Effect of *a priori* Information about the Target Location on Optimal Trajectories of Multiple Cooperative UAVs*

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This paper investigates how *a priori* localization information on a stationary ground target affects the optimal trajectories of cooperative unmanned aerial vehicles (UAVs) for estimating the target location. Each UAV is assumed to be equipped with a bearing-only sensor which always directs to the target. In order to formally incorporate the effect of *a priori* information into the estimation process, a Bayesian framework is employed: The optimal flying directions and paths of the multiple UAVs are determined by minimizing the Bayesian Cramér-Rao Lower Bound (BCRLB) with prior information on the ground target location represented via a bivariate Gaussian Probability Density Function (PDF). The effect of the prior information is examined with varying parameters that handle the size and shape of the prior PDF. It turns out that the more elliptical the prior PDF support, the more orthogonal the flying directions to the major principal axis of the ellipse in order to reduce the information imbalance in the direction of the major principal axis. In order to reduce the computational load for the calculation of the BCRLB, we employ the Bayesian Monte-Carlo integration method, which is shown to be fast enough to compute the BCRLB online in small UAVs.

Key Words: Unmanned Aerial Vehicle (UAV), Optimal Trajectory, Prior Information, Cramér-Rao Lower Bound (CRLB), Fisher Information Matrix (FIM)

Nomenclature

- θ : target location vector
- λ : prior probability density function (PDF) of target location
- p : (conditional) probability density function
- $\mathbf{x}$: state vector of UAV
- $\mathbf{z}$: measurement vector
- $\mathbf{F}$: Fisher information matrix
- σ : standard deviation
- R : distance from a UAV to the mean location of the prior PDF
- ϕ : bearing angle from a UAV to the mean location of the prior PDF
- η : information balance of the prior PDF
- ρ : relative prior information to the data
- $\mathbf{u}$: heading angle command to UAV
- v : velocity of UAV
- Subscripts
 - i : index of the i -th UAV
 - k : index of the k -th step of observation and guidance
 - B : Bayesian
 - D : measurement data
 - P : prior information
 - w : measurement noise
 - X : X -axis direction of the prior PDF
 - Y : Y -axis direction of the prior PDF

1. Introduction

Recently the demand for small unmanned aerial vehicles (UAVs) has rapidly increased in various military and civilian observation applications. In general, observation performance of a small UAV is relatively low compared to a large UAV so that multiple small UAVs can be cooperatively employed in order to enhance the observation performance. As this is directly dependent upon the geometric relation between the UAVs and the target, optimizing their flying trajectories is of great significance.

In this line of research, one of the guidance techniques in previous literature was to select the trajectories $\mathbf{x}$ of the UAVs to minimize the Cramér-Rao Lower Bound (CRLB), i.e., the optimal control input is derived from

$$\mathbf{u} = \arg \min \mathbf{F}^{-1}(\mathbf{x}, \theta), \quad (1)$$

subject to the vehicles' equations of motion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}). \quad (2)$$

Here, $\mathbf{F}$ is the Fisher Information Matrix (FIM), which is the inverse matrix of the CRLB. The input $\mathbf{u}$ usually controls the heading angle of the vehicles. For the case of two UAVs, Gu et al.¹⁾ showed that the observation performance is maximized when the UAV paths are orthogonal to each other (i.e., 90 deg relative bearing angle). Lee et al.²⁾ yielded the same result, that the determinant of one-step FIM $\mathbf{F}$

$$\det \mathbf{F} = \sin^2(\phi_1 - \phi_2) / (\sigma_1 \sigma_2 r_1 r_2)^2 \quad (3)$$

is maximal at a 90 deg relative bearing angle $|\phi_1 - \phi_2|$ for a given distance r_i to the target and the sensor noise variance σ_i^2 (see Fig. 1 for the definition of the bearing angle ϕ). Note

that the exact target location is necessary to implement the CRLB-based techniques.

In many practical situations, some information on the target location can be obtained *a priori* (e.g., from the geometrical features around the target area on the ground). This can be incorporated into the estimation process to increase the observation performance. For this reason, the current research uses the Bayesian CRLB (BCRLB) as an observation performance index in order to use such prior information by representing it as a Probability Density Function (PDF). Then the optimal trajectories for the best cooperative observation will be consequentially affected by the prior information on the target location.

