Continuous Robust Guidance Law for Intercepting Maneuvering Targets*

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This paper documents a new guidance law for tactical missiles to intercept maneuvering targets. By incorporating linear and fractional power nonlinear terms, a faster convergent and chatter-free acceleration command is derived. With the aid of Lyapunov stability criteria, we prove that the line-of-sight angular rate can converge to a small region around the origin in finite time. Since the target maneuver cannot be obtained or measured in real applications, a nonlinear finite time convergence disturbance observer for target maneuver estimation is introduced and a composite guidance law with finite time convergence is presented to improve the overall performance. Theoretical analysis and numerical simulations demonstrate the effectiveness of the proposed method.

Key Words: Missile Guidance, Sliding Mode Control, Maneuvering Targets, Chatter-free, Finite Time Convergence

1. Introduction

The proportional navigation guidance (PNG) law is the most widely used guidance law for tactical missiles in the last few decades owing to its easy implementation and efficiency for stationary or weakly maneuverable targets.^{1–3)} The basic idea behind the PNG law is that it forces the missile-target line-of-sight (LOS) angular rate to converge to zero as the missile approaches the target. Furthermore, the PNG law with a navigation ratio 3 is proven to be an optimal one in terms of minimizing both energy consumption and miss distance.³⁾ However, the performance of the PNG law deteriorates significantly and robustness is lacking as the target's maneuverability increases.³⁾

In order to achieve satisfactory results for intercepting maneuvering targets, many effective guidance laws based on modern control theories have been reported in literature. Yang and Chen⁴⁾ regarded the guidance problem as solving a partial differential Hamilton-Jacobi inequality by considering target acceleration as unknown information. Using a Lyapunov-like method, Zhou et al.⁵⁾ derived a robust L_2 gain performance index robust guidance law. With the aid of a random input description function, a novel estimation statistics-dependent guidance law with a bounded acceleration command was presented by Hexner et al.⁶⁾ As an extension of Hexner et al.,⁶⁾ a differential game-based guidance law with impact angle constraints was studied by Shaferman and Shima.⁷⁾ In Ref. 8), the authors derived a novel robust guidance law for maneuvering targets by combining integral sliding mode control theory with the classical PNG law. Shkolnikov et al.⁹⁾ applied the second-order sliding mode observer to estimate the target maneuver and augmented it with PNG law to obtain a robust, augmented PNG (APNG) law. To intercept a target performing sudden step maneuvers, Atir et al.¹⁰⁾ studied a multiple-model adaptive guidance law for

missiles with bounded acceleration command. In Ref. 11), the authors presented a novel sliding mode control and inertial delay control (SMC-IDC)-based guidance law, which do not need any prior knowledge of target acceleration.

The missile guidance laws in most of the literature mentioned above are Lipschitz continuous, which implies these closed-loop guidance systems are asymptotically stable. It is well-known that a finite time convergence guidance system can reduce miss distance and achieve higher robustness against target maneuvers. Furthermore, the whole flight time of the terminal guidance phase for most tactical missiles is very short, and therefore the guidance law for reaching finite time convergence is very meaningful for real applications.¹²⁾ In literature, Shtessel et al.^{13,14)} adopted the second-order sliding mode theory to derive a finite time convergence guidance law based on the concept of hit-to-kill, but the acceleration format is very complicated to implement. Zhou et al.¹⁵⁾ designed a novel finite time convergence guidance (FTCG) law by assuming target acceleration is bound by a known upper bound and used a discontinuous term to compensate for the target maneuvering. However, in real applications, the target maneuvering boundary cannot be obtained in advance or only a very conservative boundary can be obtained, which worsens the undesired chattering phenomenon caused by the discontinuous term since a large switching gain should be selected to guarantee finite time stability of the closed-loop guidance system.

