Sensor Selection and Optimization for Aerospace System
Health Management under Uncertainty Testing

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Prognostics and health management (PHM) has an important part in aerospace systems. Information sensing and testing are the bases of PHM, and design for testability (DFT) developed concurrently with system design is considered a fundamental way to improve PHM performance. The traditional DFT, which is only based on the requirements of fault detection and isolation, is not suitable for sensor design and optimization for PHM. Aiming to solve this problem, the intrinsic requirements of PHM for testability are firstly analyzed qualitatively and the corresponding testability indexes are defined quantitatively. Then, a sensor selection/optimization process for PHM is presented. Fault detection uncertainty is also analyzed systematically from the view of fault attributes, sensor attributes and fault-sensor matching attributes, respectively. Based on the requirements and process, the object and constraint models of sensor optimization selection problem are studied in great detail. For aerospace system health management, a sensor optimization selection model is constructed that treats sensor total cost as the objective function and the proposed testability indexes under uncertainty test as constraint conditions. Due to the NP-hard property of the model, a generic algorithm (GA) is introduced to obtain the optimal solution. The application examples show that the proposed model and algorithm are effective and feasible.

Key Words: Prognostics and Health Management, Design for Testability, Testability Index, Sensor Optimization Selection Process, Fault Detection Uncertainty, Sensor Optimization Selection Model, Generic Algorithm

Nomenclature

S: health state space
si: the ith element in the S
T: sensor set
tj: the jth sensor in the set T
F: fault set
fi: the ith fault in the fault set F
A: failure rate vector
λi: the ith element in the vector A
FC: criticality vector
fci: the ith element in the vector FC
cj: the jth element in the vector C
FR: sensor failure rate vector
rfj: the jth element in the vector FR
X: sensor selection situation vector
xj: the jth element in the vector X
Q: the upper limit of X
qj: the jth element of the vector Q
B: binary dependency matrix
bij: the ith row and jth column element of B
u: the first item of the bij
v: the second item of the bij
T: the selected sensor set
D: binary dependency matrix

1. Introduction

Prognostics and health management (PHM) can be thought of as an overall comprehensive management style used during the system lifecycle.1) PHM utilizes sensor information to monitor, diagnose, predicate and evaluate the state of system health combined with models and algorithms, and activate the optimum maintenance scheme under constraints of resource and cost.2–4) PHM is a significant factor in improving aerospace systems safety, reliability, maintainability and affordability, reducing lifecycle cost and realizing autonomic logistics.5,6) With the rapid development of the PHM concept and fault prognostics technology, PHM has become an important part in complex aerospace systems such as helicopters, aircraft engines, missiles and so on.7) At present, more and more attention is being focused on PHM architecture,8,9) PHM-related models and algorithms. Few studies focus on information sensing and

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testing of PHM. However, information sensing and testing are the premise and foundation of PHM.\textsuperscript{2,3,10} It is no exaggeration to say that data acquired from available sensors forms the foundation upon which all the PHM systems are based, and the available sensor suite directly impacts on the overall fault diagnostics and prognostics performance that can be achieved.\textsuperscript{11} Some applications and studies also show that PHM performance is more dependent on sensor information rather than on the adopted models or algorithms, and that optimized sensor types, locations and numbers can enhance fault diagnostics and prognostics capability greatly.\textsuperscript{12}

On the other hand, the traditional attached sensor design methodology has shown many disadvantages, and more and more researchers and institutes are diverting to the concurrent design for testability (DFT) methodology, and the related enable technologies and tools are becoming more advanced.\textsuperscript{13–15} Moreover, the DFT method has an important role in aerospace system fault detection and maintenance. Putting more attention on sensors and tests and using the concurrent design-based DFT methodology are deemed to be fundamental ways to improve PHM performance.

Sensor selection and optimization is one of the important elements of DFT.\textsuperscript{15} Although safety and reliability are just as important as system performance, traditionally, the sensors are primarily selected based on fault detection and isolation requirements rather than on PHM, especially in terms of fault prognostics needs.\textsuperscript{15–18} Furthermore, sensors are primarily designed through an ad hoc heuristic process rather than through a systematic method.\textsuperscript{19} In other words, for aerospace system health management, sensor selection and optimization should meet PHM requirements, and the attached sensor design methodology must give way to the systematic concurrent design approaches.

Against this backdrop, the main contents of the paper are: 1) the intrinsic requirements of PHM for testability are comprehensively analyzed qualitatively and quantitatively; 2) a sensor selection and optimization architecture is proposed that will provide a justifiable sensor suite to address PHM requirements of aerospace systems and support concurrent design methodology; 3) fault detection uncertainty is analyzed systematically; 4) A sensor selection and optimization model for aerospace system health management and the corresponding algorithm are presented in detail; 5) Two applications are introduced to validate and verify the proposed model and algorithm.

2. Prognostics and Health Management Needs for Testability

2.1. Qualitative analysis

First of all, it is very important to understand the intrinsic difference between the testability for PHM and the traditional testability theory (for fault detection and isolation). Intuitively speaking, testability constitutes a part of PHM performance, and systems being of good testability can provide sufficient state information for PHM. So it is a feasible way to analyze the PHM requirements for testability from the view of data information acquisition.

