Novel Transfer Alignment of Shipborne Gimbaled Inertial Navigation Systems

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Aiming at transfer alignment of gimbaled inertial navigation systems (INS) on a moving base, we propose an attitude matching alignment model to align the attitude of the slave platform. This method is achieved by applying an unscented Kalman filter to estimate the fixed misalignment angle, and the misalignment angle can be obtained only with attitude maneuvers, which are very easy for the ship to implement. Firstly, the frame dynamics equations are introduced. Then, frame angular error differential equations, which include the ship-body flexure and other alignment errors, are set up via the frame angle information from the master and the slave INS platform. With these frameworks, the nonlinear attitude matching alignment model is designed based on unscented Kalman filter technology. The simulation results show that the proposed method can obtain an alignment to an accuracy of 1.5° and alignment time of 50 sections.

Key Words: Gimbaled INS, Transfer Alignment, Frame Angle Matching, Unscented Kalman Filter

Nomenclature

- $i$: inertial frame
- $e$: Earth fixed frame
- $n$: navigation frame (N, E, D)
- $P$: platform coordinates system
- $f$: inner frame coordinates system
- $r$: outer frame coordinates system
- $b$: case coordinates system
- $C_{B}^{A}$: transformation matrix form $A$ frame to $B$ frame
- $\omega_{i}^{j}$: angular velocity of $i$ frame relative to the $j$ frame, coordinated in $A$ frame
- $[A \times ]$: skew-symmetric form of $A$
- $m$: master INS coordinate system
- $n$: slave INS coordinate system
- INS: inertial navigation system
- DCM: direction cosine matrix
- TA: transfer alignment

1. Introduction

Essential requirements for the guidance, navigation and control of a ship and the weapon equipped in it are position, velocity and attitude. These can be obtained by using inertial sensors—accelerometers and gyrosto sense rotational and motion with respect to a known reference frame and a navigation algorithm processor. This is known as inertial navigation. The gimbaled inertial navigation system (INS), which is mounted in a warshipborne system and the weapon equipped in it, consists of a dead-reckoning navigation system of whose operation depends heavily on the accuracy of the initial alignment and determination of the direction cosine matrix (DCM) from the body frame to the navigation frame. The simplest way to initialize the shipborne weapon’s INS is with a “one-shot” transfer using the data from the shipborne INS. However, the initialization performance is degraded due to the misalignments between the two INSs. The errors are generally caused by incorrect mounting of the weapon on the ship, the lever-arm and the ship body flexures and vibrations when the warship is maneuvering.

Aiming to estimate the misalignments, transfer alignment (TA), which is a process of aligning the weapon’s INS (called slave INS) using the accurate data supplied by host warship INS (called master INS), is one of the most accurate techniques presently. The alignment is implemented in a Kalman filter based algorithm. TA procedures are mature with theoretical research and modern navigation applications to numerous airborne, ground-based and naval systems. A marked rise in TA has been witnessed to meet the increasing demands for quick reactions in different weapon systems.

In recent decades, the study of TA technology on matching methods, lever-arm effect compensation, TA with the flexure of the carrier and the design of Kalman filters, etc. have achieved significant progress. The traditional and popular method used in TA technology is velocity match (VM). Although this method achieves accurate alignments, the vehicle is typically required to perform a designed turn or an S-turn maneuver in a span of minutes, and the maneuvers have been typically designed to last on the order of 10 min which is not acceptable for tactical weapons. In 1989, Kain and Cloutier presented a new rapid TA approach, in which attitude was added in filter measurements, allowing alignment in less than 10 sections to an accuracy of less than 1 mrad with a Wing-Rock maneuver. This combined matching scheme is known as velocity and
attitude (VAM) method. From then on, rapid TA procedures matured with extensive research and successful experiments to numerous airborne, ground-based and naval systems, such as ADKENs and other air-to-surface weapons. In 1997, Shortelle and Graham tested at VAM algorithm using F-16 flight test data. The results indicated that the VAM method achieved sub-mrad alignment accuracy in less than 10 sections. Recent papers have focused on lever-arm effect compensation. They generally add six additional states into the Kalman filter in order to define the angular flexures.

