, and the $H_\infty$ loop-shaping method is used to design the inner-loop of the unmanned helicopter according to the identified model. Finally, the simulation results prove the high standard of control performance of the unmanned helicopter system in accordance with the ADS-33E.

2. Frequency Identification Modeling Method

The identification procedure is depicted in Fig. 1. The key features of this approach are as follows: 1) Obtaining broadband excitation of the helicopter dynamics of interest via frequency sweep inputs. 2) Data analysis and process before identification, such as consistency analysis and filter design. 3) Generation of high-quality MIMO frequency response using the integrated windowing method. 4) Conducting a nonlinear search based on the DE algorithm for a state-space model that matches the frequency-response data set only in the region of highest accuracy. 5) Evaluating the accuracy of the identified model using time-domain verification method.
2.1. Integrated window method

The key step in the frequency identification procedure is the extraction of accurate frequency-response for each input/output pair. However, the selection of window size in the SISO frequency-response calculation step involves a fundamental trade-off between dynamic range and accuracy of the identification. The window length \( T \) sets the minimum frequency \( \omega_{\text{min}} \)

\[
\omega_{\text{min}} = \frac{2\pi}{T}. \tag{1}
\]

The level of random error \( \varepsilon_r \) associated with the identified frequency response is inversely proportional to the number of window segments

\[
\varepsilon_r = \frac{\sqrt{0.55[1 - \gamma^2_x]}}{|\gamma_{xy}|\sqrt{2n_d}} \tag{2}
\]

where \( n_d = T_r/T \), \( T_r \) is the concatenated run length.

As can be seen from Eq. (1), long windows are required for frequency-response identification of low-frequency dynamics. For the identification of high-frequency dynamics, considerably smaller windows are used because of the noise suppression afforded by the averaging effect in Eq. (2).

A new method has been developed that combines the conditioned frequency-response identification results obtained using several window sizes. Integrated curves of the conditioned input, output and cross-spectral functions are generated to achieve an integrated conditioned frequency response that has good coherence and low random error over the entire frequency range of interest. The final integrated values for spectral function are those that minimize the weighted least-squares cost function \( J \) at each discrete frequency

\[
J = \sum_{i=1}^{n_w} W_i \left[ \left( \frac{G_{xx} - G_{xx_i}}{G_{xx_i}} \right)^2 + \left( \frac{G_{xy} - G_{xy_i}}{G_{xy_i}} \right)^2 + \frac{5.0(\gamma_{xy} - \gamma_{xy_i})^2}{\gamma_{xy_i}} \right]^2 \tag{3}
\]

where \( n_w \) is the number of windows included in the procedure, and \( W_i \) is the weighting function dependent on random error and blended at the ends of each window’s frequency range. Subscript \( i \) refers to individual window conditioned results. Subscript \( c \) refers to “integrated” conditioned data.

After the flight-test data including the information content of the dynamic characteristics are collected, the resulting lateral coherences using window lengths of 5, 15, and 30 s are shown in Fig. 2. The 30-s window produces the best coherence at low frequency, whereas poorer results are obtained for the 15-s window. In the mid-frequency range, the 15-s window shows the best performance, although its coherence begins to oscillate in the high-frequency range due to insufficient spectral averaging. The 5-s window has poor performance in the mid-frequency range due to reduced low-frequency content, but performs well at high frequency where it allows a maximum amount of spectral averaging.

Fig. 1. Flowchart of frequency identification for the helicopter.

The frequency responses used for the state-space identification are selected from the coherence measure obtained during the multivariable frequency domain analysis (see Table 1).