In applications, PHM monitors important failure modes, detects and isolates faults, and predicts the remaining useful service life of key components. Based on the results of monitoring, diagnosis and prognosis, the state of system health can be evaluated and the corresponding maintenance decisions such as condition-based maintenance, predictive maintenance and autonomic logistics are activated. The mutual relationships are shown in Fig. 1.

It is obvious that health state evaluation (HSE) is the most important part in PHM, and condition monitoring, fault diagnostics and fault prognostics are all inputs of HSE. So DFT for PHM should guarantee that the state of system health is evaluable.

Generally speaking, HSE is a nonlinear comprehensive decision process from failure mode space to feature space and then to health state space, which can be formulated by $S = \Phi(\text{FM}, \text{KL})$. $S$ denotes health states which can be discrete health state space, $S = \{s_1, s_2, \ldots, s_n, s_{n+1}\}$ or a continuous health index, $S \in [0, 1]$. FM denotes failure modes which can be gradual faults or abrupt faults. KL denotes system domain knowledge which usually includes system structure, function, behavior and operation (external environment, load intensity, operation mode, operation time, and so forth). At the design stage, system structure and function are usually determined by system mission, and system operation is usually determined by mission section and application scenarios. Hence, in order to enable evaluation of the system health, the most important aspect at the design stage is that the system failure mode space should be observable.

Preventing fault occurrence and reducing fault effects are the main goals of PHM, so fault diagnostics and prognostics are the two key enable technologies in PHM. Fault diagnostics is an assessment about the current and past state, which usually deals with fault detection, isolation and identification based on observed symptoms of a system when abnormality occurs. While fault prognostics is an assessment of the future state, which deals with fault and degradation prediction before their occurring. So testability for PHM should enable faults detectable, isolable and predictable.
2.2. Quantitative analysis

In order to describe the requirements quantitatively, three testability indexes for PHM are defined, including universe fault detectable rate (UFDR), universe fault isolable rate (UFIR) and universe fault predictable rate (UFPR). UFDR and UFIR are defined as follows.\(^{(14)}\)

Definition 1: UFDR is generally defined as: during the stated time span, the ratio of the number of faults detected correctly by sensors to the total number of system faults.

Definition 2: UFIR is generally defined as: during the stated time span, the ratio of the number of faults isolated correctly to no more than the stated replaceable units by sensors to the number of the detected faults during the same time span.

Generally, a fault is detectable doesn’t mean the fault is predictable. Whether a fault is predictable or not depends on two basic factors: one is an objective factor, namely, the fault should be progressive in nature; the other is a necessary factor, namely, the fault should be a key fault or key component fault. Thus, possible predictable fault (PPF) can be defined as:

Definition 3: PPF is a gradual key fault or key components’ fault.

Besides, the predictability of a fault is also related to timely detection and evolution track. If a fault is detected when or after the fault leads to a failure, fault prognostics become insignificant. Furthermore, if the fault evolution process can not be tracked by some sensors, (data driven-based) fault prognostics may not be realized. So predictable fault (PF) is defined as:

Definition 4: PF is a PPF of which the early state is detectable and the evolution process is trackable.

The definition describes fault predictability through fault early state detectability and fault evolution process trackability. PFs can be obtained by fault mechanism analysis and sensor detection ability analysis. In applications, it is assumed that if a sensor can detect the early state of a fault, then it also means the sensor can track the fault evolution process.

Based on definitions 3 and 4, UFPR can be defined as:

Definition 5: UFIR is defined as: during the stated time span, the ratio of the number of PFs determined correctly by sensors to the total number of PPFs.

3. Sensor Selection/Optimization Process for Prognostics and Health Management

When the requirements analysis of PHM for testability is accomplished, sensor selection and optimization can be implemented effectively in order to meet the requirements.

As stated previously, the attached sensor design approach has brought about many disadvantages. Typically, once the system design scheme is confirmed, it is very hard and costly to add sensors or change configuration. In order to reduce system development cost and improve integration effect, model-based sensor selection and optimization methodology becomes popular at present.\(^{(15)}\) One aspect is that model-based way is very convenient to amend system design scheme according to the feedback testability analysis results; the other is that the knowledge reusability of models enables different engineers at different design stages to have consistent understandings for a system.

Testability model is the base of model-based methodology, once the system testability model is built, sensor optimization selection model can be constructed and the corresponding optimization algorithm can be designed. When the sensor suite is determined, the selected sensors can be modeled into testability model and further the sensor suite’s fault detectability, fault isolability and fault predictability can be evaluated. If the evaluation results satisfy the requirements of PHM for testability, then, an optimal sensor suite is obtained; otherwise, sensor selection scheme should be changed and the optimization process will continue until the requirements are met. Generally, the sensor selection and optimization process is shown in Fig. 2.