However, most of these studies used the strap-down INSs as the main INS and took aircraft weapon as the object. With the advent of gimbaled INS for strategic weapons equipped in warships, some of these TA methods are not viable, as the operating principle of the gimbaled INS is different from the strap-down INS. For the gimbaled INS, the gyros measure the platform rotational rate instead of body rotational rate in the strap-down INS, and the accelerometers measure the specific force in the platform frame instead of body frame in the strap-down INS. With these measurements, the missile-borne computer gives the command angular rate as output through navigation algorithm, and the platform is controlled by a servo mechanism to implement orientation in the inertial space or tracking the navigation coordinate frame. Based on accelerometer and gyro outputs, the navigation information is obtained as shown in Fig. 1. On the other hand, the specific maneuvers designed for aircraft-weapon TA may be very complex or even impossible to implement due to the huge inertia of warships in shipborne applications. In addition, the lever-arm between the weapon and warship gimbaled INS decreases the observability of attitude and increases the gyro errors through velocity measurements. This issue can be compensated to utilize the moment arm and relative orientation but the lever-arm length cannot be easily obtained as the optical measuring instruments are unavailable on a large warship.

In this paper, a TA algorithm, which uses so-called frame angle matching, is presented for warship missile initial alignment on a moving base. This TA algorithm is characterized by reducing maneuver (or even eliminating maneuver) demands and exploiting angular motions of ships induced by sea waves, which will enhance the alignment performance. A nine-state Kalman filter is designed to accurately align the attitude of a weapon-grade gimbaled INS. Under this framework, the level-arm issue can be solved as the velocity data are not used.

The reminder of this paper is organized as follows. Section 2 derives the frame dynamics equations and gives the ship flexure model. In section 3, the frame angle matching method and the Kalman filter TA model are given. Based on the model in section 2 and section 3, simulation results with different environment disturbances are analyzed in section 4. Finally, the overall conclusions are presented in section 5.

2. Theory of Attitude Matching

2.1. Frame dynamics equations

The gimbaled INS consists of a platform, inner frame and outer frame, and the motion of the platform occurs with command angular velocity and body angular velocity. In order to keep the platform orientation in the inertial space or track the navigation coordinate frame, the gimbaled INS systems is controlled by a servo mechanism through rotating the inner frame and the outer frame under the computer orders.

In order to discuss the problem of TA, a specific configuration for the inertial platform is selected somewhat arbitrarily (see Fig. 2). The coordinate frames that are relevant to the TA problem are also shown in Fig. 2. To facilitate discussion, the relative coordinate frames and quantities are defined as follows.

(a) Navigation coordinate system (n): Has its origin at the center of the platform O, and is aligned to north with the Xn axis in the direction of geographic north; Yn is upward along local the horizon plane and Zn points to the east.

(b) Platform coordinates system (P): Similar to the definition in Ref. 21), this coordinates system selects the accelerometer input axis as the reference direction. Here, it is assumed that the accelerometer input axes are orthogonal to each other. Thus, the P coordinate system is defined orthogonally and fixed relative to the platform. The Xp axis, Yp axis and Zp axis are coincident with the X accelerometer.
Y accelerometer and Z accelerometer input axis directions, respectively.

(c) Inner frame coordinates system (Ø): Has its origin at the center of the inner frame, which is assumed to be coincident with point O. The $Y_{p2}$ axis and $Z_{p2}$ axis are the revolution axes of the inner frame, and $X_{p2}$ axis complements the right-handed system. The $Z_{p2}$ axis is assumed as parallel with the $Z_b$ axis.