The BCRLB, well known as the Van Trees version of the CRLB³⁾ or the Posterior CRLB⁴⁾ (PCRLB), has been widely used in the field of sensor placement. Since a radar system has more sufficient computation resources, the BCRLB, which generally requires high computational resources, has been considered more frequently for radar systems than UAV systems. Rajagopalan et al.⁵⁾ studied optimal sensor placement for a missile defense system, and the trace of the BCRLB was used as a performance index. Godrich et al.⁶⁾ also employed the trace of the BCRLB as a target tracking performance index for a Multi-Input Multi-Output radar system with multiple transmitters and sensors, and optimized the arrangement of the transmitters and the sensors.

In the UAV system applications, the estimated target location obtained from the Extended Kalman Filter (EKF) is used to calculate the classical CRLB,⁷⁻¹⁰⁾ and the prior information is used to initialize the EKF. Ponda et al.⁷⁾ showed the CRLB calculated from the estimated target location of the EKF corresponds to the estimated covariance matrix of the EKF. On the other hand, Grocholsky et al.⁸⁾ designed a coordinated and distributed control of multiple sensor platforms by maximizing the FIM of adjacent UAVs and combined the control law with the information filter. Since the information filter is theoretically equivalent to the EKF and an estimated target location is used to calculate the FIM, this study can be classified as the same line of research.

While the classical CRLB originated from the estimation of deterministic parameters, the EKF is basically a estimation technique for random parameters. To avoid such a theoretical dissimilarity in the classical CRLB-EKF approach, the current study employs one unifying theoretical framework, the Bayesian approach: We generate and analyze the optimal trajectories of multiple UAVs by minimizing the Bayesian Cramér-Rao Lower Bound with *a priori* information on the ground target location represented via a bivariate Gaussian PDF that reflects two-dimensional localization information on the ground target. Different from the classical CRLB-EKF approach, the current approach does not require a specific type of estimator.

Each UAV is assumed to measure the bearing angle with a bearing-only sensor that always directs to the target. Although the effect of the prior information on the estimate of the target location fades away as the measurement progresses, it can remain for a considerable period of time, since

small UAVs are generally equipped with low-performance sensors.

The highlights of the results and contributions of the current study in advance are as follows: After deriving the Bayesian CRLB in Section 2, we consider the optimal UAV initial deployment for the case of using only one measurement data in Section 3:

- When the support of the two-dimensional prior PDF is circular (i.e., the two random variables representing each direction on the ground are independent), the optimal relative bearing angle $|\phi_1 - \phi_2|$ is 90 deg for the two-UAV case. This is exactly the same result as the deterministic CRLB approach^{1,2)} mentioned before (see Fig. 2(a));
 - However, when the support shape of the prior PDF is elliptical, the relative bearing angle is different from 90 deg; the more elliptical the prior support the smaller the optimal relative bearing angle (see Fig. 2(b) and (c)). This trend strongly appears when the amount of prior information is relatively large compared to that of the measurement data.
- Then, Section 4 extends this idea to derive the optimal guidance law by extending the optimal control theory^{2,11)} and analyzing the effect of the elliptical shape of the prior PDF on the optimal UAV trajectories by simulating two- and three-UAV cases with multiple measurement data:

- In order to reduce the computational burden for the BCRLB calculation, the Bayesian Monte Carlo (BMC) integral method¹²⁾ is employed, and it is shown that the calculation speed is fast enough to compute the BCRLB online in small UAVs:

In Section 5, for a two-UAV case, it was numerically found that the current Bayesian approach resulted in the same observation performance as the classical CRLB-EKF approach when the same prior information for BCRLB is used as the initial error covariance of the EKF.

2. Bayesian Cramér-Rao Lower Bound

Suppose that the parameter $\theta \in \mathbb{R}^p$ to be estimated is a random vector whose probability density function is given by $\lambda(\theta)$, and consider an estimator $\hat{\theta}$ based on a measurement vector z . If the PDFs $p(z|\theta)$ and $\lambda(\theta)$ satisfy the regularity conditions $E_{z|\theta}[\partial \ln p(z|\theta)/\partial \theta] = \mathbf{0}$ for all θ and $E_{\theta}[\partial \ln \lambda(\theta)/\partial \theta] = \mathbf{0}$, then it is known that the estimation error covariance satisfies the following Bayesian Cramér-Rao lower bound³⁾

$$E_{\theta}[E_{z|\theta}[(\hat{\theta}(z) - \theta)(\hat{\theta}(z) - \theta)^T]] \geq F_B^{-1}, \quad (4)$$