In the process, knowledge bases are mainly used to construct system testability model. Knowledge that is needed to build the testability model includes, but is not limited to, system schematics (component connectivity topology), failure modes, sensor attributes, system functional behavior, information flow (energy, material, data) and expert experience and so forth. Many approaches such as Petri net, fault tree and directed graph (DG) can be used to build system testability model. Among these various models, directed graph is widely used because DG provides a powerful representation to capture cause-effect information about the system. Further, DG doesn’t require complete quantitative description and can be developed from partial information such as system structure and function. At present, the typical DG-based testability models are multi-signal flow graph\(^{(20)}\) and information flow graph.\(^{(21)}\)

Unique properties regarding the architecture include that the sensor suite design and aerospace system design can be developed concurrently; sensor selection and optimization is based on PHM requirements rather than only on fault detection and isolation needs; sensor optimization selection process is a closed loop that can be used to validate and assess the selected sensor suite.

The sensor optimization selection model and the optimization selection algorithm are the main studies of the paper.

\[\text{Fig. 2. Sensor selection/optimization process for PHM.}\]
4. Fault Detection Uncertainty Analysis

In the existing sensor optimization selection models, it is always assumed that fault detection is uncertainty. In other words, if a sensor relates to a fault, it means the fault can be detected by the sensor with probability one. However, for complex aerospace systems, fault detection is full of uncertainty due to external environment, operation process, electromagnetic interference, and so on. Taking fault detection uncertainty into account in the sensor selection model is more rational and applicable. Generally, fault detection uncertainty closely relates to fault attributes, sensor attributes and fault-sensor matching attributes.

4.1. Fault attributes analysis

Fault detection probability is obviously affected by fault attributes.

(1) Fault sensitivity is the relative quantity of fault statistical features. Fault statistical features are generally divided into time-domain features which may be amplitude, peak, root-mean-square, margin factor, kurtosis and frequency-domain features which may be frequency, wavelet information entropy and power spectrum, etc. Fault sensitivity is very important for fault early state detection and fault prognostics.

(2) Fault stability is the fluctuation degree of fault statistical features, which can be described by stable behavior quantity.

(3) Fault resolution is the minimum measurable variation of fault symptoms.

(4) Time-to-failure (TTF) is the time duration between the initiation of a fault and the time when the failure occurs.

(5) Fault symptom duration is time duration of fault statistical features, which usually equals time-to-failure.

(6) Fault detection threshold is the minimal fault symptom quantity which enables the fault detectable. Generally speaking, at the same fault severity level, the smaller the fault detection threshold is, the higher the fault detection probability is. Of course, the probability of false alarm will increase.

4.2. Sensor attributes analysis

Obviously, sensor attributes will have direct impacts on fault detection probability.

(1) Sensor resolution is the capability that a sensor can measure the minimum variation of inputs.

(2) Sensor signal to noise ration (SSNR): a high SSNR implies fault detection uncertainty is small, while a low SSNR implies that it is hard for the sensor to detect a fault. Especially, sensors with a low SSNR are not suitable for fault early state detection.

(3) Sensor failure rate is the probability that a sensor can not execute the stated functions during the state time span and at the stated conditions.

(4) Sensor sensitivity is the ratio of the output variation at sensor static condition to the corresponding input variation.

4.3. Fault-sensor matching attributes analysis

As we know, a sensor is very suitable for certain fault detection. However, it will be bad for another fault detection. So fault-sensor matching attribute is also an important factor for sensor optimization selection for PHM.

(1) Sensor fault detection timeliness (SFDT) is the ratio of the time span between the initiation of a fault (potential failure) and the detection of the fault by the sensor (time-to-detection, TTD) to the duration between the initiation of the fault and the time when the failure occurs (time-to-failure, TTF).

A low SFDT means that the sensor can detect the fault occurrence at early stage, which is very useful for fault prognostics; while a high SFDT means the sensor needs a long time to declare the fault occurrence. If SFDT ≥ 1, which implies that the sensor detect the fault when the fault leads to a failure, and fault prognostics becomes insignificant. So sensor selection for PHM should take SFDT into account.

(2) Sensor fault detection sensitivity (SFDS) is the ratio of the sensor variation of per unit sensor resolution to the fault variation of per unit fault resolution.

(3) Sensor fault trackability (SFT) is the ratio of the time span of fault symptoms tracked by the sensor (symptom-tracking-time, STT) to the time to failure (TTF).

Generally, when a fault occurs, a sensor can track the fault until the fault evolves to failure. Sensors of these properties can be selected for fault prognostics and health evaluation. However, for certain sensors, they will return to normal measurement state after detecting and tracking the fault symptoms for a period of time span, so the sensors will not be suitable for PHM.

Based on the uncertainty analysis stated above, one can see that a fault relating to a sensor may not means the sensor can detect the fault with probability one.

Definition 6: Sensor fault detection probability (SFDP) is the extent to which a sensor can detect the presence of a particular fault, which is also called as true positive detection probability.

Generally speaking, when sensor detection results are two-value, i.e., pass and fail, and ground truth is two-value, i.e., fault and no fault, then, sensor detection situations are shown in Table 1.

Table 1. Sensor detection situations.