(d) Outer frame coordinates system (r): Has its origin at the center of the outer frame, which is assumed to be coincident with point O. Similar to the inner frame coordinates system, the $X_{p1}$ axis and $Y_{p1}$ axis are the revolution axes of the outer frame, and $Z_{p1}$ axis complements the right-handed system. The $Y_{p1}$ axis is assumed as parallel with the $Y_{p2}$ axis.

(e) Case coordinate system (b): Has as its origin at the case center of the shipborne gimbaled INS, and its $X_{b0}$ axis points along the longitudinal axis of the case; the $Z_{b0}$ axis is perpendicular to the longitudinal plane of symmetry, and $Y_{b0}$ axis complements the right-handed system. Here, it is assumed that the $X_{b0}$ axis is parallel with the $X_{p1}$ axis.

The quantities $\alpha$, $\beta$, $\gamma$, $\alpha_b$, $\beta_b$ and $\gamma_b$, which are so-called frame angles and frame angular velocity, are relative rotation angles and angular velocity of the case coordinate system, outer frame coordinate system and inner frame coordinate system, which are relative to the case rotational axis $X_{b0}$, the outer frame rotational axis $Y_{p1}$ and the inner frame rotational axis $Z_{p2}$.

Using the frame angles ($\alpha$, $\beta$, $\gamma$), the direction cosine matrix from the platform coordinate to case coordinate system is expressed as

$$C_p^b = M_1(\alpha)M_2(\beta)M_3(\gamma)$$

(1)

where the Euler angular sequence is 3-2-1($\alpha$, $\beta$, $\gamma$).

The frame angular rate can be expressed as follows

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = T_1\omega_p^b + T_2\omega_b^g$$

(2)

$T_1$ and $T_2$ are the reference coefficient matrices, and are shown as follows

$$T_1 = \begin{bmatrix} -\cos \gamma & -\sin \gamma & 0 \\ \cos \beta & \cos \beta & 0 \\ \sin \gamma & -\cos \gamma & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & \sin \alpha \tan \beta & \cos \alpha \tan \beta \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

Equation (2) is the frame dynamics equations in the gimbaled INS. The equations show the relationships among the frame angular rate, the command angular rate and the INS’s body angular rate.

### 2.2. Ship flexure model

As the ship body is very huge, the body flexure caused by wind, waves and the shift of persons and equipment will make the master and slave gimbaled INS bases generate relative angular displacements. Under ideal conditions, the fixed misalignment angle $\delta_i$ between the master gimbaled INSs and the slave gimbaled INSs can be treated as a constant. However, the ship body will undergo flexure due to sea wind or sea waves. Thus the fixed misalignment angle should consist of the mounting misalignment angle $\lambda$ and the ship body flexure angle $\theta$. Kain and Cloutier have pointed out that the fixed misalignment angle is usually expressed as a random process, which is often regarded as a second-order Gauss-Markov process. Then, the dynamic model of the fixed misalignment angle can be expressed as follows

$$\begin{bmatrix} \dot{\delta}_i \\ \ddot{\delta}_i \\ \dot{\theta}_i \\ \ddot{\theta}_i \end{bmatrix} = M_i(\alpha_b)M_2(\beta_b)M_3(\gamma_b)$$

(3)

where $\eta$ is Gaussian white noise. Details if the notions above can be found in Ref. 22.

### 3. Transfer Alignment Model

The problem of aligning the angular error of the slave INS assembly with the master INS on a moving base can be solved by the attitude matching method, which compares the frame angle output from the master INS and the slave INS to form frame angular error equations and estimates the fixed misalignment angle by a Kalman filter. Considering rapid TA, the errors of the master INS are neglected.

