The relationship between the ice accretion shape and ambient conditions is quantitatively investigated via analysis of variance (ANOVA) and is qualitatively studied using self-organization maps (SOMs). The independent variables of this study are liquid water contents (LWC), mean volumetric droplet diameter (MVD), flight speed (free stream velocity) and ambient temperature. The ranges of these variables follow appendix C of FAR part 25. The ice accretion shape is quantified through parameterization of ice accretion area, maximum thickness, ice heading and icing limits. SOMs and ANOVA can be used for parameter analysis. We determine the associated parameters, although the parameters are examined under non-linear conditions. The range of temperature, \(-40^\circ\text{C} \leq T \leq 0^\circ\text{C}\), is divided into 10 \(^\circ\text{C}\) intervals. Three distinctive temperature groups are found, depending on the response of the ice shape to the ambient conditions. They are rime (\(-40^\circ\text{C} – 20^\circ\text{C}\)), mixture (\(-20^\circ\text{C} \leq T \leq 10^\circ\text{C}\)) and glaze (\(-10^\circ\text{C} \leq T \leq 0^\circ\text{C}\)). The ice accretion area and maximum thickness are determined by free stream velocity and LWC over the entire temperature range, while ice heading is affected by the temperature, free stream velocity and LWC. Different variables affect the icing limits on the upper and lower surfaces. On the upper surface, runback dominates the formation of icing, while on the lower surface the dominant factor is where limits are impinged.

Key Words: Ice Accretion Shapes, ANOVA, Self-Organization Maps, Aircraft Icing, Ambient Conditions

1. Introduction

Aircraft icing occurs when super-cooled liquid droplets in the air collide with the surface of an aircraft and freeze. Exposed structures such as the engine inlet, wings, stabilizers, and fuselage are vulnerable to icing. In particular, icing on wings causes serious deterioration in performance as it alters the airfoil shapes and aerodynamic characteristics of the wings. Wings with icing on them usually have lower lift coefficients and higher drag coefficients. The icing also damages flight safety of the aircraft due to its effects on the stall characteristics of the wings.\(^1\)

As the cause-and-effect diagram of Fig. 1 illustrates, the flight safety of an aircraft in terms of icing is closely related to the ambient conditions. Therefore, investigation into the relationship between icing behavior and ambient conditions is also closely related to aircraft safety. Performance of anti/de-icing devices such as heat capacity, operation time, interval and duration can be determined based on an estimation of ice shape under various conditions, thus allowing information on the shape of icing to be used in designing efficient anti/de-icing devices. Furthermore, thanks to weather forecasting information from on-board or ground sensors, the aircraft can avoid areas with high chances of icing. The ambient conditions that determine ice accretion shape include liquid water content (LWC), ambient temperature, mean volumetric droplet diameter (MVD) and free stream velocity. Fuchs and Schickel compared ice accretion shapes on a cylindrical rod in terms of ambient temperature, LWC and MVD.\(^2\) Their research confirmed that LWC and ambient temperature are decisive factors in determining ice shape. However, because they relied on visual judgment, the effects of ambient conditions on ice accretion behavior and which aspects of ice accretion are affected were not clearly established. On the other hand, Wright and Potapczuk\(^3\) and Ruff and Anderson\(^4\) parameterized the shape of the ice for their analyses on the effects of ambient conditions. Wright’s research was focused on the shape of the ice horn without quantitative assessment, and the measurement of the parameters was not standardized. Ruff and Anderson\(^5\) clearly defined shape parameters that are useful for comparing the ice shapes from test results with the results of numerical analyses. However, not all these parameters were needed for analyses on the design of anti/de-icing devices and performance of the wing.\(^5\)

Parameterization of the shape of icing is useful in understanding the relationship between ice accretion behavior and ambient conditions. When it is combined with an investigation of the effects of shape parameters on aerodynamic performance, it can result in a better understanding of the relationship between the ambient conditions and aerodynamic performance.

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performance, which is the overarching purpose of research on the icing behavior of aircraft. Wright and Chung obtained information on the relationship between shape parameters and performance indices such as lift, drag and pitching moment coefficients via polynomial curve fitting of the results of an N-S based CFD analysis.5) This study parameterizes the shape of ice and identifies the relationship between the shape parameters and ambient conditions both qualitatively and quantitatively. The shape parameters are ice accretion area, maximum thickness, ice heading and icing limits. Data-mining techniques have been adopted. For the qualitative assessments, SOMs are used, and for the quantitative assessments, analysis of variance (ANOVA) is adopted.