<table>
<thead>
<tr>
<th>Sensor detection results</th>
<th>Pass (Negative)</th>
<th>Fail (Positive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground truth</td>
<td>True Negative</td>
<td>False Positive</td>
</tr>
<tr>
<td>No fault</td>
<td>False Negative</td>
<td>True Positive</td>
</tr>
</tbody>
</table>

Obviously, true negative and true positive are correct detection situations, and false negative and false positive are wrong detection situations. Further more, false positive means that a sensor gives an alarm when there is no fault, which is usually supposed as “false alarm”; false negative means that a sensor doesn’t declare a fault when there is fault, which is often referred to as “miss detection.”

Note that the false negative detection probability is the complement of the true positive detection probability; similarly, true negative detection probability is the complement.
of the false positive detection probability. So in engineering applications, we usually adopt true positive detection probability and false positive (false alarm) detection probability to describe the fault detection uncertainty.

It is possible that a fault can be detected by more than one sensor, so fault total detectable probability (FTDP) should be defined based on SFDP.

Definition 7: FTDP is defined as the extent to which the sensor scheme can detect the presence of a particular fault.

5. Sensor Optimization Selection Model for Prognostics and Health Management

5.1. Completeness description of a sensor set

Before sensor selection and optimization, the sensor set to be selected should satisfy the completeness requirements.

Definition 8: if sensor set $T$ can satisfy the testability indexes required by the system PHM, then, the sensor set is complete.

Definition 9: given the complete sensor set $T$, if completeness requirements will not be satisfied again when an arbitrary sensor is eliminated, then, the sensor set is minimal complete.

So sensor selection and optimization process should include the following three contents:

- Justify the completeness of the sensor sets to be selected;
- If the sensor set is not complete, then, a proper number of sensors should be added in order to satisfy the completeness requirements;
- Select the minimal complete sensor set among the complete sensor sets.

5.2. Objective function modeling

The PHM performance of complex aerospace systems greatly depends on sensor selection and configuration. Poor sensor suite may cause some problems. For example, some faults or states can’t be detected or the detection reliability is low, and key gradual faults can’t be detected timely and tracked effectively. At system early design stage, the available knowledge is limiting. The fault set is $F = \{f_1, f_2, \ldots, f_m\}$, the corresponding failure rate vector is $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_m]$, and the criticality vector is $FC = \{fc_1, fc_2, \ldots, fc_m\}$, $fc_i$ ($i = 1, 2, \ldots, m$) can be type I (catastrophic), type II (fatal), type III (critical) or type IV (light). The complete sensor set to be selected is $T = \{t_1, t_2, \ldots, t_n\}$, and the corresponding cost vector is $C = [c_1, c_2, \ldots, c_n]$. Cost is used in a general sense and usually includes sensor cost, test operation cost and post-processing cost. The sensor failure rate vector is $FR = [r_1, r_2, \ldots, r_m]$. The sensor selection situation vector is $X = [x_1, x_2, \ldots, x_n]$, where $x_j$ ($1 \leq j \leq n$) denotes the number of the selected sensor $t_j$, and the vector $Q = [q_j]$ denotes the upper limit of $X$, $x_j \leq q_j$, $x_j \in Z^+$. A matrix $B = [b_{ij}]_{m \times n}$ generated by reachability analysis or fault simulation denotes dependency between fault/fault evolution and sensors. The rows of $B$ correspond to faults, and the columns correspond to sensors. Element $b_{ij}$ is a two-tuple, $b_{ij} = (u, v)$. If sensor $t_j$ can detect fault $f_i$ and its early state, then $b_{ij} = (1, 1)$. If sensor $t_j$ can detect fault $f_i$ but can not detect its early state, then $b_{ij} = (1, 0)$. The reason may be that the selected sensor has no ability to detect the early state of the fault or that the fault is abrupt. If sensor $t_j$ can not detect fault $f_i$ and its early state, then $b_{ij} = (0, 0)$ ($b_{ij} = 0$ for short). Generally, if a sensor can detect the early state of a fault, it also means the sensor can detect the fault, so the case $b_{ij} = (0, 1)$ will not exist.

Suppose that there exists, at most, a single fault in the system at any given time. Given the selected sensor set is $T_s$, which is a subset of $T$, and the corresponding dependency matrix becomes $D = [d_{ij}]_{m \times n'}$, $d_{ij} = (u, v)$, $n' = |T_s|$, $|\cdot|$ denotes set cardinality.

In engineering applications, sensor selection objective functions are usually divided into three categories:

- Take sensor total cost as objective function:
  \[
  \min \sum_{i \in T} (c_ix_i) \quad (1)
  \]

- Take sensor total failure rates as objective function:
  \[
  \min \sum_{i \in T} (r_ix_i) \quad (2)
  \]

- Take the total hazard of missed detection faults as objective function:
  \[
  \min \sum_{f \in F} (1 - FD_f) \times fc_i \times \lambda_i \quad (3)
  \]

where $FD_f$ is the probability of fault $f_i$ detected by sensor suite.