In this paper, a robust continuous guidance law for intercepting targets with high maneuverability is presented to improve the performance of the FTCG proposed by Zhou et al.¹⁵⁾ By choosing the LOS angular rate as the sliding variable, a method of combining linear control and nonlinear control is adopted to obtain a faster convergence rate. Using the Lyapunov method, the finite time convergence of the LOS angular rate is proven in theory. To improve the overall performance, the higher-order sliding mode (HOSM) exact differentiator proposed by Shtessel et al.¹³⁾ and Levant¹⁶⁾ is used here as a finite time convergence disturbance observer (FTDOB) for target maneuver estimation. Detailed numeri-

cal simulations with comparisons for intercepting highly maneuverable targets demonstrate the effectiveness of the proposed guidance law.

The rest of this paper is organized as follows. Section 2 gives the statement of problem formulation. A chatter-free, finite time convergent guidance law is presented in Sec. 3. In Sec. 4, the composite guidance law is provided. Finally, the numerical simulation results and some conclusions are offered.

2. Problem Formulation

The planar homing engagement geometry between the missile and target is depicted in Fig. 1, where the subscripts M and T denote the missile and the target, γ_M and γ_T the missile and the target flight path angle, λ and r the LOS angle and the relative missile-target range, V_M and V_T the missile and target velocity, and a_M and a_T the missile and target acceleration, which are assumed normal to their corresponding velocities, respectively.

The corresponding engagement dynamic equations are formulated as

$$\dot{r} = V_T \cos(\gamma_T - \lambda) - V_M \cos(\gamma_M - \lambda) \quad (1)$$

$$\dot{\lambda} = [V_T \sin(\gamma_T - \lambda) - V_M \sin(\gamma_M - \lambda)]/r \quad (2)$$

$$\dot{\gamma}_M = a_M/V_M \quad (3)$$

$$\dot{\gamma}_T = a_T/V_T \quad (4)$$

For simplifying the design procedure, the velocities of the missile and target are assumed to be constant. Differentiating Eqs. (1) and (2) with respect to time yields

$$\ddot{r} = r\dot{\lambda}^2 + a_{Tr} - a_{Mr} \quad (5)$$

$$\ddot{\lambda} = -2\dot{r}\dot{\lambda}/r + a_{T\lambda}/r - a_{M\lambda}/r \quad (6)$$

where $a_{Tr} \triangleq a_T \sin(\lambda - \gamma_T)$ and $a_{Mr} \triangleq a_M \sin(\lambda - \gamma_M)$ denote the target and missile acceleration components along the LOS, and $a_{T\lambda} \triangleq a_T \cos(\lambda - \gamma_T)$ and $a_{M\lambda} \triangleq a_M \cos(\lambda - \gamma_M)$ the target and missile acceleration components perpendicular to the LOS, respectively.

Assumption 1. Owing to physical limitations, the target acceleration projections along and normal to the LOS are bound, and satisfy the following conditions

$$|a_{Tr}| \leq a_{Tr}^{\max}, \quad |a_{T\lambda}| \leq a_{T\lambda}^{\max} \quad (7)$$

Fig. 1. Homing engagement geometry.

where $a_{Tr}^{\max}$, $a_{T\lambda}^{\max}$ are unknown parameters that cannot be obtained in real applications due to the complexity and unpredictability of the target maneuvers.

By accepting the assumption that zeroing the LOS angular rate will lead to a perfect interception with zero miss distance, the control objective here can be summarized as designing an acceleration command such that the LOS angular rate can converge to zero or near zero in a finite time.

3. Basic Guidance Law Design

In this section, the missile guidance problem is considered based on finite time control approaches. Before giving the guidance law design result, the following finite-time convergence lemma is introduced for the convenience of analyzing the characteristics of the proposed guidance law.