This paper consists of four sections. Section 2 introduces the independent variables and numerical method for calculating ice shapes, parameterization of the shapes, and qualitative and quantitative assessment methods. Section 3 presents the results of qualitative and quantitative assessment. The ambient conditions involve LWC, MVD, free stream velocity, and ambient temperature. In addition, as the ice accretion shape is influenced greatly by ambient temperature, the influencing parameters are investigated according to different temperature ranges. The last section summarizes the paper.

2. Numerical Analysis

The relationship between ice accretion and ambient conditions is investigated via the following steps: (1) select relevant independent variables, (2) obtain accreted ice shapes from numerical simulations under the selected ambient conditions, (3) draw distinguishing shape parameters that determine the obtained ice shapes, (4) use SOMs to quantitatively relate the shape parameters and ambient conditions, and (5) conduct an ANOVA to quantify the influence of the ambient conditions.

2.1. Set of independent variables

The shape of ice on a 2D airfoil varies according to flight and atmospheric conditions. The flight conditions that affect ice accretion include altitude, speed, airfoil shape and wing shape, and atmospheric conditions include temperature, LWC and MVD. This study focuses on the atmospheric conditions. The selected sets of independent variables are free stream velocity, LWC, MVD and ambient temperature. Investigation into this group of variables yields understanding of the formation of rime and glaze ice.

In their experimental study,2) Fuchs and Schickel selected LWC and MVD as independent variables and investigated their effects on the shape of icing on a cylindrical rod. They changed one variable at a time while keeping others fixed. However, investigating the controlled effect of a single variable may not reflect the full range of icing behavior, because various factors affecting the shape of ice are nonlinearly related and interfere with each other.

In order to capture the effects of multiple independent variables acting on the ice shape at the same time, the icing results need to be investigated over a specified analysis space. This study defines the analysis space according to the icing conditions described in appendix C of FAR part 25.6) These icing conditions reflect standard conditions for airworthiness certification based on the measured atmospheric information under which icing occurs.7) Therefore, the specified analysis space reflects the actual icing environment that aircraft encounter.

FAR part 25 defines intermittent and continuous maximum conditions depending on the type of cloud. Since the intermittent maximum conditions include the continuous maximum conditions, the analysis spaces of this study for ambient temperature, LWC and MVD follow the limits defined under the intermittent maximum conditions. The limits of ambient conditions under the intermittent maximum are listed in Table 1.

The sampling points are generated in terms of free stream velocity, LWC and MVD via improved Latin hypercube sampling8) within the specified range of the ambient parameters listed in Table 1. Since the ambient temperature is a function of LWC and MVD, as shown in Fig. 2, it is determined using linear interpolation after the sampling points are generated. Some sampling points, especially when LWC and MVD are high, have temperatures outside the −40 to 0°C limits set in FAR part 25 and shown in Table 1. These outliers are eliminated from the pool, and more sampling points are generated within the limits, yielding an even distribution over the analysis space. In total, 186 sampling points are generated. In order to see the influence of ambient

<table>
<thead>
<tr>
<th>Ambient condition</th>
<th>Cases</th>
<th>Airfoil</th>
<th>( \alpha [\text{°}] )</th>
<th>( T_{\infty} [\text{°C}] )</th>
<th>LWC [g/m²]</th>
<th>MVD [µm]</th>
<th>( V_{\infty} [\text{m/s}] )</th>
<th>Spray time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>186</td>
<td>NACA0012</td>
<td>4</td>
<td>−40 – 0</td>
<td>0 – 3.0</td>
<td>15 – 50</td>
<td>15 – 100</td>
<td>360</td>
</tr>
</tbody>
</table>

Fig. 2. Sampling points.
temperature, the sampling points are divided into groups with 10°C temperature ranges, as shown in Table 2.

The icing exposure time and angle of attack effects are important factors that affect the behavior of icing. In order to investigate the influence of the atmospheric conditions, other factors are fixed, such as angle of attack, icing exposure time and so on.

The ice accretion shapes depend on the icing exposure time. Jones and Lewis\(^7\) adopted exposure times of 10, 60s and 30 min. Exposures for 10 and 60s are not long enough to capture the behavior of ice accretion area, maximum thickness and icing limits, which are influenced by runback water that runs along the surface unfrozen. On the other hand, a 30-min exposure is too long to distinguish differences in icing limits and build-up direction; only ice accretion area and maximum thickness are increased. In addition, as many variables are considered in this study, the computing load for that long exposure time is not affordable. A proper exposure time that yields efficient use of computing resources and covers the characteristics of the icing can be drawn from the experimental works of Wright and Rutkowski.\(^9\) They exposed an NACA0012 airfoil to icing conditions for 300, 360 and 420 s. With a fixed angle of attack of 4°, they obtained distinctive shapes of icing according to various ambient conditions. Following their work, this study employs an NACA0012 with an exposure time of 360 s and angle of attack of 4°. This exposure time and angle of attack are fixed throughout this study to distinguish the effects of the specified independent variables.