5.3. Constraint condition modeling

Given $\cup$ denotes the boolean variable or operation, and $\oplus$ denotes set XOR operation, when the two set are different, the operation result is true. $d_i(k)$ denotes the $k$th item of the two-tuple $d_i = (u, v)$, $k = 1, 2$. $T_h$ and $T_\theta$ denote the sensor sets which can detect fault $f_i$ and fault $f_j$, respectively, i.e., $T_h = \{t_j|d_j(1) = 1, \forall t_j\}$, $T_\theta \subseteq T$ is also called fault features of fault $f_i$. $FP \subseteq F$ denotes system PPFs. Given the ambiguity group size is $L$, then, based on the binary dependency matrix $D$, the detectable faults set $FD$, isolable faults set $F_I$ and predictable faults set $FP$ are formulated respectively by:

\[
FD = \left\{ f_i|f_i \in F, \bigcup_{i \in T} d_i(1) = 1 \right\}
\]

\[
F_I = \left\{ f_i|f_i \in FD, \bigoplus_{i \in F} T_h \oplus T_\theta \leq L, \forall f_j \in F, f_j \neq f_i \right\}
\]

\[
FP = \left\{ f_i|f_i \in FP \cap FD, \bigcup_{i \in T} d_i(1) = 1 \right\}
\]

(4)

Obviously, $F_I \subseteq FD \subseteq F$, $FP \subseteq FP \subseteq F$, $FP \subseteq FD$.

- Constraint models under perfect tests

Perfect test doesn’t take fault detection uncertainty into account. In other words, in the dependency matrix $D$,
\[ d_{ij}(k) = 1, \text{ which denotes that sensor } t_j \text{ relates to fault } f_i, \]
also means sensor \( t_j \) can detect fault \( f_i \) with probability one when fault \( f_i \) occurs. According to definition 1, 2 and 5, the constraint models are formulated respectively by:

\[
\begin{align*}
UFDR &= \frac{\sum_{j \in F_0} \lambda_j}{\sum_{j \in F}} \\
UFIR &= \frac{\sum_{j \in F_1} \lambda_j}{\sum_{j \in F}} \\
UFPR &= \frac{\sum_{j \in F_2} \lambda_j}{\sum_{j \in F}}
\end{align*}
\]

\[ \cdot \text{Constraint models under imperfect tests} \]

For complex aerospace systems consisting of mechanics, electronics and hydraulics, a sensor relating to a fault may not mean the fault can be detected by the sensor with probability one. Fault detection probability greatly depends on fault attributes, sensor attributes and fault-sensor matching attributes. Obviously, it is more rational and applicable to take those attributes into account when designing sensors in aerospace systems. Generally, sensor reliability can be featured by sensor failure rate \( r_j \) and we suppose that sensors fail independently. So the impacts on detectability and predictability of fault \( f_i \) can be formulated respectively by:

\[
\begin{align*}
R^1_i &= 1 - \prod_{t \in T_s} x_{tid}(1) \\
R^2_i &= 1 - \prod_{t \in T_s} x_{tid}(2)
\end{align*}
\]

\[ \text{SSNR, sensitivity, timely detection and symptom duration can be called “sensing probability,” which can be shown by parameter } \rho_j. \]

Here, \( \rho_j \) denotes the sensing probability of sensor \( t_j \) to fault \( f_i \).

\[
\rho_j = \begin{cases} 
(1 + e^{-10(\text{SFDS}_j - 0.5)})^{-1} & \text{if } \text{SFDS}_j < 1 \\
(1 - \text{SSNR}_j)^{0.5} \times (\text{SFT}_j)^{0.2} & \text{if } \text{SFDS}_j \geq 1 
\end{cases}
\]

where \( \text{SFDS}_j \) denotes the detection sensitivity of sensor \( t_j \) to fault \( f_i \), \( \text{SSNR}_j \) denotes the SSNR of sensor \( t_j \), \( \text{SFT}_j \) denotes the detection timeliness of sensor \( t_j \) to fault \( f_i \) and \( \text{SFT}_j \) is the trackability of sensor \( t_j \) to fault \( f_i \). Time to detection, time to failure and symptom duration time span can be obtained by fault simulation or the fault propagation timing analysis method.\(^{22}\)

Sensing probability impacts on detectability and predictability of fault \( f_i \) can be formulated respectively by:

\[
\begin{align*}
P^1_i &= \sum_{j \in T_s} \rho_j x_{jd}(1) \\
P^2_i &= \sum_{j \in T_s} \rho_j x_{jd}(2)
\end{align*}
\]

According to Eqs. (6) and (8), the total detectable and predictable probability of fault \( f_i \) can be formulated respectively by:

\[
\begin{align*}
\text{FTDP}^1_i &= R^1_i \times P^1_i \\
\text{FTDP}^2_i &= R^2_i \times P^2_i
\end{align*}
\]

According to definitions 1, 2 and 5, the constraint models under uncertainty tests are formulated respectively by:

\[
\begin{align*}
UFDR &= \frac{\sum_{j \in F_0} \lambda_j \text{FTDP}^1_i}{\sum_{j \in F}} \\
UFIR &= \frac{\sum_{j \in F_1} \lambda_j \text{FTDP}^1_i}{\sum_{j \in F}} \\
UFPR &= \frac{\sum_{j \in F_2} \lambda_j \text{FTDP}^2_i}{\sum_{j \in F}}
\end{align*}
\]