#### 3.1. Frame angular error dynamics

The platform angular rate in the $n$ coordinate system is

$$\omega_n^b = \omega_m^b = \omega_m^c + \omega_m^g$$

(4)

Considering the bias $k_0^b$ and measurement noise $e_g^2$, the gyro error modeled can be approximated as

$$\nabla_y = k_y + e_g$$

(5)

Now, the platform angular rate between the master INS and the slave INS can be expressed as

$$\omega_p^b = \omega_{cmd}$$

(6)

and the slave INS’s body angular rate can be expressed by the fixed misalignment angle and master INS angular rate

$$\omega_b^g = [I + (\delta_i \times)]\omega_m^g$$

(7)

From the frame angle differential of Eq. (2), the frame angular rate of the master INS becomes

$$\begin{bmatrix} \dot{\alpha}_m \\ \dot{\beta}_m \\ \dot{\gamma}_m \end{bmatrix} = T_1(\alpha_m, \beta_m, \gamma_m)\omega_m^g + T_2(\alpha_m, \beta_m, \gamma_m)\omega_b^g$$

(8)

and the frame angular rate of the slave INS is
Frame angular error can be obtained by subtracting Eq. (10) from Eq. (9) and the frame angular error can be easily obtained as

$$\mathbf{\dot{\psi}} = T^{-1}_1(\alpha_s, \beta_s, \gamma_s)\omega_{q_1} + T^{-1}_2(\alpha_s, \beta_s, \gamma_s)\omega_{q_2}$$  \hspace{1cm} (10)

where the symbol $\mathbf{\dot{\psi}}$ is defined as the frame angular error

$$\mathbf{\dot{\psi}} = [\alpha_m - \alpha_s, \beta_m - \beta_s, \gamma_m - \gamma_s]^T$$  \hspace{1cm} (12)

Considering Eq. (7) and Eq. (8), Eq. (11) can be rewritten as

$$\mathbf{\dot{\psi}} = (T^{-1}_1 - T^{-1}_2)\omega_{q_1} + T^{-1}_2(\mathbf{I} + (\delta_x \times)\omega_{p_2}) - T^{-1}_2\omega_{p_2}$$

$$+ (T^{-1}_1 - T^{-1}_2)\omega_{p_1}$$

Equation (11) is the frame angular error differential equation, which shows that the frame angular error is mainly caused by the fixed misalignment angle and rate error measured by the gyro.

### 3.2. System state equations

In order to take account of sufficient noise to the flexure effect, it needs to be injected into the TA Kalman filter based on past test experience if the flexure variations are not deterministically known during the TA. To achieve rapid TA, the system state vectors are chosen as follows

$$\mathbf{X} = [\delta \alpha, \delta \beta, \delta \gamma, \delta_x, \delta_y, \delta_z, \Delta \psi_x, \Delta \psi_y, \Delta \psi_z]^T$$  \hspace{1cm} (14)

where $\delta \alpha, \delta \beta$ and $\delta \gamma$ are the frame angular error, $\delta_x, \delta_y$ and $\delta_z$ are the fixed misalignment angles, $\Delta \psi_x, \Delta \psi_y, \Delta \psi_z$ are the biases of gyro.

The time of the whole TA is so short that the drift of the gyro can be considered as a constant. Thus

$$\mathbf{\dot{\Delta} \psi}_i = 0 \quad i = x, y, z$$  \hspace{1cm} (15)

Now, the state equations of TA can be expressed as:

$$\mathbf{\dot{X}}(t) = f(\mathbf{X}(t)) + \mathbf{w}$$  \hspace{1cm} (16)

where $\mathbf{w}$ is the Gaussian white process noise.

### 3.3. System measurement equations

The system measurement equations of the attitude matching model use frame angular errors as the measurement vector. The measurement model is a linear model, and the measurement variable $\mathbf{z}(t)$ can be defined as

$$\mathbf{z}(t) = [\alpha_m - \alpha_s, \beta_m - \beta_s, \gamma_m - \gamma_s]^T$$  \hspace{1cm} (17)

where the frame angles can be easily obtained from angular sensors, which are installed on the gimbaled system of master and slave gimbaled INSs.

As the measurement vector doesn’t include the accelerometer measurement, the level arm can’t affect the estimation performance of the Kalman filter based on the attitude matching TA method.

