Study on a Calibration Method for Shape Control Parameters of a Self-Sensing Reflector Antenna Equipped with Surface Adjustment Mechanisms*

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In this study, a novel method for the on-orbit calibration for shape control parameters of a self-sensing reflector antenna equipped with surface adjustment mechanisms is developed and verified. These control parameters are related to the control inputs for shape control and changes in the antenna gains caused by intentional deformations. In the antenna system, intentional deformations are added to the reflector surface using surface adjustment mechanisms, and the corresponding changes in the antenna gains are measured. The control inputs for correcting the shape of the deformed reflector are determined directly from information on the changes in the strengths of received radio waves using the calibrated shape control parameters. Some numerical simulations are performed to investigate the feasibility of the developed method. A demonstrator equipped with surface adjustment mechanisms is under development, and the corresponding numerical model is employed for the numerical simulations. The results of these simulations show that the parameters are calibrated appropriately and the deformation of the antenna reflector is properly corrected by the developed method. The results clearly indicate that the developed method is an effective means of controlling the shape of a reflector antenna equipped with surface adjustment mechanisms.

Key Words: Space Antenna, Reconfigurable Antenna, Shape Control, Antenna Gain Analysis

1. Introduction

Large and high-precision reflector antennas are necessary for future satellite communications and space observations. Reconfigurable antenna systems are good candidates for such high-performance space antennas.1–3) The surface shapes of reflectors and beam shapes of reconfigurable antenna systems can be controlled by surface adjustment mechanisms.

Shape controls for space reflector antennas have been investigated by several research groups.3–8) Many of these studies have focused on structural shape controls and have assumed ideal shape measurement. Some studies have employed radio holographic analyses7,8) to diagnose reflector distortions. Photogrammetry measurements are often used to carry out ground adjustments on space reflector antennas.9)

Radio holographic analyses10–12) are very powerful methods for diagnosing and measuring distortions in reflector antennas. The measurement accuracies of these methods rely on the radio wavelengths of the antennas. Therefore, it is more appropriate to measure the surface deformations of a reflector antenna, rather than determining the accuracy of the antenna surface based on the antenna’s wavelength. However, these methods require special equipment such as near-field scanners or other reference antennas. They also involve special procedures such as precise scanning. Therefore, it is difficult to adopt such methods for space antennas.

Photogrammetry is widely used to measure the shapes of structures.13,14) It is a noncontact measurement method based on triangulation. It requires a large number of cameras for precise measurement, and the measurement accuracy is poor for large structures. Therefore, photogrammetry is not appropriate for the measurement of a high-precision antenna in orbit.

In order to overcome the difficulties involved in measuring surface deformations, we developed a self-sensing high-precision reconfigurable antenna system,15) as shown in Fig. 1. In this antenna system, the surface adjustment mechanisms, which are primarily used to control the shape of the reflector antenna, are also used to estimate the surface errors. Further, no special equipment is required, which simplifies the antenna system. In this antenna system, intentional deformations are added to an antenna surface using the surface adjustment mechanisms, and then, the corresponding changes in the antenna gains are measured. The original deformation of the antenna surface is estimated, and the control inputs are determined using the equations derived from the relationships among the antenna surface errors, intentional deformations and changes in the antenna gains.

The shape control parameters used in the developed method are calculated on the basis of the shapes of the intentional deformations and the aperture illumination. Therefore, the resultant surface accuracy of the controlled reflec-
tor relies on the prediction accuracies for the shapes of the intentional deformations and the aperture illumination. On the other hand, large space reflector antennas are generally flexible and have structural nonlinearity. Accordingly, it is difficult to accurately predict the shapes of the intentional deformations. Moreover, accurate prediction of the on-orbit antenna illumination is also difficult before a launch. These difficulties may lead to a worse shape control performance.

In this study, a method is developed for calibrating the shape control parameters in orbit. The parameters are calibrated from changes in the antenna gains based on the relationships between the antenna surface errors, antenna illumination and changes in the gains. Accurate shape control is achieved using the shape control parameters calibrated in orbit.

Some numerical simulations are performed to investigate the feasibility of this method. A demonstrator equipped with surface adjustment mechanisms is under development, and the corresponding numerical model is employed for the numerical simulations.