where

$$F_B \triangleq E_{\theta} F_D] + F_P, \quad (5)$$

$$F_D \triangleq -E_{z|\theta}[\nabla_{\theta}^2 \ln p(z|\theta)], \quad (6)$$

$$F_P \triangleq -E_{\theta}[\nabla_{\theta}^2 \ln \lambda(\theta)]. \quad (7)$$

Here ∇_{θ}^2 is used to indicate the second-order partial derivative operator $\nabla_{\theta}^2 \triangleq (\partial/\partial \theta)(\partial/\partial \theta)^T$ with $\partial/\partial \theta = [\partial/\partial \theta_1 \ \partial/\partial \theta_2 \ \cdots \ \partial/\partial \theta_p]^T$, F_D and F_P are the FIMs of the measurement and the prior information, respectively. In this

Fig. 1. Ground target localization with the prior information (see Eq. (23) for the definition of the polar coordinate expression).

paper, F_B is called the Bayesian FIM (BFIM) to distinguish it from F_D .

A stationary ground target is assumed to lie on a two-dimensional plane and *a priori* information on the target location $\theta = [\theta_x \ \theta_y]^T$ is provided in the form of a probability density function $\lambda(\theta)$. In this study, we choose the zero-mean bivariate Gaussian distribution with the variance σ_x^2 of the x -axis and the variance σ_y^2 of the y -axis (see Fig. 1) given by

$$\lambda(\theta) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{\theta_x^2}{2\sigma_x^2} - \frac{\theta_y^2}{2\sigma_y^2}\right]. \quad (8)$$

We consider N -UAV cases, each of which measures the target bearing angle $\phi_i[k]$ obtaining $z_i[k]$ ($i = 1, 2, \dots, N, k = 1, 2, \dots, M$) using a gimbaled bearing-only sensor installed in each UAV so that the sensor always points to the ground target (see Fig. 1 for $N = 2$ case). Here i and k denote the indices of each UAV and the measurement sequence, respectively. It is assumed that the altitude effect between the target and the UAVs can be ignored, and the exact location of each UAV $\mathbf{x}_i[k] = [x_i[k] \ y_i[k]]^T$ is known to all UAVs. The measured bearing signal $z_i[k]$ is modeled as a sum of the true bearing angle $h(\theta, \mathbf{x}_i[k])$ and a white noise sequence $w_i[k]$

$$z_i[k] = h(\theta, \mathbf{x}_i[k]) + w_i[k], \quad (9)$$

where $w_i[k]$ has a zero-mean Gaussian distribution with the variance σ_w^2 .

The conditional PDF of the M measurements $\mathbf{z} = [z_1[1] \ z_1[2] \ \dots \ z_1[M]]^T$ for a given θ is represented as

$$p(\mathbf{z}|\theta) = \prod_{k=1}^M \prod_{i=1}^N p(z_i[k]|\theta), \quad (10)$$

where

$$p(z_i[k]|\theta) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp\left[-\frac{1}{2\sigma_w^2}(z_i[k] - h(\theta, \mathbf{x}_i[k]))^2\right], \quad (11)$$

$$h(\theta, \mathbf{x}_i[k]) = \tan^{-1}\left(\frac{\theta_y - y_i[k]}{\theta_x - x_i[k]}\right). \quad (12)$$

Then plugging Eq. (10) into Eq. (6) yields the FIM (see Appendix A),

$$\mathbf{F}_D \triangleq \begin{bmatrix} F_{D,11} & F_{D,12} \\ F_{D,21} & F_{D,22} \end{bmatrix}, \quad (13)$$

where

$$F_{D,11} = \sum_{k=1}^M \sum_{i=1}^N F_{D,11,i}[k], \quad (14)$$

$$F_{D,12} = \sum_{k=1}^M \sum_{i=1}^N F_{D,12,i}[k] = F_{D,21}, \quad (15)$$

$$F_{D,22} = \sum_{k=1}^M \sum_{i=1}^N F_{D,22,i}[k], \quad (16)$$

with

$$F_{D,11,i}[k] = \frac{1}{\sigma_w^2} \frac{(\theta_y - y_i[k])^2}{[(\theta_x - x_i[k])^2 + (\theta_y - y_i[k])^2]^2}, \quad (17)$$

$$F_{D,12,i}[k] = -\frac{1}{\sigma_w^2} \frac{(\theta_x - x_i[k])(\theta_y - y_i[k])}{[(\theta_x - x_i[k])^2 + (\theta_y - y_i[k])^2]^2}, \quad (18)$$

$$F_{D,22,i}[k] = \frac{1}{\sigma_w^2} \frac{(\theta_x - x_i[k])^2}{[(\theta_x - x_i[k])^2 + (\theta_y - y_i[k])^2]^2}. \quad (19)$$