Hence, the dimensional measurement equations can be given as follows

$$\mathbf{z}(t) = \mathbf{H}(t)\mathbf{X}(t) + \mathbf{v}$$  \hspace{1cm} (18)

where $\mathbf{H}(t) = [\mathbf{I}_3 \ 0_{3 \times 3} \ 0_{3 \times 3}]$, and $\mathbf{v}$ is the Gaussian white measurement noise.

### 3.4. Kalman filter model

As the system model is a nonlinear model, the extended Kalman filter (EKF) and unscented Kalman filter (UKF) can be used to estimate the state vector. Compared with the UKF, the EKF is more widely used for nonlinear systems. However, as pointed out, due to use of linearization, the EKF is difficult to implement and tune, and only reliable for systems which are almost linear on the time scale of the updates. To overcome these limitations, the UKF is developed. It is more precise and easier to implement, but the filter may take too much time, which might not meet the rapid TA demands. Here, for the sake of comparing the performance of these two methods, both Kalman filter methods are applied to the attitude matching TA problem.

In order to use the EKF, a first-order differential equation for system state equations should be derived first.

From Eq. (13), frame angular error can be described as

$$\mathbf{\dot{\psi}} = (T^{-1}_1 + \delta T_1)(\omega_{q_1} + \Delta \psi_1) - T^{-1}_2\omega_{p_2}$$

$$+ (T^{-1}_2 + \delta T_2)(\mathbf{I} + (\delta_x \times)) + T^{-1}_2\omega_{p_2}$$  \hspace{1cm} (19)

Expanding the right hand side of Eq. (19), and omitting the second-order item, the frame angular error can be simplified as

$$\mathbf{\dot{\psi}} = A\mathbf{\psi} + T^{-1}_2\mathbf{\Delta} \psi_2 - T^{-1}_2(\omega_{p_2} \times)\delta_x$$  \hspace{1cm} (20)

where

$$A = \begin{bmatrix}
\frac{\partial T_1}{\partial \alpha} \omega_{q_1} + \frac{\partial T_2}{\partial \beta} \omega_{p_2} \\
\frac{\partial T_1}{\partial \beta} \omega_{q_1} + \frac{\partial T_2}{\partial \alpha} \omega_{p_2}
\end{bmatrix}$$

With Eq. (19), the EKF model can be given as

$$\left\{ \begin{array}{l}
\mathbf{\dot{X}}(t) = f(\mathbf{X}(t)) + \mathbf{w} \\
\mathbf{z}(t) = \mathbf{H}(t)\mathbf{X}(t) + \mathbf{v}
\end{array} \right.$$

where

$$F(t) = \begin{bmatrix}
\frac{\partial f}{\partial \mathbf{X}} & -T_2^{-1}(\omega_{p_2} \times) \\
0_{6 \times 3} & 0_{6 \times 3}
\end{bmatrix}$$  \hspace{1cm} (22)

and the matrices $A$, $-T_2^{-1}(\omega_{p_2} \times)$ and $T_2^{-1}$ are illustrated as above.

Unlike the EKF, the UKF doesn’t need the first-order differential of the system state equations. Applying the
unscented transformation to propagate mean and covariance through nonlinear transformations, UKF can be used to very easily and to accurately estimate the state vector. In order to keep the paper brief, relative notations and formulations are not introduced; if needed, please see Ref. 23).

4. Simulation Results and Analysis

Illustrative TA simulation results are presented in this section to demonstrate the effectiveness and applicability of the attitude matching TA method. This section is divided into three subsections. The first subsection presents the proposed attitude matching procedures, and the second subsection presents the simulation conditions. Finally, the sensitivity analysis to sea conditions and IMU performance are presented in the third subsection.