### 2.2. Calculation of ice accretion shapes

An in-house, 2D numerical analysis code is used to calculate the shape of icing.\(^{10}\) The code consists of an aerodynamic solver, impingement model, thermodynamic model and ice growth model. The aerodynamic solver employs the source and doublet panel method used by other icing analysis codes such as NASA’s LEWICE\(^{11}\) and DRA’s TRAJICE.\(^{12}\) The impingement model determines the amount of liquid water that flows onto the airfoil surface via the Lagrangian approach, which tracks water droplets with drag, gravity and buoyancy forces acting on them. The thermodynamic model employs Messinger’s model,\(^{13}\) which is also used in LEWICE and TRAJICE. The thermodynamic model is needed to calculate heat transfer between the liquid water and airfoil surface under glaze conditions, in which the liquid water runs along the airfoil surface while it freezes.

The code is verified in Ref. 10, which compares the ice accretion shapes achieved by various methods and icing wind tunnel tests under the conditions. The code predicts ice accretion shapes as accurately as other numerical codes under both the rime and glaze conditions.

### 2.3. Parameterization of ice accretion shapes

Direct comparison of ice accretion shapes with respect to Cartesian coordinates, which many early studies employed to verify other codes, presents difficulties in quantitative assessment of the influence of ambient conditions. As a solution to this problem, shape parameters have been defined to characterize icing behavior. Wright and Potapczuk\(^7\) employed length, width and direction of the ice horn as the parameters of the shape of icing.\(^3\) These parameters are focused only on the characteristics of the ice horn and lack elements for quantitative assessment of overall icing behavior on the wing. Furthermore, because measurement of these parameters is not standardized, their values vary from case to case. Ruff and Anderson\(^4\) added more parameters, such as impingement width, stagnation thickness and maximum width to Wright’s parameters\(^3\) and standardized the measurement. However, these parameters are not useful in a performance analysis of an iced wing or in the design of anti/de-icing devices. Therefore, this study defines additional parameters that quantify the overall shape, including that of the ice horn, and standardize measurement of the parameters.

The selected parameters are ice accretion area, maximum thickness, ice heading and icing limits. The ice accretion area is calculated via Green’s theorem, which relates to a closed curve and its area as shown in Fig. 3. The closed curve of the icing is defined as a curve that consists of the exposed ice surface and the part of the airfoil surface that borders the ice. The maximum thickness and ice heading are newly defined as shown in Fig. 4. Maximum thickness is calculated by the difference of the maximum distance and airfoil distance. The maximum distance is the farthest distance between the reference point (0.25, 0) and the ice point, and airfoil distance is the distance between the reference point and intersection point of the airfoil surface and the maximum distance line. The ice heading is defined by the angle between x-axis and the maximum distance line. The icing limits are the bounds of the accreted ice on the wing surface, as is demonstrated in Fig. 5. They are mea-
sured as the integrated distance along the airfoil surface, with a positive sign for the upper surface and a negative sign for the lower surface.

2.4. SOMs

Since there are many independent variables associated with the shape of icing, the relationship between them cannot be systematically determined by investigating the individual influence of a single variable at a time, as early studies did. In addition, such methods cannot give the relative amount of influence of each parameter. This study employs SOMs to comprehensively understand the behavior of shape parameters in terms of ambient conditions.

A SOM is a neural net based on similarities in data. SOMs can project data of higher dimensions into lower dimensions, usually two dimensions, through unsupervised learning of the neurons.

The characteristic vector, which represents the relationship between the input vector and neurons, is trained to produce a 2D map that shows the characteristics of high-dimensional data. Through this projection, the neighboring data in higher dimensions are also placed on the low-dimensional map. In this research, meteorological and flight parameters (i.e., ambient temperature, free stream velocity, LWC and MVD) are depicted on the 2D maps, and then the location of shape parameters (i.e., ice accretion area, ice heading and icing limits) are determined.

The characteristic vectors concerned with neurons are expressed as shown in Eq. (1). Here \( n \) is equal to the dimension of input vector and \( M \) is the number of neurons.

\[
m_i = [m_{i1}, m_{i2}, \cdots, m_{in}], \quad (i = 1, \cdots, M)
\]  

Each neuron is connected to adjacent neurons in a neighborhood relationship and usually form 2D rectangular or hexagonal topology as shown in Fig. 6.