Using Eqs. (7) and (8), the FIM of the Gaussian distribution is given by,

$$\mathbf{F}_P = \begin{bmatrix} 1/\sigma_x^2 & 0 \\ 0 & 1/\sigma_y^2 \end{bmatrix}. \quad (20)$$

Finally the BFIM becomes

$$\mathbf{F}_B \triangleq \begin{bmatrix} F_{B,11} & F_{B,12} \\ F_{B,21} & F_{B,22} \end{bmatrix}, \quad (21)$$

where

$$F_{B,rs} = \iint F_{D,rs} \lambda(\theta_x, \theta_y) d\theta_x d\theta_y + F_{P,rs} \quad (r = 1, 2, s = 1, 2). \quad (22)$$

3. UAV Deployment for Optimal Observation

We investigate the effect of *a priori* information about the target location on the optimal UAV deployment maximizing the BFIM, Eq. (21). F_D is dependent on θ and N -UAV locations $\mathbf{x}_i[k]$ ($i = 1, 2, \dots, N, k = 1, 2, \dots, M$), and F_P is a constant matrix for a given $\lambda(\theta)$. After F_D is integrated over $\lambda(\theta)$, F_B , the inverse of the BCRLB, is dependent only on $\mathbf{x}_i[k]$. Thus, the observation performance is maximized (i.e., the BCRLB is minimized) by proper UAV deployment. Then, the optimal UAV deployment is achieved by minimizing the determinant of the BCRLB, which can be converted to maximizing the determinant of the BFIM (note $\det(\mathbf{A}^{-1}) = (\det \mathbf{A})^{-1}$).

In order to see the difference between the previous studies^{1,2)} using the classical FIM and current Bayesian FIM approach, we consider a simple two-UAV case with one-step measurement ($M = 1, N = 2$ in Eqs. (14)–(16)) and the mean target location at the origin, as shown in Fig. 1. Then,

F_B becomes a function of the initial UAV locations $x_i[k]$ ($i = 1, 2, k = 1$), and we can find the optimal geometric relations between the two UAV locations and the mean target location of a given prior PDF by maximizing $\det(F_B)$.

Each UAV location is expressed as in the polar coordinate system, for $i = 1, 2$,

$$\begin{aligned} x_i &= R_i \cos(\phi_i + \pi), \\ y_i &= R_i \sin(\phi_i + \pi), \end{aligned} \quad (23)$$

where R_i is the distance from the i -th UAV to the origin, which is equal to the mean target location, and ϕ_i is its bearing angle to the origin. The index k is omitted for a brief presentation.

In order to analyze the effect of the various shapes and sizes of the prior PDF on observation performance, we define the following two major normalized parameters:

- The first parameter is termed as the *prior information balance* and defined as

$$\eta \triangleq \frac{\sigma_x^2}{\sigma_y^2} \quad (24)$$

which is the ratio of the x -axis variance to the y -axis variance of the prior PDF. For example, in the case of $\eta = 1$, the support of the prior PDF is circular. A $\eta > 1$ case has an elliptical PDF support whose main axis is parallel to the x -axis and provides more information in the y -axis direction than in the x -axis direction.

- The second parameter is termed as the *relative prior information to the data* and defined as

$$\rho \triangleq \frac{A_D}{A_p} \quad (25)$$

which represents the relative amount of the prior information to the one-step measurement data.

Here $A_D \triangleq \pi(\det[F_D]_{at k=1, \Delta\phi=\Delta\phi^*})^{1/2}$ is the area of the $1-\sigma$ confidence region of the one-step measurement at the optimal relative bearing angle $\Delta\phi^*$. For example, the two-UAV case has 90° of $\Delta\phi^*$ and $A_D = \sigma_w^2 R_1 R_2 \pi$.

$A_p \triangleq \sigma_x \sigma_y \pi$ is the area of the $1-\sigma$ confidence region of the prior PDF. Therefore, in the case of a big ρ , the amount of prior information is relatively large compared to the one-step measured information.

Then, the optimal deployment problem of multiple UAVs is to find the optimal set of bearing angles ϕ_i , $i = 1, 2, \dots, N$ for given parameter values σ_x , σ_y , σ_w , and R_i , ($\phi_1, \phi_2, \dots, \phi_N$)

$$= \arg \max \det F_B(\sigma_x, \sigma_y, \sigma_w, R_1, R_2, \dots, R_N). \quad (26)$$

Within this framework, we consider a two-UAV case for the different parameter sets and investigate the effect of each parameter change.