4.1. Proposed attitude matching procedures

The proposed attitude matching method is achieved by a nine-state UKF, which can perform well to estimate the state vectors if the command angular rate and the body angular rate of the master INS are known. In fact, the command angular rate could be given by the computer as shown in Fig. 1. Meanwhile, the body angular rate can be obtained from the differential of attitude angle dates which are measured by the master gimbaled INS and the slave gimbaled INS. However, the attitude angle dates are measured with noise, and these errors could be enlarged when computing the time derivative of these dates in short sampling period (0.02). Due to this reason, we use a set of strap-down gyros, which is mounted in the case of the master gimbaled INS and the slave gimbaled INS. As noted here, the accuracy of the strap-down gyros needn’t be very high (detailed report in section 4.3.2) and medium-cost strap-down gyros can meet the TA demands.

The attitude matching scheme of the transfer alignment simulation is shown in Fig. 3. It contains three major steps which are briefly described in the following.

Step 1: Design the maneuver through the observability analysis on TA models. The designed alignment maneuver for the warship’s TA will be disturbed by the environment disturbances, including natural sources (such as sea waves, ocean wind) and other human activities (such as the movement of sailors around the master or the slave gimbaled INS). Among these sources, sea waves are the main factor, while human activities and other small disturbances can be seen as Gaussian white noise. Sea wave motion is assumed to be generated by a combination of sinusoidal waves

\[ \eta_i = A_i \sin(2\pi f_i t + \alpha_i) \quad i = x, y, z \tag{23} \]

where \( \eta_i \) is the sea wave angle, and \( A_i \) and \( f_i \) are the amplitude and the frequency of the \( i \)th harmonic, respectively.

The ship pitch, roll and yaw angular motions, which are caused by the maneuver and the effects of sea waves and ocean wind, can be described as a combination of sinusoidal waves

\[ \Phi_i = A_i \sin(2\pi f_i t + \alpha_i) + B_i \sin(2\pi k_i t + \beta_i) + \phi_{0i} \]

\[ i = x, y, z \tag{24} \]

where \( \phi_{0i} \) is the initial Euler angle, \( \Phi_i \) represents the real time Euler angle, \( \beta_i \) is the initial phase angle, and \( k_i \) and \( B_i \) are the frequency and amplitude corresponding to angular motions along the Euler angle, respectively.

Step 2: Computer navigation dates both of the master and slave gimbaled INSs considering the ship-body flexure and the error of inertial instrument. According to subsection 2.2, this module calculates the linear and angular flexure components of the slave gimbaled INS case during the designed maneuver. Due to the elastic body motion, which results from the ship-body structural flexures and vibrations, there is relative motion in the case frame between the base points on the main and the slave gimbaled INSs. This error is considered in the slave gimbaled INS navigation. Meanwhile, the gyros errors can also increase the navigation error of the slave gimbaled INS navigation.

Step 3: With the above procedures, the frame angles of the master and slave gimbaled INS, the command rate and the case angular rate at the master gimbaled INSs can be easily obtained. Combined with the frame dynamics equations, this module estimates the value of the fixed angle, the gyros bias and the frame angle errors using a Kalman filter.

4.2. Simulation conditions

The simulation parameters are set as follows, which will not change in the following simulations except special demonstrations.
The parameters for typical sea conditions are presented in Table 1. The inertial instrument errors simulated for the slave gimbaled INSs are listed in Table 2, and the master gimbaled INS’s inertial sensors are assumed errorless. The performance of the strap-down gyros is also presented in Table 2.

The ship speed is adjusted to be approximately 20 knots per hour and the attitude motion parameters as follows. The amplitude of roll angle is $10^\circ$ and cycle is 12 s; the amplitude of pitch angle is $15^\circ$ and cycle is 10 s; the amplitude of yaw angle is $5^\circ$ and cycle is 20 s. An initial slave gimbaled INS case orientation with respect to a master of $0.8^\circ$ in the yaw, pitch and roll plane is simulated.

### 4.3. Sensitivity analysis

A TA simulation result is presented in this subsection for an angular maneuver lasting 30 s, which is performed at E 110.23, N 31.34. This maneuver is especially suitable for the rapid TA used in targets of opportunity missions as it requires only an angle motion. Note that the flexure magnitude is assumed to be known only for the data simulation process.