In the simulations, the initial position errors of surface adjustment mechanisms are assumed to be disturbances. The deformed reflector surface is additionally deformed using the surface adjustment mechanisms, and the changes in the gains are measured. The parameters are then calibrated using the developed method. The control inputs are calculated using the calibrated parameters, and the original deformation due to the disturbance is corrected. The results of these simulations show that the shape control parameters are calibrated appropriately, and the deformation of the antenna reflector is corrected properly by the developed method.

2. Surface Error Estimation and Correction Method

2.1. Relationships between antenna surface errors and changes in antenna gains

The surface error estimation and correction methods are based on the relationships between the antenna surface errors and changes in the antenna gains. Let us consider a reflector antenna deformed by a disturbance. The antenna gain of the deformed reflector, $G_d$, is estimated from the Ruze equation as follows

$$G_d = G_0 + \Delta G_f = C \left( \frac{\Delta e_{\text{rms},d}}{\lambda} \right)^2$$

(1)

and

$$C = 160\pi^2 \log_{10}(e)$$

(2)

Here, $G_0$ is the antenna gain of an ideal parabolic reflector and $\Delta G_f$ is the reduction in the antenna gain caused by factors other than the antenna surface deformation. $\lambda$ is the wavelength, and $\Delta e_{\text{rms}}$ is the RMS of the half-path-length error of the reflector. In this paper, the RMS of the half-path-length error is called the “effective surface accuracy.” Although the Ruze equation assumes random errors, it is useful for estimating the gain degradation due to systematic errors. Hereafter, the gains are expressed in decibel units.

The square of the effective surface accuracy of the deformed antenna surface, $\Delta e_{\text{rms},d}$, is

$$\Delta e_{\text{rms},d}^2 = \frac{1}{\lambda^2} \int (\Delta e_a)^2 w \, dA$$

(3)

Here, $\Delta e_a$ is the half-path-length error of the antenna surface deformed by the disturbance, and $A$ is the area of the aperture. $w$ is the weighting function, which includes the effect of the illumination distribution. The deformation will be estimated and corrected.

We now consider adding an intentional deformation to the original antenna surface. $\Delta e_i$ is the half-path-length error corresponding to this intentional deformation. From Eq. (3), the square of the effective surface accuracy, $\Delta e_{\text{rms},i}$, is obtained as follows

$$\Delta e_{\text{rms},i}^2 = \frac{1}{A^2} \int (\Delta e_i)^2 w \, dA = \Delta e_{\text{rms},d}^2 + \frac{1}{A} \int (\Delta e_a)^2 w \, dA + \int (\Delta e_a)^2 w \, dA$$

(4)

The antenna gain, $G_i$, is expressed as follows

$$G_i = G_0 + \Delta G_f = C \left( \frac{\Delta e_{\text{rms},d}}{\lambda} \right)^2$$

$$- \frac{C}{A^2} \left[ 2 \int \Delta e_i \Delta e_d w \, dA + \int (\Delta e_a)^2 w \, dA \right]$$

(5)

From Eqs. (1) and (5), we are able to determine the changes resulting from the intentional deformation. The changes in the antenna gains, $\Delta G_i$, are determined as follows

$$\Delta G_i = G_i - G_d = - \frac{C}{A^2} \left[ 2 \int \Delta e_i \Delta e_d w \, dA + \int (\Delta e_a)^2 w \, dA \right]$$

As can be seen from this equation, $\Delta G_i$ is eliminated. Therefore, the antenna deformation is distinguished from other causes of antenna performance degradation.
2.2. Surface error estimation of reflector antenna

The surface deformation of a reflector antenna is estimated from shape information about the intentional deformations and the changes in the antenna gains. Some modes of intentional deformations are added using the surface adjustment mechanisms on the original antenna surface, which is deformed by a disturbance. Then, the change in the antenna gain or power of the received radio waves caused by each mode of the intentional deformation is measured.