5.4. Sensor optimization selection model for aerospace system health management

Sensor selection and optimization for aerospace system health management should take both the requirements of fault diagnostics (fault detection and fault isolation) and fault prognostics into account. Further, aerospace systems are usually complex electromechanical systems, so fault detection uncertainty should be also considered. The proposed optimization model of the paper takes sensor total cost as optimization object, and UFDR, UFIR and UFPR under imperfect tests as constraint conditions. The model can be formulated by:

\[
\begin{align*}
T^*_s &= \text{argmin}_{T_s} \sum_{j \in T_s} c_j x_j \\
\text{s.t.} \quad UFDR &\geq \gamma_1 \\
UFIR &\geq \gamma_2 \\
UFPR &\geq \gamma_3
\end{align*}
\]

where \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are testability requirements that aerospace system PHM will satisfy. The distinct features of the proposed model consist of three aspects: 1) the objective function contains sensor configuration \( x_j \); 2) the constraint conditions add UFPR, which enables DFT to satisfy PHM requirements rather than only fault detection and isolation needs; 3) fault detection uncertainty is taken into account, which enable the model to be applicable in engineering.

5.5. Sensor optimization selection based on generic algorithm

The sensor selection and optimization problem is a combination optimization problem and is related to the NP-hard property. The optimal solution can be obtained using a generic algorithm (GA).

Step 1: Parameter initialization, including population size, PopSize, generic crossover and mutation probability, \( p_c, p_m \), maximum iterative number, \( N_{\text{max}} \). The initialization population, \( \text{Pop} = (x_{ij})_{n \times n} \), is randomly generated, where \( n \) denotes the number of sensors to be selected. When \( t_j \) is selected, \( x_{ij} = 1 \), otherwise, \( x_{ij} = 0 \).

Step 2: Define the fitness function:
FitFun = \( C_0 \left( \sum_{j \in T_s} c_j + \sum_{j \in T_a} c_j \right) \) 
\[ - C_1 \cdot \max(0, \gamma_1 - \text{UFDR}) \]
\[ - C_2 \cdot \max(0, \gamma_2 - \text{UFIR}) \]
\[ - C_3 \cdot \max(0, \gamma_3 - \text{UFPR}) \]  \( \quad (12) \)

where \( C_0, C_1, C_2 \) and \( C_3 \) are constant. Individual fitness is calculated according to Eq. (12).

Justify whether the iterative number satisfies the max. iterative number, and if true, output the optimal individual and the corresponding optimal solution, and end the program; otherwise, go to step 3.

Step 3: Select individuals using the roulette wheel selection method based on individual fitness, and execute cross-over operation with probability \( p_c \), hence producing population \( \text{Pop}' \).

Step 4: Execute mutation operation with probability \( p_m \) on the individuals in population \( \text{Pop}' \), hence producing a new population \( \text{Pop}'' \), then return to step 2.

6. Case Studies

6.1. Stable tracking servo platform

The stable tracking servo platform (STSP) has been widely used in advanced aerospace systems such as cruise missiles and fighters. The structure of some STSPs is shown in Fig. 3. The failure modes information and sensors information are listed in Tables 2 and 3, respectively. And it is assumed that the resolution of all the faults is one, the SSNR, upper limit and resolution of all the sensors are 10 dB, 5 and 0.01, respectively.

The fault/fault evolution-sensor dependency matrix is shown in Table 4.\(^{25}\)

According to definition 3, \( F_{PP} = \{f_1, f_2, f_6, f_7, f_8 \} \). The required testability indexes are: UFDR is no less than 0.98, UFIR is no less than 0.95 and UFPR is no less than 0.99. The sensor optimization selection model is:

\[
T_s^* = \arg\min_{T_s} \sum_{j \in T_s} c_j x_j \\
\text{s.t.} \quad \text{UFDR} \geq 0.98 \\
\text{and} \quad \text{UFIR} \geq 0.95 \\
\text{and} \quad \text{UFPR} \geq 0.99
\]  \( \quad (13) \)

A GA is used to solve the problem, and the parameters are set as PopSize = 40, \( p_c = 0.7, p_m = 0.02, N_{\max} = 50, C_0 = 10, C_1 = C_2 = C_3 = 0.5 \). The optimization results are shown in Table 5.

Table 5 shows that selection scheme I can satisfy STSP testability requirements for PHM with a small number of sensors, and sensor resources can be economized greatly, so the GA is effective for the sensor optimization selection problem. In order to further validate the rationality of the proposed model, STSP is used again as a case example. The optimization objective is still sensor total cost but the constraint conditions are only UFDR and UFIR, and the fault detection uncertainty is not considered. The LINGO software package, which is specially used for solving programming problems, is used to obtain the optimal selection scheme. The results are shown in Table 6.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Failure rate (/1000 h)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level signal detection (t1)</td>
<td>0.001</td>
<td>6.0</td>
</tr>
<tr>
<td>Vibration sensor (t2)</td>
<td>0.001</td>
<td>6.6</td>
</tr>
<tr>
<td>Current detection (t3)</td>
<td>0.001</td>
<td>4.5</td>
</tr>
<tr>
<td>Optical-electricity encoder (t4)</td>
<td>0.001</td>
<td>7.0</td>
</tr>
<tr>
<td>Temperature sensor (t5)</td>
<td>0.001</td>
<td>8.2</td>
</tr>
<tr>
<td>Vibration sensor (t6)</td>
<td>0.001</td>
<td>5.7</td>
</tr>
<tr>
<td>Optical-electricity encoder (t7)</td>
<td>0.001</td>
<td>8.6</td>
</tr>
<tr>
<td>Rate gyroscope (t8)</td>
<td>0.001</td>
<td>15.2</td>
</tr>
<tr>
<td>Strap-down inertial navigation system (t9)</td>
<td>0.001</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Table 4. Fault/fault evolution-sensor dependency matrix of the STSP.