Lemma 1.^{17,18)} Supposing that $V(t)$ is a smooth positive definite function on $U \subset \mathbb{R}^n$ and satisfies the differential inequalities

$$\dot{V}(t) + \beta_1 V^{\beta_2}(t) \leq 0 \quad (8)$$

$$\dot{V}(t) + \beta_3 V(t) + \beta_1 V^{\beta_2}(t) \leq 0$$

respectively, where $\beta_1, \beta_3 > 0$ and $0 < \beta_2 < 1$ are constants. Then, for any given t_0 , there exists an area $U_0 \subset \mathbb{R}^n$ such that any $V(t)$ which starts from U_0 can reach $V(t) \equiv 0$ in finite time, and the settling time t_r satisfies

$$t_r = t_0 + \frac{V^{1-\beta_2}(t_0)}{\beta_1(1-\beta_2)} \quad (9)$$

$$t_r = t_0 + \frac{1}{\beta_3(1-\beta_2)} \ln \frac{\beta_3 V^{1-\beta_2}(t_0) + \beta_1}{\beta_1}$$

respectively.

Based on the control objective mentioned above, the sliding surface is selected as

$$s = \dot{\lambda} \quad (10)$$

The control law is designed as

$$a_M = [-2\dot{r}\dot{\lambda} + k_1 s + k_2 |s|^\rho \text{sgn}(s)]/\cos(\theta) \quad (11)$$

where $k_1, k_2 > 0$, $0 < \rho < 1$ and $\theta \triangleq \lambda - \gamma_M$.

Theorem 1. Consider the dynamic system Eq. (6), subject to Assumption 1, and the sliding surface Eq. (10). If the control law is selected as Eq. (11), then the system trajectory will converge into the region

$$\Omega \triangleq \left\{ s \mid |s| \leq \min \left(a_{T\lambda}^{\max}/k_1, (a_{T\lambda}^{\max}/k_2)^{1/\rho} \right) \right\} \quad (12)$$

in finite time.

Proof. Differentiating Eq. (10) with respect to time and substituting Eqs. (6) and (11) into it yields

$$\dot{s} = [a_{T\lambda} - k_1 s - k_2 |s|^\rho \text{sgn}(s)]/r \quad (13)$$

Consider the following Lyapunov function candidate

$$V = s^2/2 \quad (14)$$

Evaluating the derivative of V along trajectory Eq. (13) gives

$$\begin{aligned}\dot{V} &= s\dot{s} \\ &= s(a_{T\lambda} - k_1s - k_2|s|^\rho \text{sgn}(s))/r \\ &\leq (a_{T\lambda}^{\max}|s| - k_1s^2 - k_2|s|^{\rho+1})/r\end{aligned}\quad (15)$$

When $k_1 > a_{T\lambda}^{\max}/|s|$ (or $|s| > a_{T\lambda}^{\max}/k_1$), then Eq. (15) becomes

$$\dot{V} \leq -k_2|s|^{\rho+1}/r = -k_2(2V)^{(\rho+1)/2}/r \quad (16)$$

Without loss of generality, we can assume that there exists a positive constant $r_{\max}$ such that $r < r_{\max}$ holds during the flight. Then, Eq. (16) becomes

$$\dot{V} \leq -k_2(2V)^{(\rho+1)/2}/r_{\max} \quad (17)$$

Thus, according to Lemma 1, the region $|s| \leq a_{T\lambda}^{\max}/k_1$ can be reached in finite time. Similarly, for the case of

$$k_2 > a_{T\lambda}^{\max}/|s|^\rho,$$

one can conclude that the region

$$|s| \leq (a_{T\lambda}^{\max}/k_2)^{1/\rho}$$

can also be reached in finite time, which completes the proof.

