Regulated Point Mass Flight Dynamics Model for a Nonlinear Aircraft Tracking Filter*

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A novel flight dynamics-based model for a nonlinear aircraft tracking filter is proposed. The point mass flight dynamics (PMFD) model proposed in a previous study is elaborated. This study details the case for which the estimation of the state variables is essentially needed but the estimation of the target-dependent control variables is not. The present model, the regulated PMFD (RPMFD), adopts the target-independent control variables instead of the target-dependent variables. In other words, angle of attack and normalized thrust, which are treated as states with random processes in the original PMFD model, are replaced with normal and longitudinal load factors, respectively. The replacement of the target-dependent variables with the target-independent ones enables the unknown aircraft parameters to be removed from the dynamic model. The simulation indicates that RPMFD, which has an advantage of not requiring the unknown parameters, results in the same tracking performance as PMFD. This is because both are basically the same when viewed through implementation of the attitude effects on translational acceleration.

Key Words: Filter, Flight Dynamics, Nonlinear, Point Mass, Tracking

1. Introduction

Many studies on dynamic models for target tracking filters have attracted significant attention for the past several decades. This is due to the fact that the dynamic model is critical to successful target tracking. A flight dynamics-based model is one of the sophisticated models that have been proposed by several studies in order to improve modeling accuracy through exploring detailed flight dynamics relationships with the help of additional measurements. Andrisani et al.1) and Mook and Shyu2) proposed typical six-degrees-of-freedom (DOF) flight dynamics models for their tracking problems, which assumed velocity and attitude observations. Jeon et al.3) extended these studies by proposing a three-DOF point mass flight dynamics (PMFD) model to account for the attitude effects on translational acceleration with a limited number of states. However, there were still several unknown aircraft parameters to be set, even though they are limited in quantity and their impact on the estimation of meaningful states is small.

The objective of this study is to propose a practical model for air traffic control systems, which require accurate estimates of position, speed and course, but do not need estimates of target-dependent control variables. The regulated PMFD (RPMFD) model that is proposed in this study maintains the tracking performance of PMFD, while it does not account for the target-dependent control variables used for PMFD. The removal of the target-dependent control variables in the model is followed by the removal of other unknown aircraft parameters.

This paper is organized as follows: A brief summary of the PMFD model is presented in Section 2. Next, the RPMFD model is derived in Section 3. The simulation results that are compared with those conducted by the original PMFD model are presented in Section 4, and the concluding remarks are given in Section 5.

2. PMFD Model

The PMFD model for nonlinear aircraft target tracking is defined by, with state vector $x = [X, Y, Z, V, \gamma, \psi, \alpha, \phi, T_f]^T$,3)

$$
\dot{x} = C^Tf(x) + v
$$

$$
f(x) = 
\begin{bmatrix}
V \cos \gamma \cos \psi \\
V \cos \gamma \sin \psi \\
-V \sin \gamma \\
T \cos \alpha - D \\
m \sin \gamma \\
T \sin \alpha + L \\
m V \cos \gamma \\
\cos \phi - g \cos \gamma \\
V \\
T \sin \alpha + L \\
m V \cos \gamma \\
\cos \phi 0 \\
0 \\
0
\end{bmatrix}
$$

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where \( x \) is the state vector for the tracking filter. \( X, Y, \) and \( Z \) are the orthogonal target positions in the earth-centered earth-fixed (ECEF) coordinates. \( V \) is the target speed and \( \gamma \) is the flight path angle. \( \alpha, \phi, \) and \( T_f \) are the angle of attack, the roll angle, and the normalized coefficient for thrust setting, respectively. These three variables represent the control variables, with which the flight trajectory can be determined. \( L, D, T, m, \) and \( g \) represent lift, drag, thrust, aircraft mass, and gravity acceleration, respectively. \( v \) is the process noise term that is assumed to be zero-mean Gaussian and white. \( C_{D_0}, \) is the zero-lift drag coefficient, \( C_{L_0}, \) is the lift coefficient at zero angle of attack, \( C_{L_n}, \) is the coefficient of the ratio of lift to angle of attack, \( A R \) is the wing aspect ratio, \( e \) is the Oswald’s efficiency factor, \( W/S \) is the wing loading, and \( T_0/W \) is the thrust-to-weight ratio. As \( C_{D_0}, C_{L_0}, C_{L_n}, A R, W/S, \) and \( T_0/W \) are unknown, they should be set as the design parameters based on the properties of the aircraft to be tracked. \( \rho \) is the standard atmospheric air density calculated as a function of altitude \( h. \) In Eq. (1), \( C \) is a conversion matrix defined as follows:

\[
C = \begin{bmatrix} U & O_{3 \times 6} \\ O_{6 \times 3} & I_6 \end{bmatrix}
\]

where \( O_{m \times n} \) represents an \( m \times n \) null matrix, \( I_n \) is an \( n \times n \) identity matrix, and \( U \) is the global-to-local coordinate conversion component, which is described in Jeon et al. \(^3\)

The main idea of the PMFD model is that the angle of attack, roll angle, and normalized thrust are attributed as control variables and are treated with random processes in order to model the control inputs for vertical, lateral/directional, and longitudinal control, respectively. These control variables, in conjunction with the attitude measurements, reflect the effect of attitude on translational acceleration. This results in improved tracking performance.

### 3. RPMFD Model

The main drawback of the PMFD model is that the adoption of these target-dependent control variables in the model inevitably requires the target-dependent design parameters to be set, and the incomplete knowledge of these parameters may result in inaccurate state estimations. The RPMFD model is motivated by the facts that, although the main contributor of the PMFD model is the reflection of attitude effect on translational acceleration with control variables treated with random processes, these control variables are not necessarily target-dependent. The RPMFD replaces these target-dependent variables in the PMFD model with target-independent variables and maintains the estimation accuracy for meaningful states such as position, speed, and course. The removal of the target-dependent variables entails removal of unknown target-dependent design parameters from the model. A detailed description of the RPMFD model is presented below.