The learning algorithm of SOMs begins with finding the best-matching unit \((m_c)\), which is closest to the input vector \((x)\), as follows.

\[
|x - m_c| = \min |x - m_i|, \quad (i = 1, \cdots, M)
\]  

Once the best-matching unit is determined, the weight adjustments are performed not only for the best-matching unit but also for its neighbors to organize the topological mapping. The adjustment depends on the distance (similarity) between the input vector and the neuron. Based on the distance, the best-matching unit and its neighboring neurons become closer to the input vector as shown in Fig. 7.

In this investigation, SOMs are generated using commercial software Viscovery® SOMine plus 4.0 produced by Eudaptics GmbH. Although, SOMine is based on the general SOM concept and algorithm, it employs an advanced variant of unsupervised neural networks, Kohonen’s Batch SOM.

The vertical and horizontal axes of the SOM, which represent the relationship between ambient conditions and ice accretion shape parameters based on the similarity of the ambient conditions, are not necessarily physically meaningful. Therefore, the similarity can be identified by looking at the relative locations of the data on a plane. The neighboring vectors form groups called clusters, which are used for analyzing the resultant SOMs. As shown in Fig. 8(a), 31 clusters are formed and each cluster is numbered to identify its location.

2.5. ANOVA

Although the SOM yields visualized evaluation of the relationship between the independent variables and the ice
A Kriging model is developed to conduct the ANOVA as a response surface model, and the statistical data of the model is investigated.

\[ \hat{y}(x) = \hat{\mu} + r' R^{-1} (y - \hat{\mu}) \]  

accretion shape parameters, it is only qualitative, and quantitative comparison is still not easy. This study employs ANOVA to quantify the effects of the independent variables on the shape parameters. The key concept of ANOVA is well summarized in Ref. 16) by Jeong et al.

Fig. 8. Results of SOMs.
The vector parameter ($\theta$) for the model fitting is determined by the maximum condition of likelihood function.

$$L(\hat{\mu}, \hat{\sigma}^2; \theta) = -\frac{n}{2} \ln \hat{\sigma}^2 - \frac{1}{2} \ln |R|$$  \hspace{1cm} (7)

$$\hat{\sigma}^2 = \frac{(y - \hat{\mu})R^{-1}(y - \hat{\mu})}{n}$$  \hspace{1cm} (8)

The effects of the ambient and flight variables on the shape parameters can be calculated by decomposing the total variance of the model into the variance component due to the ambient and flight conditions using the obtained Kriging parameter. Decomposition is done by integrating variables out of the model $\hat{y}$. The total mean and the variance of the model are as follow.

$$\hat{y}_{\text{total}} = \int \ldots \int \hat{y}(x_1, \ldots, x_n) \, dx_1, \ldots, dx_n$$  \hspace{1cm} (9)

$$\sigma^2_{\text{total}} = \int \ldots \int [\hat{y}(x_1, \ldots, x_n) - \hat{y}_{\text{total}}]^2 \, dx_1, \ldots, dx_n$$  \hspace{1cm} (10)

The main effect of variable $x_i$ and the two-way interaction effect of variables $x_i$ and $x_j$ are given as:

$$\hat{\mu}_i(x_i) = \int \ldots \int \hat{y}(x_1, \ldots, x_n) \, dx_1, \ldots, dx_{i-1}, dx_{i+1}, \ldots, dx_n - \hat{\mu}_{\text{total}}$$  \hspace{1cm} (11)

$$\hat{\mu}_{i,j}(x_i, x_j) = \int \ldots \int \hat{y}(x_1, \ldots, x_n) \, dx_1, \ldots, dx_{i-1}, dx_{i+1}, \ldots, dx_{j-1}, dx_{j+1}, \ldots, dx_n - \hat{\mu}_{\text{total}}$$  \hspace{1cm} (12)

$\hat{\mu}_i(x_i)$ and $\hat{\mu}_{i,j}(x_i, x_j)$ quantify the effect of variable $x_i$ and interaction effect of $x_i$ and $x_j$ on the shape parameters.

The variance due to the ambient and flight variables $x_i$ is given as

$$\sigma^2 = \int [\hat{\mu}_i(x_i)]^2 \, dx_i.$$  \hspace{1cm} (13)

The proportion of the variance due to the ambient and flight variables $x_i$ to total variance of the model can be calculated as

$$\frac{\sigma^2_i}{\sigma^2_{\text{total}}} = \frac{\int [\hat{\mu}_i(x_i)]^2 \, dx_i}{\ldots \int [\hat{\mu}(x_1, \ldots, x_n) - \hat{y}]^2 \, dx_1, \ldots, dx_n}.$$  \hspace{1cm} (14)

This value indicates the effect of ambient and flight variables $x_i$ on the shape parameters.