In this section, it is assumed that the intentional deformation is predictable. Here, we rewrite Eq. (6) as follows

$$\int \Delta e_o \Delta e_i w \, dA = - \int \frac{(\Delta e_o)^2 w \, dA}{2} - \frac{\lambda^2 A \Delta G_a}{2C}$$

(7)

By dividing the antenna surface into small elements, the following equation is derived from Eq. (7).

$$\sum_j w_j \Delta e_o,j \Delta e_i,j \Delta A_j = - \int \frac{(\Delta e_o)^2 w \, dA}{2} - \frac{\lambda^2 A \Delta G_a}{2C}$$

(8)

Here, the subscript $j$ is the element number and $\Delta A_j$ is the area of element $j$. Equation (8) is calculated for each mode of the intentional deformation. Hence, Eq. (8) is written in a matrix form as follows

$$S \text{diag}(w_j \Delta A_j) \Delta e_o = d + g$$

(9)

Here,

$$S_{i,j} = \Delta e_{o,i,j}$$

(10)

$$d_i = - \int \frac{(\Delta e_o)^2 w \, dA}{2}$$

(11)

and

$$g_i = - \frac{\lambda^2 A \Delta G_{a,i,j}}{2C}$$

(12)

The subscript $i$ is the mode number of the intentional deformation. Therefore, the half-path-length error due to the original deformation is estimated as follows

$$\Delta e_o = \text{diag} \left( \frac{1}{w_j \Delta A_j} \right) S^T (d + g)$$

(13)

Here, $S^T$ is the pseudo-inverse of matrix $S$.

Adequate measurements of the changes in the antenna gains or power values of the received radio waves are important in estimating the antenna deformations using this method. We can see from Eq. (1) that the antenna gains change considerably for a short wavelength. Therefore, even small deformations of an antenna can be accurately estimated using an adequate radio wavelength.

In this method, the deformation is expressed as a combination of intentional deformation modes. Therefore, the number and shapes of the intentional deformations have to be adequately selected. Consequently, the allocation of surface adjustment mechanisms is designed by considering the antenna deformations in orbit. This study focuses on investigating the feasibility of the developed method, and the optimal allocation of actuators was discussed in a previous paper.18)

2.3. Surface error correction of reflector antenna

The shape control inputs are obtained using a sensitivity matrix that relates to the control inputs and corresponding surface deformations. Now, we employ the intentional deformations, which are used for the surface error estimation, for shape control. Consequently, the sensitivity matrix is given as the transpose of matrix $S$. Therefore, the relationships between the control inputs and surface deformations are given as follows

$$S^T p = \Delta e_o \cong -\Delta e_o$$

(14)

where $p$ is the column vector of the control inputs, and $\Delta e_o$ is the column vector of the achievable deformations due to shape control. The control inputs of the conventional control methods are usually obtained using Eq. (14).

Substituting Eq. (14) into Eq. (9) yields

$$S \text{diag}(w_j \Delta A_j)S^T p = Hp = -d - g$$

(15)

Here, we define matrix $H$ as follows

$$H = S \text{diag}(w_j \Delta A_j)S^T$$

(16)

Each element of this matrix is obtained by the following equation.

$$H_{i1,i2} = \sum_k w_k \Delta e_{o1,k} \Delta e_{o2,k} \Delta A_k$$

(17)

Here, the subscripts $i1$ and $i2$ denote the mode numbers of the intentional deformations. Matrix $H$ is usually positive definite. From Eq. (15), the control inputs are determined as follows

$$p = -H^{-1}(d + g)$$

(18)

In the shape control method, the control inputs are directly obtained from the changes in the antenna gains without iterations.

As can be seen from Eqs. (11) and (17), vector $d$ and matrix $H$ contain terms relating to the shapes of the intentional deformations $\Delta e_o$ and weighting function, which includes the effect of the illumination distribution. Therefore, errors in the prediction of the intentional deformations and the antenna illumination lead to inaccurate shape control. The shapes of the intentional deformations and the illumination distribution are able to be estimated before launch through numerical analyses or ground experiments. However, it is difficult to accurately predict the shapes of the intentional deformations because of the non-linearity of the flexible antenna structure. These difficulties may lead to worse shape control performance.

Therefore, in order to achieve high-precision shape control, accurate values have to be obtained for vector $d$ and matrix $H$ of a reflector antenna in orbit. Furthermore, Eq. (18) indicates that accurate shape control is achieved using accurate values for vector $d$ and matrix $H$, even if...
the shapes of the intentional deformations and the illumination distribution are unknown.