2.2. State-space model identification algorithm

State-space models are often the desired end product of system identification. In the current frequency-response approach, stability-derivative identification is achieved directly through iterative MIMO matching of the identified conditioned frequency responses with those of the following linear model.
The frequency ranges for the identification criterion are selected individually for each input/output pair according to their individual ranges of good coherence. An overall average cost function that achieves $J_{ave} = J/n_{TF} \leq 100$ is generally considered as reflecting an acceptable level of accuracy for flight dynamics modeling.\(^7\)

The $n_p$ parameters to be identified in the model matrices $A$, $B$ are collected into an identification vector $\Theta = [\theta_1 \ \theta_2 \ \cdots \ \theta_{np}]$. A nonlinear search method based on the DE algorithm is much better suited for this application and is the method used in this paper. The DE algorithm has three evolutionary operations, namely mutation, recombination and selection. Suppose that the initial solution population has NP individuals, and the search space is D-dimensional; the DE’s population of the $n_{th}$ iteration can be represented by $X(n) = [x_1 \ x_2 \ \cdots \ x_{NP}]$, where $x_i$ is a candidate solution in D-dimensional solution space. The basic operators of DE are described as follows.\(^8\)

### 2.2.1. Mutation

The mutation operator is to randomly select three different individuals from the current population to generate a new mutate individual.

$$v_i = x_{i,j} + F \cdot (x_{i,r} - x_{i,t})$$  \hspace{1cm} (7)

where $F$ is a control parameter usually chosen from the interval $[0, 2]$. The mutation process is shown in Fig. 4.

### 2.2.2. Recombination

The recombination operation generates the new individual by copying components from the mutation vector $v_i$ and the target vector $x_j$ as follows

$$u_{ij} = \begin{cases} v_{ij}, & \text{if } \text{randb} \leq C_R \text{ or } j = r_j \\ x_{ij}, & \text{if } \text{randb} > C_R \text{ or } j \neq r_j \end{cases}$$  \hspace{1cm} (8)

where the random number randb $\in [0, 1]$, the recombination control parameter CR is a constant in the interval $[0, 1]$, and $r_j$ is an integer randomly chosen from $[1, D]$. The recombination process is shown in Fig. 5.

### 2.2.3. Selection

The selection operator, as the deterministic process in the DE algorithm, is implemented to choose better individuals with lower fitness function value between the target vector and the trial vector, which is inherited by the next generation, expressed as
other control inputs to maintain the helicopter within the remote control unit. While doing so, the pilot uses the control sequence to one of the four control inputs via the

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\[ x_i^{t+1} = \begin{cases} u_i^t & J(u_i^t) < J(x_i^t) \\ x_i^t & \text{otherwise.} \end{cases} \] (9)

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selected operating condition. The same experiment is repeated several times to gather sufficient data. Figure 7 shows a sample of flight data of the lateral vehicle axes for two concatenated lateral frequency sweeps conducted in hover.

2.4. Identification results and model verification
Comparisons of the identified model fit to the flight data at hover are shown for the on-axis responses in Fig. 8. These figures indicate that the model and the flight data are in good agreement.

The identification model is verified in the time domain to ensure that it can accurately predict the aircraft dynamic response. The pilot inputs are used to excite the model, and the model responses and aircraft responses are then compared. If the responses match, then the model has good predictive accuracy. The lateral verification results for hover are given in Fig. 9. As seen in the figure, the model matches the flight data well in the time domain.
3. Robust Controller Design

The $H_\infty$ loop-shaping method is used to design the inner-loop of the unmanned helicopter based on the identified model and is described in Ref. 4).

3.1. Requirements for the inner-loop

The inner loop (Fig. 10) is an attitude-command attitude-hold (ACAH) response type controller. References inputs are pitch attitude ($\theta$), roll attitude ($\phi$), vertical velocity ($w$) and yaw rate ($r$). The same four variables were used for feedback. We did not include pitch rate ($q$) and roll rate ($p$) in the measurement set because the stabilizer bar already acts as a lagged-rate feedback system.\(^\text{10}\)

After completing the controller design, it is checked against some specifications adopted in the full-scale military helicopter community, ADS-33E. Although a higher level of performance is generally expected from small-scale helicopters, the Raptor50 is quite limited in achievable performances; we think that, given the relevant payload added by the instrumentation, the use of a set of full-scale helicopter’s specifications may be justified. Nevertheless, we were able to satisfy the requirements with a large margin for the examined bounds.