The results of the ANOVA are shown in Fig. 9 as bar charts and line graphs that compare relative amounts of influence.

3. Analysis Results

The results of the SOMs and ANOVA are presented in terms of the independent variables. The SOMs are shown in Fig. 8, and the results of the ANOVA are shown in Fig. 9. These bar charts include line graphs that present the results over the temperature range of $-40$–$0^\circ$C, grouped into $10^\circ$C intervals. The temperature range also represents the rime, mixture and glaze ice conditions.

3.1 Total temperature region

The SOMs of Fig. 8 show that the free stream velocity and LWC severely influence the ice accretion area. The high-valued clusters of ice accretion area shown in Fig. 8(f) (13, 14, 21, 22, 12, 19, 20) are similarly matched with the clusters of high free stream velocity (12, 18, 19, 20, 22, 23) in Fig. 8(b), and the clusters of low LWC (18, 20, 29, 30, 31) in Fig. 8(d) correspond to the clusters of small ice accretion area (18, 20, 29, 30, 31). The results of the ANOVA, shown in Fig. 9(a), also confirm that free stream velocity and LWC are the factors that primarily influence ice accretion area within the temperature bounds of $0$–$40^\circ$C, which are specified in FAR part 25. Actually, Figs. 10(a) and 11(a) show the ice accretion area in different icing conditions. These are calculated swept conditions about the free stream velocity and LWC, respectively. These results show that the free stream velocity and LWC severely influence the ice accretion area. The maximum thickness is also affected by the free stream velocity and LWC. The clusters 12, 22, 23, 18, 19, and 20 on the left and right sides of Fig. 8(b) indicate high values for the free stream velocity, and at similar locations in Fig. 8(g) (clusters 13, 14, 21, 22, 12, 19 and 20), the maximum thickness is large. On the other hand, at the bottom right-hand corner of the SOMs of Fig. 8(d) and (g), clusters 18, 20, 29, 30 and 31 show low values for LWC and low maximum thickness. The pattern in the SOM of maximum
thickness is similar to that of the ice accretion area. Both maps have large values on the left-hand side (clusters 13, 14, 21 and 22) and right-hand side (clusters 18, 19 and 20), and they show generally similar distribution. The results of the ANOVA, shown in Fig. 9(b), also confirm that the free stream velocity and LWC are primary parameters affecting the maximum thickness. As shown in Figs. 10(c) and 11(c), the maximum thickness is also affected by the free stream velocity and LWC. Figures 10(c) and 11(c) show the maximum thickness and its direction on the polar coordinate system. The maximum thickness of ice grows according to the increased LWC and free stream velocity.

The causes of these results are analytically determined as follows: Set the control volume between the ice layer on the airfoil surface and the boundary layer. Then, the mass and energy rates in the control volume are conserved as

\[
\begin{align*}
\text{(a) Ice accretion area} & \quad \text{(b) Maximum thickness} & \quad \text{(c) Ice heading} \\
\text{(d) Upper icing limit} & \quad \text{(e) Lower icing limit} \\
\text{(i) 33 m/s} & \quad \text{(ii) 48 m/s} & \quad \text{(iii) 64 m/s} & \quad \text{(iv) 81 m/s} & \quad \text{(v) 101 m/s} & \quad \text{(vi) 110 m/s} & \quad \text{(vii) 129 m/s} \\
\text{(a) Ice accretion shapes} & \quad \text{(b) Ice accretion area} & \quad \text{(c) Ice heading} & \quad \text{(d) Ice distributions} \\
\end{align*}
\]
Eqs. (15) and (16). Subscripts com, in, ice, out, eva and conv are impinging water, run in water, freezing ice, runback water, evaporation and convection, respectively.

\[ \dot{m}_{\text{com}} + \dot{m}_{\text{in}} = \dot{m}_{\text{ice}} + \dot{m}_{\text{out}} + \dot{m}_{\text{eva}} \tag{15} \]

\[ \dot{E}_{\text{com}} + \dot{E}_{\text{in}} = \dot{E}_{\text{ice}} + \dot{E}_{\text{out}} + \dot{E}_{\text{eva}} + \dot{E}_{\text{conv}} \tag{16} \]