Now, with 50 repeated Monte-Carlo simulations, the respective simulation results using the UKF and EKF are evaluated and depicted in Figs. 4–6, which show the estimated error curve of the fixed misalignment angle. The figures and the tables show that the biggest estimated error of the fixed misalignment angle in the UKF curve converges to no more than $7000$ within 25 s, while the EKF curve is divergent. The UKF results are listed in Table 3 where the data denote the parameter estimate errors when the steady-state estimations are obtained. This result illustrates that the attitude matching TA method may achieve a higher accuracy with the UKF compared with the EKF as the system is a nonlinear system. Hence, for the attitude matching, we choose the UKF to estimate the state vectors as it is more accurate and easier to implement.

The TA technique is not the only factor determining post-launch navigation performance of a guided weapon. Other important factors include the mounting of the weapon aboard the host aircraft, maneuvers during the alignment and the autonomous phases, alignment duration and the grade of weapon inertial sensors.

#### 4.3.1. Sensitivity analysis to sea conditions

When a ship undergoes roll, pitch and yaw motions due to sea waves or sea wind, the body of the ship will be flexural which may add a flexural angle to the gimbaled INSs and cause a navigation error. However, the maneuverability of a ship is quite limited, and the attitude matching TA method may fail without any attitude movement. Thus, ocean waves as a driving source can make the ship swing which will help TA.

With respect to the sea conditions, the effect of the TA is analyzed with the following simulations. Two schemes
according to the grade of the waves are designed and the corresponding attitude motion amplitude are set as follows: Rough sea (amplitude is set as \((10^\circ 10^\circ 10^\circ)\)) and calm sea (amplitude is set as \((0.1^\circ 0.1^\circ 0.1^\circ)\)). Figure 7 shows the estimated error curves of the fixed misalignment angle, and Table 4 gives the corresponding data. From the curves, it can be seen that the present method is robust under different sea condition. In contrast, the estimated errors of the fixed misalignment angle are below \(30^\circ\) under the rough sea and \(150^\circ\) under the calm sea conditions. These results show that the rough sea condition is helpful to transfer alignment on board.

### 4.3.2. Sensitivity analysis to different strap down gyro performance

As illustrated above, the attitude matching method needs a set of strap-down gyros to measure the case angular rate, which is used to estimate the fixed misalignment angles. Therefore the strap-down INS error will affect TA performance. Figures 8(a) and (b) and Table 5 respectively show the Monte Carlo simulation results using the present method and assuming the bias of strap-down gyros are increased from \(20^\circ/h\) to \(50^\circ/h\). Comparing the two figures, the two schemes perform similarly with different bias of the strap-down gyros, as the estimated error between two schemes is not marked. That means a middle grade strap-down can meet the TA demand.

### 5. Conclusion

In order to apply the TA to gimbaled INSs on a ship, a novel TA method based on the frame angle matching was addressed in this paper. Here, it is important to note that the proposed method aligned only attitude. Considering the difference between the gimbaled INSs and the strap-down INSs, the frame angle dynamics equations were derived first. Subsequently, the attitude matching, which considers the main inertial instrument errors and the flexure of the ship body, was designed in detail. This method enables rapid attitude alignment with just a simple attitude maneuver, which does not require the ship to change its trajectory. The simulation results showed that the presented attitude matching scheme had acceptable precision and it can meet the need of the gimbaled INS on a moving base and the sea conditions can enhance the filter in terms of converging faster and more precisely.

However, the method should be verified by actual data from a ship, and investigating how to improve the speed of the TA will be a future work.

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<th>Table 4. The simulation data under different sea conditions.</th>
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Fig. 7. Simulation curve under different sea condition.

(a) Rough sea

(b) Calm sea

![Fig. 7. Simulation curve under different sea condition.](image)

Fig. 8. Simulation curve.

(a) Low degree gyros

(b) Middle degree gyros

![Fig. 8. Simulation curve.](image)
References