<table>
<thead>
<tr>
<th>( f_1 )</th>
<th>( 1, 1 )</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_2 )</td>
<td>0</td>
<td>( 1, 0 )</td>
<td>( 1, 1 )</td>
<td>( 1, 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>0</td>
<td>0</td>
<td>( 1, 0 )</td>
<td>( 1, 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 1, 0 )</td>
<td>( 1, 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_5 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 1, 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_6 )</td>
<td>0</td>
<td>( 1, 1 )</td>
<td>0</td>
<td>( 1, 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_7 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 1, 1 )</td>
<td>( 1, 0 )</td>
<td>( 1, 1 )</td>
<td>( 1, 1 )</td>
<td>( 1, 1 )</td>
</tr>
<tr>
<td>( f_8 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 1, 1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( f_9 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 1, 1 )</td>
<td>( 1, 0 )</td>
<td>( 1, 0 )</td>
</tr>
</tbody>
</table>
Table 5. Sensor optimization selection results for the STSP.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimize</td>
<td></td>
</tr>
<tr>
<td>total sensor</td>
<td>Total cost: 68.7</td>
</tr>
<tr>
<td>cost</td>
<td></td>
</tr>
<tr>
<td>UFDR 0.98</td>
<td>UFDR 0.9951</td>
</tr>
<tr>
<td>UFIR 0.95</td>
<td>UFIR 0.9827</td>
</tr>
<tr>
<td>UFPR 0.99</td>
<td>UFPR 0.9992</td>
</tr>
</tbody>
</table>

S, sensor; N, number.

Table 6. Sensor optimization selection results with UFDR and UFIR constraints.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimize</td>
<td></td>
</tr>
<tr>
<td>total sensor</td>
<td>Total cost: 56.5</td>
</tr>
<tr>
<td>cost</td>
<td></td>
</tr>
<tr>
<td>UFDR 0.98</td>
<td>UFDR 0.9973</td>
</tr>
<tr>
<td>UFIR 0.95</td>
<td>UFIR 0.9921</td>
</tr>
</tbody>
</table>

S, sensor; N, number.

From Table 6, one can see that the cost of scheme II is lower than that of scheme I. The reasons are: 1) The fault detection uncertainty is not considered. That is, a sensor can detect a fault with a probability of one when the fault occurs, so higher UFDR and UFIR can be reached with fewer sensors; 2) Scheme II does not take UFPR as a constraint. That is, fault early state detectability and fault evolution process trackability of the sensor are not necessary, so the sensors with low cost will have priority for selection. Scheme II is very suitable for fault detection and isolation of digital systems. However, as stated previously in the paper, the practical attributes of the sensors used in complex aerospace systems should be taken into account. Furthermore, for aerospace system PHM, testability should provide state information for fault prognostics besides satisfying fault diagnostics requirements, that is, UFPR should be considered in the optimization selection model. In the STSP system, \( F_{PP} = \{f_1, f_2, f_6, f_7, f_8\} \), but in scheme II, due to sensors \( t_5 \) and \( t_7 \) not having the ability of fault early state detection and/or fault evolution process tracking, fault prognostics will not be realized for the key faults \( f_6 \) and \( f_8 \). In other words, although scheme II can satisfy fault diagnostics requirements with lower cost, it cannot satisfy PHM needs. The comparison analysis results show the proposed model, which adds UFPR to the constraint conditions and takes sensor practical attributes into account, can guide sensor selection and optimization for aerospace system PHM very well and hence can provide sufficient state information for PHM.

In order to further demonstrate the proposed model and algorithm, a horizon system is used as another example.

6.2. Horizon system

The attitude and heading system of the helicopter mainly consists of the aviation horizon, combined compass, magnetic compass and course position indicator. Aviation horizon is one of the important components in the helicopter attitude and heading system, and it is usually used to measure the pitch angle and inclination angle of the helicopter. At the same time, the aviation horizon is also the key component resulting in the reduction of reliability and availability of the helicopter, so it is of great significance to develop the PHM system for the aviation horizon.

The schematic diagram of one aviation horizon is shown in Fig. 4. The aviation horizon is composed of a gyroscope, static converter, circuit board, corrective mechanism, quick righting mechanism, synchronic generator, indicative mechanism and so forth.

Using multi-signal modeling methodology, the testability model of the horizon, which has eight modules and seven sensors, is shown in Fig. 5, and the corresponding physical meanings are shown in Table 7.