3. Calibration of Shape Control Parameters

In order to achieve high-precision shape control, a method to calibrate matrix $H$ and vector $d$ is developed. In this method, calibrations are carried out as follows. First, deformations that are twice as large as the intentional deformations originally used are added to the surface of the reflector antenna, and vector $d$ is calibrated from the corresponding changes in the antenna gains. Next, some intentional deformation modes are simultaneously added to the reflector, and matrix $H$ is calibrated.

In the first step, deformations that are twice as large as the intentional deformations are added. As a consequence of these deformations, the antenna gains are changed as follows

$$\Delta G_{2a} = G_{2a} - G_d$$

$$= -\frac{C}{\lambda^2 A} \left[ 4 \Delta e_a \Delta e_a w \ dA + 4 \int (\Delta e_a)^2 w \ dA \right]$$

The following equation is obtained by substituting Eq. (6) into Eq. (19).

$$\Delta G_{2a} = 2 \Delta G_a - \frac{2C}{\lambda^2 A} \int (\Delta e_a)^2 w \ dA$$

By rearranging the expression and substituting into Eq. (11), the $i$-th element of vector $d$ is given as follows

$$d_i = \frac{\lambda^2 A (\Delta G_{2a,i} - 2 \Delta G_{a,i})}{4C}$$

In the next step, the $i1$-th and $i2$-th intentional deformations are simultaneously added to the antenna surface. The corresponding antenna gain is changed as follows

$$\Delta G_{a,i1+i2} = G_{a,i1+i2} - G_d$$

$$= \Delta G_{a,i1} + \Delta G_{a,i2} - \frac{2C}{\lambda^2 A} \int \Delta e_{ai1} \Delta e_{ai2} w \ dA$$

The following equation is derived by rearranging the expression.

$$\int \Delta e_{ai1} \Delta e_{ai2} w \ dA = \frac{\lambda^2 A}{2C} (\Delta G_{a,i1} + \Delta G_{a,i2} - \Delta G_{a,i1+i2})$$

By dividing the antenna surface into small elements and substituting into Eq. (17), the $i1,i2$-th element of matrix $H$ is given as follows

$$H_{i1,i2} = \frac{\lambda^2 A}{2C} (\Delta G_{a,i1} + \Delta G_{a,i2} - \Delta G_{a,i1+i2})$$

From Eqs. (21) and (24), matrix $H$ and vector $d$ are calibrated using the changes in the measured antenna gains caused by the intentional deformations. The superscript $c$ denotes a calibrated parameter. Although these parameter calibrations need to be carried out periodically in orbit, it is not necessary to perform them for every shape control. The control inputs are obtained using the calibrated parameters as follows

$$p = -(H^T)^{-1}(d^T + g)$$

4. Numerical Simulations

4.1. Model for analysis

In order to clarify the feasibility of the developed method, some numerical simulations are performed. We are currently developing a demonstrator equipped with surface adjustment mechanisms. The aim of this demonstrator is to clarify the feasibility of the developed shape control method. Therefore, commercial off-the-shelf components are being employed. A schematic representation of the demonstrator is shown in Fig. 2. Figure 3 shows photographs of the demonstrator under development. The reflector of the demonstrator is a membrane surface supported by 12 radial ribs. These radial ribs have parabolic shapes and are connected to a central hub. The membrane surface is shaped using these radial ribs. Therefore, the membrane has a ruled surface. Accordingly, the reflector has surface errors in the parabolic shape in the initial configuration. These surface errors are shown in Fig. 4.
Each radial rib is equipped with a linear actuator as a surface adjustment mechanism, and the rib is able to rotate out of the plane of the reflector around a hinge on the hub. The coordinate system is shown in Fig. 2. Here, the $Z$ direction is the boresight direction. The specifications of the reflector antenna are summarized in Table 1.

The corresponding numerical model of the demonstrator is employed for the numerical simulations. It is assumed that the reflector surface is illuminated by a Gaussian beam with a $-10$-dB edge illumination. The surface accuracy of the reflector antenna is $3.969$ mm RMS, and the antenna gain is $34.035$ dBi. The surface accuracy in the initial configuration does not meet the required level.

### 4.2. Calibration of shape control parameters

In the antenna system, the linear actuators rotated the radial ribs around the hinge on the central hub, and these rotations caused surface deformations. The corresponding surface deformations are considered to be the intentional deformations. Therefore, the $12$ intentional deformations are considered to be the control modes. These intentional deformations are used to calibrate matrix $H$ and vector $d$ and correct the antenna deformations. Figure 5 shows a typical shape for the intentional deformation.