The requirements for the inner-loop controller are as follows.\(^\text{5}\)

3.1.1. Stability

In single-loop analysis, gain margin shall be $\geq 6$ dB and phase margin $\geq 45$ deg to guarantee a reasonable level of robustness.

3.1.2. Bandwidth

Closed-loop bandwidth $\omega_{BW}$ shall satisfy level 1 target acquisition and tracking specifications. For the $\theta$, $\phi$ and $r$ channel, ADS-33E defines $\omega_{BW}$ as the frequency where the phase of the closed-loop response is $-135$ deg. $\omega_{BW}$ should be larger than 2.0, 2.5 and 3.5 rad/s for the $\theta$, $\phi$ and $r$ channels, respectively. For the $w$ channel, ADS-33E gives the desired speed of response in terms of bounds on a time constant $RT_w \leq 5.0$ and a time delay $\tau_w \geq 0.2$ of a first-order transfer function

$$T_{FIT} = \frac{e^{\tau_w}}{RT_w s + 1}$$

that is used to fit the step response of the $w$ channel. ADS-33E considers the fit successful when the following cost coefficient is between 0.97 and 1.03

$$r^2 = \frac{\|w_{FIT} - \bar{w}\|^2}{\|w - \bar{w}\|^2}$$

where $\bar{w}$ is the average of $w$ during the step.

3.1.3. Decoupling

Off-axis responses due to on-axis requests $\phi/\theta_r, \theta/\phi_r, w/r$ shall comply with level 1 aggressive agility specifications. For the first two transfer functions, ADS-33E imposes the following bounds: where the numerators are the differences between the values (peak within 4 s and trim) of the off-axis variable caused by a step change of the on-axis variable, and the denominators are the differences between the values (at 4 s and at trim) of the on-axis variable.

$$-0.25 \leq \frac{\Delta \theta_{rk}}{\Delta \theta_{r4}} \leq 0.25, \quad -0.25 \leq \frac{\Delta \phi_{rk}}{\Delta \phi_{r4}} \leq 0.25.$$

For the third transfer function, ADS-33E imposes bounds on $r$ following a step request in $w$ where $r_1$ is the first peak of $r$ before 3 s or the value of $r$ at 1 s $r(1)$ if no peak occurs, and

$$r_3 = \begin{cases} r(3) - r_1: & r_1 > 0 \\ -r(3) + r_1: & r_1 < 0. \end{cases}$$

The units for the correct estimation of the bounds are deg/s for $r$, and ft/s for $w$.

3.1.4. Disturbance rejection

Response of $w$, $r$, $\phi$ and $\theta$ to pulse disturbances injected directly at the inputs of the plant shall return to within 10% of the peak value in less than 10 s.

3.2. Weight matrix selection

The method of choosing a proper weighting matrix is a key factor and the most difficult one when we design the $H_\infty$ controller following the above steps. It is well recognized in the practice that design of loop-shaping weights $W_1$ and $W_2$ to achieve a desired loop-shape is not always straightforward. As of now, there is no systematic design method to solve this problem, mostly it depends on the engineer practice of the designer.\(^\text{11}\)

The weights $W_1$ and $W_2$ are usually chosen to be a diagonal matrix. $W_2$ contains low-pass filters for noise rejection and lead-lag filters for improving robustness. $W_1$ contains proportional plus integral (PI) filters. The integrators are used to boost the low frequency gain, and thus improve output decoupling, tracking and disturbance rejection at both the plant input and output. The proportional matrix gain is used to reduce the phase lag introduced by the integrators around crossover. The overall gains of $W_1$ and $W_2$ are used to specify the desired loop bandwidth.