3.1. Effect of prior information balance η

For a fixed initial distance from each UAV to the mean target location $R_1 = R_2 = 500$ m, Fig. 2(a), (b), and (c) show the optimal bearing angles obtained by solving Eq. (26) for $\eta = 1, 1.5$, and 2.5 , respectively. Other parameters used are $\sigma_w = 0.1$ rad ($\approx 5.7^\circ$), and $\rho = 2$ ($A_D = 2500\pi \text{ m}^2$,

Fig. 2. Optimal deployments of two equidistant UAVs ($\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m).

Fig. 3. Relative angle of optimal deployments for $\eta \geq 1$ using two equidistant UAVs ($\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m).

Fig. 4. Optimal relative angle of optimal for $\eta \geq 1$, $\rho = 1, 2$, and 4 deployments using the two equidistant UAVs ($\sigma_w = 0.1$ rad, $R = 500$ m).

$A_p = 1250\pi \text{ m}^2$). In the figures, the ellipse represents the $1-\sigma$ line of each prior PDF, and the straight line is the optimal line-of-sight from each UAV to the mean target location (the origin).

In the case of $\eta = 1$ (Fig. 2(a)), observation performance is maximized when the relative bearing angle $\Delta\phi$ is 90° . This result is exactly the same as that of the classical CRLB approaches.^{1,2)} However, the cases of $\eta = 1.5$ and 2.5 have relative angles $\Delta\phi$ less than 90° (Fig. 2(b) and (c)). As shown in Fig. 3, as η increases, the optimal relative bearing angle $\Delta\phi$ decreases. UAVs tend to be deployed to get more information in the x -axis direction since the uncertainty in this direction is higher than that of the y -axis. For approximately $\eta > 2.7$, the optimal $\Delta\phi$ is 0° , which means that UAVs are deployed to gather only x -axis direction information. The behavior of UAVs will be further explained in Section 4.

3.2. Effect of relative prior information to the data ρ

Figure 4 describes the changes of the optimal relative bearing angle $\Delta\phi$ for various values $\rho = 1, 2$, and 4 ($A_p = 2500\pi \text{ m}^2$, 1250π , 625π for a fixed $A_D = 2500\pi \text{ m}^2$).

The results show that the bigger the relative prior information ρ , the faster the optimal relative bearing angle changes.

This means that the optimal deployment of the UAVs is more influenced by the shape of the prior PDF in the cases with more abundant prior information.

4. Bayesian Information-based Guidance Law

The previous section focused only on finding a strategy for the optimal initial two-UAV deployment with one-step measurement ($M = 1$) for a given distance from each UAV to the mean target location. The main issue of this section is to further analyze the effect of the prior information about the target location on UAV trajectories generated by Bayesian information-based guidance with multiple measurement data. Here a guidance law for small UAVs, considered as point masses, is designed in a similar manner to the previous study²⁾ in which the guidance law generates the control inputs for the UAV heading angles using one-step FIM. But, the current guidance law uses one-step BFIM to generate the heading angle input instead of using one-step FIM. Posterior PDF at the current stage, which reflects each measurement data, is used as the prior PDF in the next stage.

We assume that the UAVs are perfectly controlled by the guidance, and all UAVs have a constant velocity v , and the dynamics of N -UAVs can be written as

$$\dot{\mathbf{x}}_i(t) = v \begin{bmatrix} \cos u_i(t) \\ \sin u_i(t) \end{bmatrix}, \quad i = 1, 2, \dots, N. \quad (27)$$

It is also assumed that the location of each UAV is known exactly, and the communication between the UAVs is perfect so that all information on the UAV locations and the sensor measurement data are shared immediately. Furthermore, all sequences of the UAV measurements and guidance inputs are synchronized.