Remark 1. For Eq. (11), it is clear that the control law is continuous and hence chatter-free. Additionally, no negative fractional power is involved in Eq. (11). Therefore, it is also nonsingular. Moreover, the advantage of the proposed control formula is that the linear term k_1s dominates over the corresponding nonlinear term $k_2|s|^\rho \text{sgn}(s)$ for $|s| \geq 1$, thereby yielding faster asymptotical convergence of the sliding variable. The nonlinear term $k_2|s|^\rho \text{sgn}(s)$, on the other hand, dominates over the linear term k_1s for $|s| < 1$, yielding finite time convergence. Accordingly, a faster convergent rate can be obtained. Analyzing Eqs. (6), (10) and (11), assume that $|s| \ll \mu \ll 1$, $\mu > 0$. The term $k_1s + k_2|s|^\rho \text{sgn}(s)$ in the control law Eq. (11) can be rewritten as

$$(k_1 + k_2|s|^{\rho-1})s.$$

Then, the choice of the gain $k_2 \gg k_1\mu^{1-\rho}$ provides domination of the nonlinear term $k_2|s|^\rho \text{sgn}(s)$ that yields finite time convergence in the domain $|s| \ll \mu \ll 1$. It is worth noting that control law Eq. (11) with $\rho = 1$ can only guarantee asymptotical stability of the guidance loop.

Remark 2. The term $-2\dot{r}\dot{\lambda}/\cos(\lambda - \gamma_M)$ is the well-known PNG law with a navigation ratio of 2. Thus, the proposed guidance law, Eq. (11), can be interpreted as a pseudo-PNG law with a navigation ratio of 2 and some additional robust terms.

Remark 3. It is worth pointing out that singularity occurs when $\theta = \pm\pi/2$. In order to handle this problem, a slight change is made as

$$a_M = [-2\dot{r}\dot{\lambda} + k_1s + k_2|s|^\rho \text{sgn}(s)]/\cos(\bar{\theta}) \quad (18)$$

where $\bar{\theta} \triangleq \bar{\lambda} - \bar{\gamma}_M$ satisfies $\bar{\theta} \neq \pm\pi/2$. In this way, of course, it generates an error between the value of $\bar{\theta}$ and the actual value of θ . However, this error can be lumped together with the unknown target acceleration. Therefore, control law

Eq. (11) still works well, and the finite time convergence characteristic still holds.

Remark 4. In Ref. 15), the authors presented a novel FTCL law, which is

$$a_M = [-N\dot{r}\dot{\lambda} + l_1\text{sgn}(s) + l_2|s|^\eta \text{sgn}(s)]/\cos(\bar{\theta}) \quad (19)$$

where $N > 2$, $l_1 > a_{T\lambda}^{\max}$, $l_2 > 0$ and $0 \leq \eta < 1$ are the design parameters. The term $l_1\text{sgn}(s)$ is used to compensate for target maneuvering, while the term $l_2|s|^\eta \text{sgn}(s)$ is used to ensure finite time convergence. However, noting from Eq. (19), a discontinuous sign function exists. Therefore the undesired chattering phenomenon will be excited in practical implementation. This, in turn, will harm the performance of the missile guidance and control system, and may even lead to system instability.

4. FTDOB-based Composite Guidance Law

In reality, target maneuvers cannot be obtained or measured in advance. Therefore, the value of k_1 or k_2 should be selected large enough to compensate for target maneuvering. However, missile acceleration response, on the other hand, is limited by the actuator's capability. Thus, too large of a value for k_1 or k_2 will make the actuator saturate quickly at the beginning of the terminal guidance phase, which, in turn, can affect the performance of the proposed guidance law. In order to avoid this, FTDOB is used in this section to estimate target maneuvering $a_{T\lambda}$ and a composite robust guidance law is presented.