The ice accretion area is proportional to the mass of ice accreted on the total airfoil surface, and the maximum thickness is proportional to the mass of ice accreted in the control volume. The mass of inflow water from the neighboring grid cell \((\dot{m}_{\text{in}(i\rightarrow j)})\) is balanced with the mass of runback water \((\dot{m}_{\text{out}(i)})\), which is unfrozen and flows from the current control volume to the next one. If the difference between the inflow mass of liquid water and outflow mass of runback water between grid cells is negligible, the mass flow is balanced as

\[ \dot{m}_{\text{in}(i\rightarrow j)} - \dot{m}_{\text{out}(i)} = 0. \tag{17} \]

Therefore, the mass rate of accreted ice on the airfoil surface is determined by the difference between the water inflow rate onto the surface and the mass rate of evaporated and sublimated water. However, because the evaporated and sublimated water mass is negligible compared to the inflow water mass, the accreted mass rate can be considered a function of the inflow mass rate only. The inflow rate of water mass onto the airfoil surfaces is determined by the collection efficiency \(\beta\), free stream velocity and LWC. The collection efficiency is defined as the ratio of the distance between droplets in the free space to the distance between attached droplets.

\[ \dot{m}_{\text{com}} = V_{\infty} \cdot \text{LWC} \cdot \beta \cdot \Delta s \tag{18} \]

Therefore, as shown in Eq. (18), the ice accretion area and maximum thickness, which are related to the water inflow onto the surface, are also influenced by free stream velocity and LWC.

The ice heading is mainly affected by ambient temperature, LWC and MVD. The SOMs in Fig. 8 show that the patterns of ice heading (Fig. 8(h)), ambient temperature (Fig. 8(c)), LWC (Fig. 8(d)), and MVD (Fig. 8(e)) are similar. The upper left corner (1, 13, 14) and the lower right corner (28, 29, 30, 31) have high values for ice heading, and upper clusters (5, 6, 7, 8) have higher values than lower clusters. The ambient temperature values are high in clusters 4, 5, 6, 7, 8 and 9, and LWC values are high in the clusters 1, 2, 3 and 13, where the ice heading is large. The MVD has high values in clusters 28, 29, 30 and 31, where the ice heading also has high values. The results of the ANOVA, shown in Fig. 9(c), confirm that the temperature, LWC and MVD are main contributors of influence on ice heading.

The physical mechanisms of the ambient conditions’ influence on the ice heading are as follows: The water droplet, which contacts the airfoil surface from free space, releases heat as it freezes. As the released latent heat is cooled by convection, the accreted mass rate of ice on the airfoil surface increases. The force convection energy\(^{11}\) in the boundary layer is calculated as

\[ \dot{E}_{\text{conv}} = h_c \left[ T_{\text{sur}} - \left( T_{\text{edge}} + \frac{r_c V_{\text{edge}}}{2C_p} \right) \right] \Delta s \tag{19} \]

where \(h_c\), \(r_c\), \(C_p\), \(T_{\text{sur}}\), \(T_{\text{edge}}\) and \(V_{\text{edge}}\) are convective heat transfer coefficient, recovery factor, specific heat, temperature of surface, temperature at the edge of the boundary layer and velocity at the edge of the boundary layer, respectively.

If the ambient temperature increases, the temperature in the boundary layer also increases. Therefore, assuming that the heat transfer coefficient and the speed at the boundary layer remain constant, convective cooling diminishes. An increase in ambient temperature reduces the ice accretion.
rate on the surface, leaving some water unfrozen. The unfrozen water (i.e., runback water), flows from the leading edge toward the trailing edge along the airfoil surface due to the flow around it. As a result, the ice heading increases, meaning that it approaches vertical.

Figure 12 shows the results of swept conditions and ambient temperature. Ice accretion shapes (Fig. 12(a)), ice accretion area (Fig. 12(b)), ice heading (Fig. 12(c)) and ice distributions (Fig. 12(d)) are represented. The results are obtained in different ambient temperatures, but the other parameters are fixed. Due to the increased temperature at the edge of the boundary layer, the thickness of ice is reduced, and the generated runback water leads to an increased value of the ice heading.

Under glaze icing conditions, the convective heat transfer determines the thickness of the ice. The increase in LWC, which is not affected by the boundary layer temperature or velocity, causes an increase in water inflow onto the airfoil surface and produces runback water. The runback water, just as in the case of increased temperature, flows along the airfoil surface and increases the ice heading.

The SOMs of Fig. 8 show clear correlation between the icing limits and the free stream velocity and ambient temperature. However, no clear similarity with the map of LWC is found. For the upper surface in Fig. 8(i), large icing limits appear in the right-hand (11, 12, 10, 18, 19, 20) and left-hand (13, 21) clusters, and upper clusters (4, 5, 7) have larger values than lower ones. For the lower surface, as the values are negative, a lower value denotes a larger limit of accretion. As Fig. 8(j) shows, the pattern of the icing limits is identical to the upper surface with the opposite sign. The results of the ANOVA, shown in Fig. 9(d) and (e), also indicate that the free stream velocity and ambient temperature are primary contributors to the influences on the icing limits. Figures 10(d) and 12(d) show the distribution of ice thickness along the airfoil surface. In the figures, we can confirm the expended region covered with ice in the high temperature and free stream velocity condition.