The new cost function using the one-step BFIM is given by

$$J = -\det \mathbf{F}_B(t + \Delta t) + \int_t^{t+\Delta t} \ell^T(\mathbf{f} - \dot{\mathbf{x}})d\tau, \quad (28)$$

where $\ell = [\ell_{x1} \ \ell_{y1} \ \dots \ \ell_{xN} \ \ell_{yN}]^T \in \mathbb{R}^{2N}$ denotes a Lagrange multiplier vector, $\mathbf{x} = [\mathbf{x}_1^T \ \mathbf{x}_2^T \ \dots \ \mathbf{x}_N^T]^T$ denotes the location vector of the UAVs, and Δt denotes the time horizon. As insisted in Lee et al.,²⁾ the resulting trajectories using this one-step time horizon will not be globally optimal, but sufficiently efficient and proper commands could be generated. For the effects of time horizon on path planning, readers are referred to Ousingsawat and Campbell.¹⁰⁾

The solution of the optimal problem minimizing Eq. (28) is obtained by applying the optimal control theory¹¹⁾ (see Appendix B for the derivation).

$$\dot{\ell} = \mathbf{0}, \quad (29)$$

$$\partial \ell^T \mathbf{f} / \partial \mathbf{u} = \mathbf{0}, \quad (30)$$

$$\ell^T(t_f) = \partial(-\det \mathbf{F}_B) / \partial \mathbf{x}|_{t_f}, \quad t_f = t + \Delta t. \quad (31)$$

Then, solving Eqs. (29)–(31) yields

$$\ell = \text{constant}, \quad (32)$$

$$u_i = \tan^{-1}(\ell_{iy}/\ell_{ix}), \quad i = 1, \dots, N, \quad (33)$$

$$\ell^T = - \left[F_{B,11} \left(\frac{\partial F_{B,22}}{\partial \mathbf{x}} \right) + \left(\frac{\partial F_{B,11}}{\partial \mathbf{x}} \right) F_{B,11} - 2F_{B,12} \left(\frac{\partial F_{B,12}}{\partial \mathbf{x}} \right) \right]. \quad (34)$$

Since this result is a continuous-time solution, the guidance command u_i is discretized at each observation sequence and kept constant during each period.

When a new measurement is available, the prior PDF is updated to a posterior PDF, which in turn, can be used as a prior PDF in the next stage. Thus, the prior PDF at the k -th stage can be written as

$$\lambda_k(\boldsymbol{\theta}) = \lambda(\boldsymbol{\theta}|z[1], \dots, z[k]) = \frac{1}{n_k} p(z[1], \dots, z[k]|\boldsymbol{\theta}) \lambda_0(\boldsymbol{\theta}), \quad (35)$$

where

$$n_k = \int p(z[1], \dots, z[k]|\boldsymbol{\theta}) \lambda_0(\boldsymbol{\theta}) d\boldsymbol{\theta}. \quad (36)$$

Since each measurement is independent, Eq. (35) can be rewritten as

$$\begin{aligned} \lambda_k(\boldsymbol{\theta}) &= \frac{1}{n_k} \lambda_0(\boldsymbol{\theta}) \prod_{j=1}^k p(z[j]|\boldsymbol{\theta}) \\ &= \frac{n_{k-1}}{n_k} p(z[k]|\boldsymbol{\theta}) \left[\frac{1}{n_{k-1}} \lambda_0(\boldsymbol{\theta}) \prod_{j=1}^{k-1} p(z[j]|\boldsymbol{\theta}) \right] \\ &= \frac{n_{k-1}}{n_k} p(z[k]|\boldsymbol{\theta}) \lambda_{k-1}(\boldsymbol{\theta}). \end{aligned} \quad (37)$$

Then, the $(k+1)$ -th BFIM of the prior PDF becomes

$$\begin{aligned} F_P[k+1] &= E_{z,\boldsymbol{\theta}}[\nabla_{\boldsymbol{\theta}}^2 \ln \lambda_k(\boldsymbol{\theta})] \\ &= E_{z,\boldsymbol{\theta}} \left[\nabla_{\boldsymbol{\theta}}^2 \left(\ln \frac{n_{k-1}}{n_k} + \ln p(z[k]|\boldsymbol{\theta}) + \ln \lambda_{k-1}(\boldsymbol{\theta}) \right) \right] \\ &= E_{\boldsymbol{\theta}}[F_D[k]] + F_P[k]. \end{aligned} \quad (38)$$

Here, $E_{\boldsymbol{\theta}}[F_D[k]]$ is obtained from Eqs. (14)–(16) by taking the expectation over the $\lambda_{k-1}(\boldsymbol{\theta})$. Then, the elements of the BFIM in Eq. (22) for Eq. (34) at the k -th stage can be rewritten as

$$F_{B,rs}[k] = \iint \left(\sum_{i=1}^N F_{D,rs,i}[k] \right) \lambda_{k-1}(\theta_x, \theta_y) d\theta_x d\theta_y + F_{P,rs}[k], \quad (39)$$

and the partial derivative terms in Eq. (34) are given by

$$\begin{aligned} \frac{\partial F_{B,rs}[k]}{\partial x_i} &= \iint \frac{\partial F_{D,rs,i}[k]}{\partial x_i} \lambda_{k-1}(\theta_x, \theta_y) d\theta_x d\theta_y, \\ (r = 1, 2, s = 1, 2). \end{aligned} \quad (40)$$

Finally, the discretized guidance command $u_i[k]$ in Eq. (33) can be calculated through Eqs. (34)–(40).