4.1. FTDOB design

Consider the following single-input-single-output (SISO) system:

$$\dot{\sigma} = f(t) + u \quad (20)$$

where σ is the system state, u is the control signal, $f(t)$ is an uncertain but sufficiently smooth function that satisfies

$$|\dot{f}(t)| < L,$$

where $L > 0$ is a Lipschitz constant. In view of the arbitrary-order exact robust differentiator proposed by Shtessel et al.¹³⁾ and Levant,¹⁶⁾ we obtain:

Lemma 2.^{13,16)} For the system Eq. (20), by constructing the following second-order FTDOB

$$\begin{aligned}\dot{z}_0 &= v_0 + u, \quad v_0 = -\beta_0 L^{1/3}|z_0 - \sigma|^{2/3} \text{sgn}(z_0 - \sigma) + z_1 \\ \dot{z}_1 &= v_1, \quad v_1 = -\beta_1 L^{1/2}|z_1 - v_0|^{1/2} \text{sgn}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\beta_2 L \text{sgn}(z_2 - v_1)\end{aligned}\quad (21)$$

where z_i ($i = 0, 1, 2$) is the observer output. With the proper selection of β_i and L , z_1 will converge to $f(t)$ in finite time.

Remark 5. A possible selection of observer parameters can be found in Shtessel et al.,¹³⁾ that is, $\beta_0 = 2$, $\beta_1 = 1.5$ and $\beta_2 = 1.1$. Moreover, from Eq. (21), it can be observed that the discontinuous term $\text{sgn}(z_2 - v_1)$ hides in the

higher-order term, which means that the chattering phenomenon (e.g. typically occurring in the traditional sliding mode observer) is eliminated by integration.

Remark 6. For the Lipschitz constant L , the convergence rate of disturbance estimation response increases as L increases. However, by the same token, large overshoot also occurs due to the initial error between the observer and the real system. Therefore, this parameter should be selected under certain circumstances.

Let $x = r\dot{\lambda}$, then Eq. (6) can be rewritten as

$$\dot{x} = -\dot{r}\dot{\lambda} - a_{M\lambda} + a_{T\lambda} \quad (22)$$

Suppose that $a_{T\lambda}$ can be differentiated with a Lipschitz constant L . For the system Eq. (22), according to Lemma 2, the FTDOB for estimating the target maneuver $a_{T\lambda}$ can be constructed as

$$\begin{aligned} \dot{z}_0 &= v_0 - \dot{r}\dot{\lambda} - a_{M\lambda}, \\ v_0 &= -2L^{1/3}|z_0 - x|^{2/3}\text{sgn}(z_0 - x) + z_1 \\ \dot{z}_1 &= v_1, \quad v_1 = -1.5L^{1/2}|z_1 - v_0|^{1/2}\text{sgn}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -1.1L\text{sgn}(z_2 - v_1) \end{aligned} \quad (23)$$

Using Lemma 2, one can imply that z_1 will converge to $a_{T\lambda}$ in finite time with the proper selection of L .

4.2. Composite guidance law design

The main results of this section are summarized in the following theorem.

Theorem 2. For the system Eq. (6) subject to Assumption 1, and the sliding surface Eq. (10), if the control law is chosen as

$$a_M = [z_1 - 2\dot{r}\dot{\lambda} + k_1s + k_2|s|^\rho\text{sgn}(s)]/\cos(\theta) \quad (24)$$

then, the LOS angular rate will converge to zero in finite time.

Proof. Suppose the settling time of FTDOB is T . We first prove that the closed-loop guidance system is bound during the flight period $t \in (0, T]$. To this end, assume that there exists a positive constant r_0 such that $r > r_0$ during $t \in (0, T]$. Then, evaluating the derivative of V gives

$$\begin{aligned} \dot{V} &= s(a_{T\lambda} - z_1 - k_1s - k_2|s|^\rho\text{sgn}(s))/r \\ &\leq |a_{T\lambda} - z_1||s|/r_0 \end{aligned} \quad (25)$$