In order to achieve high-precision shape control, matrix $H$ and vector $d$ are calibrated by the developed method, as described in section 3. The elements of the ideal and calibrated matrix $H$ are shown in Figs. 6 and 7, respectively. These figures indicate that matrix $H$ is calibrated properly using this method. Figure 8 shows the ideal and calibrated vector $d$. It is observed from the figure that vector $d$ is calibrated with little offset. The effect of the offset is investigated in the next section.

**Table 1. Specifications of the reflector antenna.**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum diameter of aperture</td>
<td>1470 mm</td>
</tr>
<tr>
<td>$F/D$ ratio (focal length)</td>
<td>$0.4$ (588 mm)</td>
</tr>
<tr>
<td>Frequency (wavelength)</td>
<td>$4$ GHz/C-Band ($\lambda$: 75 mm)</td>
</tr>
<tr>
<td>Number of ribs</td>
<td>12</td>
</tr>
<tr>
<td>Surface accuracy at initial configuration</td>
<td>$3.969$ mm RMS</td>
</tr>
<tr>
<td>Required surface accuracy</td>
<td>$\sim3.75$ mm RMS ($=1/20$)</td>
</tr>
</tbody>
</table>
4.3. Improvement in reflector surface for nominal configuration

It is observed from Fig. 4 that the reflector surface in the initial configuration is above the ideal parabola everywhere because the reflector has a ruled surface shaped by the parabolic-shaped ribs. Therefore, the surface accuracy of the reflector surface is improved by rotating the ribs in the minus $Z$-direction. The shape of the reflector is modified using the shape control parameters calibrated in the previous section, and we set the modified surface as the nominal surface. The resultant surface errors are shown in Fig. 9. The surface accuracy of the corrected surface improves to 2.716 mm RMS, and the antenna gain recovers to 34.540 dBi.

4.4. Shape control of reflector antenna

In order to validate the developed calibration method, shape control simulations are carried out using the calibrated parameters. In these simulations, the initial position errors of the surface adjustment mechanisms are assumed to be disturbances. Thus, the corresponding surface deformations are correctable. We set the surface deformations from the nominal shape caused by the disturbances to have a surface accuracy of 1 mm RMS. The numerical simulations are carried out 1,000 times. The results are summarized in Fig. 10, which shows a histogram of the root mean square (RMS) of the residual deformations from the nominal shape. Figure 11 shows the antenna gains with disturbances and those under control. We can see from these figures that the deformed surfaces are properly corrected and the antenna gains are recovered to almost the nominal values, even though calibrated vector $d$ has some offsets.

In the control method, measurement errors in the antenna gains affect the control performance of the system. In order to clarify the effect of the measurement errors, numerical simulations are performed while changing the measurement errors of the system with well calibrated control parameters. The numerical simulations are carried out 1,000 times for each measurement condition. The results are summarized in Fig. 12, which shows the relationships between the measurement errors in the antenna gains and the RMS of the residual deformations from the nominal shape. The error bars indicate the minimum and the maximum of the residual deformations. This figure indicates that good control performance is obtained by the developed method when the measurement errors are smaller than 0.01% of the maximum antenna gain.

These results clearly indicate that the developed method is effective at controlling the shape of a reflector antenna equipped with surface adjustment mechanisms when the antenna gain is measured with sufficiently small errors.
5. Conclusion

A novel method for calibrating the shape control parameters to correct the reflector surface deformations was developed and verified. In the developed method, the shape control parameters were calibrated from changes in the antenna gains based on the relationships between the antenna surface errors and changes in the gains. Deformations in the antenna surface were corrected based on the changes in the antenna gains caused by intentional deformations using the parameters calibrated by this method.

Some numerical simulations were performed to investigate the feasibility of the developed method. The numerical model, which corresponded to a demonstrator under development, was employed for the numerical simulations. In these simulations, the initial position errors of the surface adjustment mechanisms were assumed to be disturbances. The results of the numerical simulations showed that the shape control parameters were appropriately estimated, and the deformations were properly corrected using the developed method. These numerical simulations clearly indicated that the developed method is an effective means for controlling the shape of a reflector antenna equipped with surface adjustment mechanisms when the antenna gain is measured with sufficiently small errors.

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