This fomulation can be extended to a three-dimensional problem with additional computational burden. In this case,

Fig. 5. Two UAV trajectories with different prior information: $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 45$ deg, $\phi_2 = 135$ deg.

Fig. 6. Prior information balance η_k at the k -th step (two-UAV case): $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 45$ deg, $\phi_2 = 135$ deg.

the prior information can be given as an ellipsoid shape, measurements can consist of bearing angle and vertical angle, and UAV dynamics are given based on the three-dimensional coordinates system.

In order to reduce the computational load required for the multiple integrations, the Bayesian Monte Carlo (BMC) integral method¹²⁾ is applied. The run time of the guidance calculation of each step for a UAV is 0.025 s on average in a MATLAB environment using an Intel Core i7-3770 CPU.

A set of 100-run Monte Carlo simulations were performed for different prior information cases ($\eta = 0.04, 0.2, 1, 5$, and 25), and the mean trajectories and prior information balance $\eta_k \triangleq F_{P,11}[k]/F_{P,22}[k]$ at the k -th observation stage are shown in Figs. 5 and 6, respectively. The simulations were conducted under the same conditions as in Section 3: The mean target location is fixed at the origin, but, two UAVs start from the fixed locations at $R = 500$ m, $\phi_1 = 45$ deg, $\phi_2 = 135$ deg. The speed of UAVs is 20 m/s, and the guidance commands are updated at 1 Hz. The final time of each simulation is chosen such that one of the UAVs approaches the estimated target location within 20 m.

In Fig. 5, the regions indicated as ① and ② represent the areas of $|y| > |x|$ and $|y| < |x|$, respectively, and ③ denotes the line of $y = \pm x$. Based on Eqs. (17)–(19), a UAV flying in region ① can gather more x -axis information than that of the y -axis from the measurement data (i.e., $F_{D,11} > F_{D,22}$). On the contrary, for a UAV flying in region ②, there is more y -axis information than that of the x -axis (i.e., $F_{D,11} < F_{D,22}$). If a UAV is on the line of $y = \pm x$, then the information amounts of each axis are the same.

Fig. 7. Three UAV trajectories with different prior information: $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, $\phi_3 = 270$ deg.

Therefore, for the case of $\eta = 1$, the optimal UAV trajectories are generated along the line of $y = \pm x$. For $\eta > 1$, the UAV trajectories are curved inwards in the early stage to gather more x -axis information. On the contrary, for the cases of $\eta < 1$, the UAV trajectories are curved outwards in the early stage so that more y -axis information can be gathered.

These trends of the optimal UAV trajectories can be explained in connection with the change of the prior information balance η_k at the k -th observation stage as shown in Fig. 6. As the cooperative observation progresses, η_k approaches to 1 in all cases. This means that the guidance tends to reduce the information imbalance between the two directions.

From the perspective of balancing the information, this trend can be explained as follows: If there is information imbalance in a direction, in the early phase, the flying paths of the UAVs deviate from the line of $y = \pm x$ to reduce the information imbalance and return to the line of $y = \pm x$ after balancing the amount of information in both directions.

Figure 7 shows a three-UAV case for different prior information balances. In this case, A_D is redefined as $\pi(\det[F_D])_{at k=1, \Delta\phi=120\text{ deg}}^{-1/2}$ since the optimal relative bearing angle for three UAVs is 120 deg in the classical approach.

UAV1 and UAV2 show similar tendencies to those of the previous two-UAV cases, but the trajectories of UAV3 do not change for different η values, which causes UAV3 to gather only x -axis information. For non-unity η cases, the amount of y -axis information is more sensitive to UAV1 and UAV2 than UAV3. For these reasons, UAV3 is allocated to contribute only to x -axis information, and UAV1 and UAV2 are arranged to obtain y -axis information in order to maximize the total information amount. The prior information balance η_k of this case is depicted in Fig. 8. Similar to Fig. 6, η_k approaches to 1 in all cases.