In the PMFD model shown in Eqs. (1) and (2), the target-dependent terms include not only design parameters such as \( C_{D_0}, C_{L_0}, C_{L_n}, A R, W/S, \) but also control variables \( \alpha \) and \( T_f \). Therefore, these control variables used in Eq. (2) should be replaced with the target-independent variables in order to remove the target-dependent terms from the model.

In this study, control variables \( \alpha \) and \( T_f \) are replaced with \( n_a \) and \( n_l \), respectively, which are defined as follows:

\[
n_a = \frac{T \sin \alpha + L}{mg} \quad (4)
\]

\[
n_l = \frac{T \cos \alpha - D}{mg} \quad (5)
\]

where \( n_a \) is the normal load factor, and \( n_l \) is the longitudinal load factor. Once these control variables are substituted for \( \alpha \) and \( T_f \), the design parameters, \( C_{D_0}, C_{L_0}, C_{L_n}, A R, W/S, \) and \( T_0/W \) are no longer needed in the model.

Now, a RPMFD model can be defined with a new state vector \( x = [X, Y, Z, V, \gamma, \psi, \phi, n_a, n_l]^T \) as follows:

\[
\dot{x} = C^T f(x) + v \quad (6)
\]
The RPMFD model derived in Eq. (7) is different from the one shown in Eq. (2) because the former excludes the detailed dynamic relations based on target-dependent control variables and parameters. Despite this, both models are fundamentally identical from the viewpoint of implementing the attitude effects on translational acceleration. Therefore, it is expected that the performance of the estimation of meaningful states, which include position, speed, and course, will be maintained. This is demonstrated through simulations in the next section.

4. Simulation Results

In comparison with the original PMFD model, the simulations of aircraft tracking with the RPMFD model are performed for the scenario described in Table 1. This scenario is identical to scenario IV in Jeon et al. A target starts flight at an altitude of 40,000 ft and a speed of 250 knots, and it is assumed that there is a transition time of 10 s between the consecutive segments. The aircraft data used for the simulation are based on a Boeing 747-100, shown in Table 2.

The sensor’s measurement is assumed to provide the position of each of the three Cartesian axes, horizontal speed, heading angle, and roll angle. The root mean square (RMS) error of the position measurement for each of the three Cartesian coordinates is assumed to be 35.5 ft. The errors in horizontal speed and vertical speed are 1.2 ft/s and 1.7 ft/s, respectively. Both of the errors in heading angle and roll angle are assumed to be 2°. The intervals between the samples are 1 s. The continuous-discrete unscented Kalman filter that consists of the measurement equations and covariance adopted in Jeon et al. is used for the tracking.

The PMFD model, which is used as a reference model in this study, is modified as Eqs. (8) and (9) in order to estimate the design parameters reflecting that the aircraft types are unknown in the air traffic control environment.

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} X, Y, Z, V, \gamma, \alpha, \phi, T_f, C_{D_0}, C_{L_0}, C_{L_0} \end{bmatrix}^T, \\
&= \begin{bmatrix} C \ O_{9 \times 6} \ O_{6 \times 6} \end{bmatrix} \begin{bmatrix} f(x) \ v_{AC} \end{bmatrix} + \begin{bmatrix} v \end{bmatrix} \\
&\quad \text{where } W/S \text{ and } T/W \text{ denote } W/S \text{ and } T_0/W, \text{ respectively. } v_{AC}
\end{align*}
\]

Table 2. Sets of the design parameters.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>(C_{D_0})</th>
<th>(C_{L_0})</th>
<th>(C_{L_0})</th>
<th>(AR)</th>
<th>(W/S)</th>
<th>(T_0/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B747-100</td>
<td>0.0430</td>
<td>0.654</td>
<td>4.92</td>
<td>7.0</td>
<td>115.0</td>
<td>0.25</td>
</tr>
<tr>
<td>Beech 99A</td>
<td>0.0270</td>
<td>0.201</td>
<td>5.48</td>
<td>7.6</td>
<td>39.3</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Fig. 1. Tracking performance comparison.
Figure 1 and Table 3 present the RMS errors of the target tracking via 100 Monte Carlo simulations in terms of position, speed, and course error. They show that both models work well and yield almost the same tracking performance in terms of position, speed, and course. The bank angle, longitudinal load factor, and normal load factor are attributed as the control variables, and are also compared with their corresponding true values in Fig. 2. As shown in the figure, the estimated states for the control variables correspond well with the true values, even though the target-dependent control variables such as angle of attack and thrust settings, which are estimated using the original PMFD model, are not available in the RPMFD model. It should be noted that, as Table 3 shows, the RPMFD model requires fewer state variables, and hence, less computation cost than the PMFD model because the RPMFD model does not need to estimate the design parameters.

5. Conclusion

A new flight dynamics-based dynamic model, the RPMFD model, for a nonlinear aircraft tracking filter is proposed. The proposed model employs normal and longitudinal load factors instead of the angle of attack and normalized thrust, which are used in the original PMFD model. This is done in order to remove the necessity for the unknown design parameters. The simulation results indicate that the RPMFD shows the same tracking performance as the PMFD in spite of the absence of the target-dependent design parameters.

In summary, it can be concluded that the RPMFD model can be a propitious solution to tracking applications such as air traffic control systems that do not require the estimation of target-dependent control variables.

Acknowledgments

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