Based on the engineer design practice, the diagonal gains of $W_1$ and $W_2$ are tuned to have crossover frequencies of the open-loop system at 7 rad/s.

$$W_1(s) = \text{diag} \begin{bmatrix} \frac{3(s + 3)}{s}, & \frac{1.5(s + 5)}{s}, \\ \frac{20(s + 0.5)}{s}, & \frac{35(s + 0.5)}{s} \end{bmatrix}$$
\[
W_2(s) = \text{diag} \begin{bmatrix}
  2750 \cdot \frac{s^2 + 80s + 2500}{s^2 + 80s + 2500}, \\
  1.2, \\
  2035 \\
\end{bmatrix}.
\]

Figure 11 shows the singular values of the plant with and without the shaping. It satisfies the requirements that the gain should be large at low frequency but small at high frequency. The crossover frequencies of the open-loop system mostly satisfy the demand.

After we finished the loop shaping design, Eq. (1) is maximized to achieve the 24-order controller \( K_{\infty} \), and the stability margin \( \epsilon = 0.3947 \). Figure 12 shows Bode plots with gain and phase margins for single-loop analysis; all the loops satisfy the robustness specifications.

### 3.3. A new method for weight matrix selection

Engineer designers usually obtain good loop-shaping weights and controllers by choosing proper diagonal weights \( W_1 \) and \( W_2 \) to achieve desired performance. However, the design experience shows that the diagonal weights cannot satisfy the requirements very well, especially for plants with strong cross-coupling, such as helicopters.

The non-diagonal weights are required to obtain good performance.\(^{12}\) The paper uses the greatest common right divisor (GCRD) method to transfer the transfer function matrix from the real system \( G \) to the target system \( G_s \) (see Ref. 13 for the details of the theorems).

First, we obtain the target system
\[
G_s(s) = \text{diag} \begin{bmatrix}
  \frac{7}{s}, \\
  \frac{7.5}{s}, \\
  \frac{9}{s}, \\
  \frac{8}{s}
\end{bmatrix}
\]

based on the requirement that the crossover frequencies of the open-loop system should not be less than 7 rad/s.

Next, we use the GCRD method to obtain the transfer matrix \( W \) from the real system \( G \) to the target system \( G_s \).

Figure 13 shows perfect fit of the singular values of the real system \( G \) and the target system \( G_s \).

After finishing the loop-shaping design, the cost function is maximized to achieve the 24-order controller \( K_{\infty} \), and the stability margin \( \epsilon = 0.6957 \). Figure 14 shows Bode plots with gain and phase margins for single-loop analysis; all
3.4.2. Decoupling

The last specification is disturbance rejection. Figures 17 and 18 show the responses for each of the four axes when sharp pulses are applied directly at the corresponding plant’s input; they all clear the 10% bound well before the 10 s required. Moreover, the response time of the system with the new method is faster, and the oscillatory is smaller.

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3.4.4. Conclusion

In this paper, we designed a robust automatic flight control system for a small-scale UAV helicopter based on system identification and $H_{\infty}$ loop-shaping methods. 

1) The frequency identification method was used to extract an accurate helicopter model in the hover condition and the control system is designed based on the identified model.
2) We conduct a nonlinear search based on a DE algorithm for a six degrees-of-freedom linear state-space model that matches the frequency-response data set.

3) The $H_\infty$ loop-shaping method was used to design the inner-loop in order to satisfy the requirements of decoupling and stabilization as well as the high bandwidth of the control system.

4) The GCRD method was used to transfer the transfer function matrix from the real system $G$ to the target system $G_s$ in choosing a proper weighting matrix in the $H_\infty$ loop-shaping. Compared with the traditional method, the system using the new method had a larger robust stability margin. The decoupling and the bandwidth of the system also improved greatly.

5) The simulation results proved the high standard of control performance of the unmanned helicopter system in accordance with ADS-33E.

References