Due to the principle of FTDOB, the estimation error $\varepsilon \triangleq a_{T\lambda} - z_1$ is bound for $t \in (0, T]$, and hence we can assume that a positive constant α exists such that $|a_{T\lambda} - z_1|/r_0 \leq \alpha$ holds during the flight period $t \in (0, T]$. Then, Eq. (25) becomes

$$\dot{V} \leq \alpha|s| \quad (26)$$

Solving Eq. (26) gives

$$V \leq \left(V^{1/2}(0) + \sqrt{2}/2\alpha t\right)^2 \leq \left(V^{1/2}(0) + \sqrt{2}/2\alpha T\right)^2 \quad (27)$$

Since V is bound for $t \in (0, T]$, the LOS angular rate will not escape in finite time. When $t \geq T$, we have

$$\begin{aligned} \dot{V} &= -(k_1s^2 + k_2|s|^{\rho+1})/r \\ &= -[k_1V + k_2(2V)^{(\rho+1)/2}]/r \\ &< -[k_1V + k_2(2V)^{(\rho+1)/2}]/r_{\max} \end{aligned} \quad (28)$$

According to Lemma 1, the LOS angular rate will converge to zero in finite time, which completes the proof.

Remark 7. From a practical point of view, although the estimation error of the FTDOB cannot precisely converge to zero in finite time due to the measurement noise and sampling rate, theorems 6 and 7 of Levant¹⁶⁾ prove that the observer Eq. (23) also ensures ε is bound in a small region. Since $0 < \rho < 1$, the value of $(\varepsilon/k_2)^{1/\rho}$ can be tuned much closer to zero compare to ε/k_1 . For example, if the estimation error of FTDOB is 2 m/s², k_2 and ρ are selected as 100 and 0.5, respectively, then we can obtain $(\varepsilon/k_2)^{1/\rho} = 0.0004$.

5. Simulations

In this section, the effectiveness of the proposed composite guidance law is demonstrated through numerical simulations. The initial conditions and other data required in Eqs. (1)–(4) are selected as: 1) missile-target initial relative range: 9000 m; 2) initial LOS angle: 20°; 3) missile initial flight path angle: 60°; 4) target initial flight path angle: 140°; 5) missile velocity: 1000 m/s; and 6) target velocity: 600 m/s. The parameters needed in the proposed guidance law and FTDOB are selected as: $k_1 = 150$, $k_2 = 150$, $\rho = 0.5$ and $L = 800$.

In order to make better illustrations, the proposed guidance law is compared to the augmented PNG (APNG) law,¹⁾ SMC-IDC guidance law¹¹⁾ and FTCL law.¹⁵⁾ To implement APNG, target maneuvering is required and we suppose that this information is perfectly known. Note that the APNG with a perfect target maneuver profile should be ideal. Furthermore, the navigation ratio for APNG is chosen as $N = 4$. The SMC-IDC law is defined as

$$\begin{aligned} a_M &= -N\dot{r}\dot{\lambda} + r\text{sgn}(\dot{\lambda}) + r\sigma + \frac{r\sigma}{\tau} \\ &\quad + \frac{1}{\tau} \int (r\sigma - \dot{r}\sigma)dt \end{aligned} \quad (29)$$

where

$$\begin{aligned} \sigma &= \dot{\lambda} + z, \quad z(0) = -\dot{\lambda}(0), \\ \dot{z} &= k\dot{\lambda} + l\text{sgn}(\dot{\lambda}), \quad k > 0, \quad l > 0 \end{aligned} \quad (30)$$

The parameters that are required for implementing the SMC-IDC guidance law are taken from Phadke and Talole¹¹⁾ and readers can refer to that literature for more details. The idea behind this guidance law is that the target maneuvers are estimated by IDC, and a linear term $r\sigma$ is adopted to guarantee the asymptotical stability of the guidance system.