### 3.2. Influence of temperature

As depicted in Fig. 12, which shows the shape of icing in the $-21$–$-3^\circ C$ range, the ambient temperature has a huge impact on the shape of the ice. As the shape of icing varies, the flow field also varies, resulting in even greater changes in the icing behavior. Figure 13 shows the mechanism of influences on the ice accretion shape under rime conditions and glaze conditions. A higher ambient temperature reduces the amount of ice accreted on the airfoil surface and produces runback water. The behavior of the runback greatly alters the shape of the ice. The changed shape of the leading edge of the airfoil affects water inflow rate as well as flow field around the airfoil. The influence of the ambient temperature is investigated using the results of an ANOVA, shown in Fig. 9 for four sections in 10°C intervals as well as for the whole range.

According to the results of the ANOVA, three greatly dis-
tinct types of icing behavior are found, and the relative influences of the ambient conditions are very different in each group. The three groups are

1. -10°-0°C, glaze conditions
2. -20°-10°C, mixture conditions
3. -40°-20°C, rime conditions

The rime conditions occupy both the -40°-30°C and -30°-20°C intervals, where the effects of the ambient conditions are similar. Under the rime conditions, no significant effect of exposure time is found. Furthermore, because the water freezes as soon as it collides with the surface, the influence of the change in ice shape on the flow field is relatively small. Therefore, ice accretion shape varies very little, although it is influenced by the amount of water inflow onto the airfoil surface. As shown in Eq. (18), the accreted ice mass depends on LWC, free stream velocity and collection efficiency. However, because the shape of the leading edge does not change much, the effect of collection efficiency is slight. Therefore, under rime conditions, as shown by the ANOVA in Fig. 9, only LWC and free stream velocity are dominant factors that change the ice accretion shape parameters, namely ice accretion area, maximum thickness, ice heading and icing limits. The results indicate that any design of anti/de-icing devices that operate under rime conditions needs to take the flight speed and LWC into account.

Under glaze conditions, where the ambient temperature appears to be relatively high, less ice can be produced with the same control volume, and runback occurs. Therefore, thin and wide ice is formed, as depicted by the -3°C shape in Fig. 12. In this case, the free stream velocity and LWC are the primary factors that affect the ice accretion area in Fig. 9(a) and maximum thickness in Fig. 9(b). This is a consequence of the relationship between the change in water inflow rate and change in ice accretion area and maximum thickness. The effect of free stream velocity increases under lower temperatures because higher velocity increases the effect of convective cooling.

Under glaze conditions, the ice heading of Fig. 9(c) is mainly affected by the ambient temperature and MVD. The ambient temperature is the primary contributor of the upper surface icing limit (Fig. 9(d)) as well. As a result of runback due to high temperature, the ice expands beyond where the water droplet collides and sticks. Figure 9(d) and (e) shows that, under higher temperatures, the contribution of ambient conditions to the icing limits of the upper surface differs more from that of the lower surface. As the ambient temperature and LWC become dominant, runback determines the icing limit of the upper surface, while the free stream velocity and MVD are main contributors on the lower surface.

The MVD affects the trajectory of water droplets. Under rime conditions, because water droplets freeze as soon as they collide with the airfoil surface, their trajectories determine the icing limits. The trajectory of a water droplet affects the icing limits under glaze conditions as well, because the icing limit of the upper surface expands further than where the water droplets collide and the icing limit of the lower surface is within the collision area. The mechanism of MVD’s influence on the trajectory of a water droplet is as follows: As the diameter of the water droplet increases, the Reynolds number, which is based on the relative speed of the droplet and air, also increases, as does the droplet’s diameter. At the same time, the drag coefficients (Cd) of Eqs. (20) and (21) decrease.19)

\[ C_d = \frac{24}{R_e_d} \quad \text{(Stokes, } 0.1 < R_e_d < 10) \tag{20} \]
\[ C_d = \frac{12}{\sqrt{R_e_d}} \quad \text{(Empirical, } 10 < R_e_d < 1000) \tag{21} \]