According to the dependency model, the dependency matrix along with conditional probabilities of failure sources is shown in Table 8.

According to definition 3, \( F_{PP} = \{f_2, f_4, f_6, f_{10}, f_{11}, f_{12}\} \). The required testability indices are: UFDR is no less than 0.95, UFIR is no less than 0.90 and UFPR is no less than 0.98. GA is used to solve the problem, and the parameters are set as \( \text{PopSize} = 50, p_c = 0.8, p_m = 0.02, N_{\text{max}} = 100, C_0 = 10, C_1 = C_2 = C_3 = 0.5 \). Supposing that all sensor cost is one unit, SSNR is 10 dB, the upper limit of each sensor is five, and the resolution of faults and sensors is one. The optimization results are shown in Table 9.

7. Conclusions

PHM is of great significance to improve safety, availability and maintainability of aerospace systems. Aerospace systems with good testability can improve PHM capability fundamentally. This study mainly looked at sensor selection and optimization for PHM of aerospace systems. The main contributions of the paper are:

1. The intrinsic requirements of PHM for testability are analyzed qualitatively, and then quantitative testability indexes for PHM are defined, i.e., FDR, FIR and FPR.
2. A general sensor optimization selection frame is proposed that would provide a justifiable sensor suite to address PHM requirements of aerospace systems and hold up concurrent design methodology.
(3) Fault detection uncertainty is analyzed from the view of fault attributes, sensor attributes and fault-sensor matching attributes.

(4) Based on requirements analysis, the general frame and uncertainty analysis, the objective models and constraints models are formulated in great detail; a sensor optimization selection model for PHM is constructed, which adds FPR.

<table>
<thead>
<tr>
<th>Module number</th>
<th>Module name</th>
<th>Interface number</th>
<th>Interface name</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>28 VDC power</td>
<td>1</td>
<td>DC</td>
</tr>
<tr>
<td>b</td>
<td>Static converter</td>
<td>1</td>
<td>DC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>AC</td>
</tr>
<tr>
<td>c</td>
<td>26 VAC power</td>
<td>1</td>
<td>AC</td>
</tr>
<tr>
<td>d</td>
<td>Gyroscope</td>
<td>1</td>
<td>AC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Disturbed torque</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Righting torque</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>Corrective torque</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>State signal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>Self-rotation axes</td>
</tr>
<tr>
<td>e</td>
<td>Synchronic generator</td>
<td>1</td>
<td>Self-rotation axes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Fly attitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>26 VAC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>Pitch signal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>Inclination signal</td>
</tr>
<tr>
<td>f</td>
<td>Quick righting mechanism</td>
<td>1</td>
<td>Pull force</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Righting torque</td>
</tr>
<tr>
<td>g</td>
<td>Corrective mechanism</td>
<td>1</td>
<td>AC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Vertical line</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Self-rotation axes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>Corrective torque</td>
</tr>
<tr>
<td>h</td>
<td>Indicating mechanism</td>
<td>1</td>
<td>Self-rotation axes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Fly attitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Attitude indication</td>
</tr>
</tbody>
</table>

Table 7. Corresponding physical meanings of items in Fig. 5.

<table>
<thead>
<tr>
<th>Module name</th>
<th>Interface number</th>
<th>Interface name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbed torque</td>
<td>1</td>
<td>DC</td>
</tr>
<tr>
<td>Fly attitude</td>
<td>2</td>
<td>26 VAC</td>
</tr>
<tr>
<td>Pull force</td>
<td>3</td>
<td>Pitch signal</td>
</tr>
<tr>
<td>Righting torque</td>
<td>4</td>
<td>Inclination signal</td>
</tr>
<tr>
<td>Gravity</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Pitch angle signal</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Inclination angle signal</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Fly attitude indication</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Test matrix, fault probability for the horizon.

<table>
<thead>
<tr>
<th>Module name</th>
<th>Interface number</th>
<th>Interface name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbed torque</td>
<td>1</td>
<td>DC</td>
</tr>
<tr>
<td>Fly attitude</td>
<td>2</td>
<td>26 VAC</td>
</tr>
<tr>
<td>Pull force</td>
<td>3</td>
<td>Pitch signal</td>
</tr>
<tr>
<td>Righting torque</td>
<td>4</td>
<td>Inclination signal</td>
</tr>
<tr>
<td>Gravity</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Pitch angle signal</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Inclination angle signal</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Fly attitude indication</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Table 9. Sensor optimization selection results for horizon.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Optimization</th>
<th>Total cost: 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimize</td>
<td>total sensor cost</td>
<td></td>
</tr>
<tr>
<td>UDFR</td>
<td>0.95</td>
<td>0.9841</td>
</tr>
<tr>
<td>UFIR</td>
<td>0.90</td>
<td>0.9521 S</td>
</tr>
<tr>
<td>UFPR</td>
<td>0.98</td>
<td>0.9973 N</td>
</tr>
</tbody>
</table>

S, sensor; N, number.
to constrain conditions and considers fault detection uncertainty, and a generic algorithm is designed to obtain the optimal sensor suites.

(5) Two cases are provided to validate & verify the proposed model and algorithm.

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References