For the FTCL law, to avoid the chattering resulting from the discontinuous term, the saturation function $\text{sat}(x)$ is used to replace the discontinuous sign function in the second term of Eq. (19), where

Fig. 2. Simulation results for Case 1 under various guidance laws: (a) Missile and target trajectories; (b) Missile acceleration commands; (c) LOS angular rate profiles; and (d) Uncertainty estimation by FTDOB.

$$\text{sat}(x) = \begin{cases} \text{sgn}(x), & |x| > \delta \\ x/\delta, & |x| \leq \delta \end{cases} \quad (31)$$

where δ is a small positive constant and is selected as 0.002/57.3 rad/s in simulations. The design parameters for FTGC in Eq. (19) are selected as: $N = 3$, $l_1 = 160$, $l_2 = 30$ and $\eta = 0.5$.

In simulations, the following two cases are considered. Case 1. $a_T = 160 \sin(3t) \text{ m/s}^2$; Case 2. $a_T = -160 \text{ m/s}^2$ for $0 < t \leq 2 \text{ s}$, $a_T = 0$ for $2 \text{ s} < t \leq 4 \text{ s}$, $a_T = 160 \text{ m/s}^2$ for others.

The simulation results for Case 1 and Case 2 under these four different guidance laws are presented in Figs. 2 and 3, respectively. The miss distances of these two cases are summarized in Table 1. From Figs. 2(a) and 3(a), it can be seen that, to some extent, interception is achieved under these four guidance laws. It can be further noted, as seen in Figs. 2(c) and 3(c), that the performance of the proposed guidance law in driving the LOS angular rate to zero is superior to that of other three guidance laws. Although the lumped uncertainty is estimated by IDC in the SMC-IDC guidance law, the LOS angular rate does not really converge to zero in Case 1, as is expected in this guidance law. Noting in the local zoomed-in graph of Fig. 3(c), the proposed guidance law has a faster convergence rate compared to that of the SMC-IDC guidance law. Moreover, in Figs. 2(b) and 3(b), it can be seen that the acceleration command for performing the proposed guidance law demands less than the SMC-IDC

guidance law and FTGC law.

For FTGC, since the boundary layer is used to address the chattering problem, only bounded motion around the origin can be ensured. Moreover, implementing the FTGC law needs the upper boundary for the target maneuver, which cannot be obtained in reality, and usually the maximum maneuverability of the pre-designated target is chosen as this upper boundary in real applications. However, due to the unpredictability and complicity of the target maneuver, a highly conservative boundary requires excessive control energy consumption and large chattering will be excited, while a lower upper boundary will result in performance degradation. As a comparison, the proposed continuous guidance law adopts FTDOB to estimate the target maneuver normal to the LOS; thus, the associated drawbacks of FTGC are overcome.

In Figs. 2(d) and 3(d), the uncertainty means target maneuvers normal to the LOS; that is, $a_{T\lambda}$. One can see that the FTDOB has quite satisfactory performance in uncertainty estimation whatever the target maneuver is. For obtaining better transient behavior of the FTDOB estimation response, higher-order FTDOB (larger than two)¹⁶⁾ can also be used.

6. Conclusion

In this paper, a new robust continuous guidance law for intercepting highly maneuverable targets is presented based on

Fig. 3. Simulation results for Case 2 under various guidance laws: (a) Missile and target trajectories; (b) Missile acceleration commands; (c) LOS angular rate profiles; and (d) Uncertainty estimation by FTDOB.

Table 1. Miss distance of the two cases.

Guidance law	FTCG	SMC-IDC	APNG	Proposed
Case 1	0.2742 m	0.3013 m	0.4655 m	0.2017 m
Case 2	0.1931 m	0.2908 m	0.3566 m	0.1817 m

sliding mode control theory. Using a non-smooth but continuous fractional power term, the finite time convergence of the LOS angular rate can be guaranteed. To improve the original guidance law further, a FTDOB-based composite guidance law, which do not need target maneuver information, is derived. Future works include extending this result to 3D cases.

Acknowledgments

The authors are deeply grateful to the editor and the associate editor for the time and effort spent in handling the paper, and to the anonymous reviewers for their valuable comments and constructive suggestions with regard to the revision of the paper.

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