The change in drag coefficient affects the droplet’s trajectory, as indicated by the following governing equation of motion of the droplet.11)

\[ m \ddot{a} = (\rho_d - \rho_{air})V_d \ddot{V} + \frac{1}{2} \rho_{air} A_d C_d \left| \overrightarrow{V} \right| \left( \overrightarrow{V} \right) \tag{22} \]

The subscripts of d and air represent the droplet and air, and \( \overrightarrow{V} \) is relative velocity between air and the droplet. \( V_d \) is the volume, and \( A_d \) is the area of droplet. The first term on the right-hand side indicates the effect of gravity and buoyancy, and the second term indicates the effect of drag. Since the drag coefficient affects the droplet’s acceleration, velocity and position, its trajectory and the location of collision on the airfoil surface depend on MVD. As a consequence, icing limits are changed. Figure 14 shows the effects of the MVD. Due to the increased droplet size, upper and lower icing limit are changed. We swept the value of MVD from 20 to 70μm. However, the effect of MVD within the 15–50μm range specified in FAR part 25 is not as large as the effects of free stream velocity and ambient temperature.

In the -20°-10°C interval, the shape of the leading edge changes significantly, and runback freezes intensively where the convective cooling is strong. Therefore, free stream velocity greatly influences the ice accretion area, maximum thickness and ice heading. The airflow accelerates at the leading edge and gains even more speed as the ice thickens, resulting in increased frontal area on which water droplet can collide. Therefore, more points on the surface have positive collection efficiency, and ice accretion area increases. Figure 9(a) and (b) shows that the effect of MVD increases most in this temperature range.

The findings in this section identify some aspects to be considered for design of anti/de-icing devices. The heat capacity of a heated anti-icing device needs to be controlled and \( \overrightarrow{V} \) is relative velocity between air and the droplet. A de-icing device needs to be turned on more often under high-speed, high-LWC conditions. The ice heading indicates where the maximum heat capacity should be applied for a heated anti-icing device. As the ice heading is affected by the temperature and MVD under a high-temperature regime and free stream velocity and LWC under a low-speed regime, all of the variables need to be considered in determining the location of maximum heat capacity. Considering the
behavior of the icing limit, anti/de-icing devices for glaze conditions need wide application areas. A heated device needs to distribute different heat capacities along the airfoil surface, with higher heat capacity on the upper surface than on the lower surface for higher ambient temperatures. The expansion of accretion due to runback needs to be accounted for on the upper surface, while the expansion of accretion due to collection of water droplets and airfoil shape needs to be considered for the lower surface.

4. Conclusions

The relationship between the shape of icing and ambient conditions was investigated via SOMs and ANOVA under the conditions specified in appendix C of FAR part 25. The investigation via SOMs and ANOVA yielded qualitative and quantitative comparisons and prioritized the effects of variables. The results of the investigation confirm physical findings.

1) In the investigation, the ice accretion area and maximum thickness were affected primarily by free stream velocity and LWC, and ice heading was affected by free stream velocity, ambient temperature and LWC.

2) Three distinctive ambient temperature groups were found, depending on the response of the ice shape to the ambient conditions. They were rime ice conditions of $-40$--$-20^\circ$C, mixture ice conditions of $-20$--$-10^\circ$C and glaze ice conditions of $-10$--$0^\circ$C.

3) The ice accretion area and the maximum thickness were primarily affected by free stream velocity and LWC under both the rime and glaze conditions. However, under mixture conditions, the influence of MVD was greater than that of LWC. The heat capacity, which is related to the ice accretion area and maximum thickness, can be estimated regardless of the temperature because it has little effect.

The heat capacity of a heated anti-icing device needs to be controlled according to flight speed and LWC, and a de-icing device needs to be operated more frequently under high-speed and high-LWC conditions.

4) The free stream velocity and LWC greatly influenced ice heading regardless of temperature. The ambient temperature and MVD had greater influence on the ice heading under higher temperatures, and free stream velocity and LWC increased in importance under lower temperatures. Therefore, all the variables need to be considered in an estimation of the location of maximum heat capacity.

5) The icing limit of the upper surface was affected more by runback, while the icing limit of the lower surface was more affected by where the water droplet sticks. This behavior can be seen clearly under high temperatures ($-10^\circ$C or above). Since the location in which water droplets stick depends on the airfoil shape, anti/de-icing devices need to consider the airfoil shape of the lower surface and runback of the upper surface when determining their application area.

To analyze the relationship between the ice accretion shape parameters and ambient parameters, SOMs and ANOVA are employed. This approach can be adopted to design field and parameter analysis. In the design field, important design variables can be extracted and the effects of those design variables to the objective function can be evaluated. In case of the parameter analysis, we can discover the associated parameters, although the parameters are examined under the non-linear conditions.

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References