The optimal trajectories of UAV1 and UAV2 in the three-UAV case are compared with those of a two-UAV case for

Fig. 8. Prior information balance η_k at the k -th step (three-UAV case): $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, $\phi_3 = 270$ deg.

Fig. 9. Comparison of two- and three-UAV trajectories with Configuration-I initial locations $(\phi_1, \phi_2, \phi_3) = (45, 135, \text{and } 270 \text{ deg})$ and the prior information shape: $\eta = 1$, $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m.

two different sets of initial locations of UAV1 and UAV2 (Configuration-I and Configuration-II). Initial UAV1 and UAV2 locations in Configuration-I and Configuration-II, respectively, correspond to the optimal initial locations of two-UAV (Fig. 5) and three-UAV (Fig. 7) cases, while UAV3 initial location is fixed for both configurations of the three-UAV case.

Figures 9 and 10 describe the differences of the optimal trajectories of Configuration-I and Configuration-II, respectively. As for prior information, $\eta = 1$ and $\rho = 2$ are used for both configurations. It is commonly observed that the optimal trajectories of UAV1 and UAV2 of the three-UAV case are closer to the x -axis than those of the two-UAV case, so that UAV1 and UAV2 of the three-UAV case can concentrate on gathering y -axis information of the target location. This is because UAV3 can gather x -axis information more effectively than UAV1 and UAV2, and can share the information with UAV1 and UAV2.

Figure 11 shows the trajectories of a different three-UAV case with $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, and $\phi_3 = 320$ deg. The trajectories of the UAVs show similar trends to the previous cases (Figs. 5 and 7). In cases where the prior information has more y -axis information than x -axis information ($\eta < 1$ case), the trajectories are curved toward the x -axis to gather more y -axis information in the early observation

Fig. 10. Comparison of two- and three-UAV trajectories with Configuration-II initial locations $(\phi_1, \phi_2, \phi_3) = (30, 150, \text{and } 270 \text{ deg})$ and the prior information shape: $\eta = 1$, $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m.

Fig. 11. Three-UAV trajectories with different prior information: $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, $\phi_3 = 320$ deg.

Fig. 12. Prior information balance η_k at the k -th step (three-UAV case): $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, $\phi_3 = 320$ deg.

stage. In the early observation stage of $\eta \geq 1$ cases, the trajectories are curved toward the y -axis to reduce the information imbalance caused from the initial locations (all UAVs are in area ② of Fig. 5) and the prior information.

Figure 12 shows the prior information balance η_k of this case which shows a similar tendency to those of Figs. 6 and 8 in that η_k returns back to 1 as the observation pro-

Fig. 13. Confidence area comparison with/without prior information (two-UAV, BCRLB and CRLB-EKF approaches): $\rho = 2$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 45$ deg, $\phi_2 = 135$ deg.

Fig. 14. Confidence area comparison with/without prior information (two-UAV, BCRLB and CRLB-EKF approaches): $\rho = 10$, $\sigma_w = 0.1$ rad, $R = 500$ m, $\phi_1 = 45$ deg, $\phi_2 = 135$ deg.

gresses. From this result, we can reconfirm that the the guidance tends to reduce the information imbalance regardless of the number of UAVs or initial locations.

5. Performance Comparison

The observation performance of the current Bayesian approach was compared with the classical CLRB-EKF approach²⁾ for the two-UAV case. In order to endow the classical CLRB-EKF approach with the same level of prior information, the covariance of the bivariate Gaussian prior PDF for the Bayesian approach was used as the initial error covariance for the EKF of the classical CRLB-EKF approach.

Figures 13 and 14 show the comparison results of both approaches for various levels of prior information. Jumping to the conclusion first, both approaches yielded the same observation performance for equal levels of prior information used. The difference was very minor and seems to be due to the BMC integral method used for the current Bayesian approach. However, it must be said that the performance equality is only based on numerical simulation results without formal proof, for which a continuing effort is necessary. It should also be noted that the current Bayesian framework does not require any specific estimator and provides a unifying framework for analyzing the prior information effect on optimal guidance.

In the figures, observation performance was represented as the area of the 1- σ confidence region of the estimated target location of the posterior PDF (for the Bayesian approach) and the error covariance of the EKF (for the classical CRLB-EKF approach) for the same level of prior information (Figs. 11 and 12, respectively, depict a $\rho = 2$ case and a $\rho = 10$ case).

The area of the 1- σ confidence region for the EKF is equal to $\pi(\det[\Sigma_{\text{EKF}}])^{1/2}$, where Σ_{EKF} is the error covariance of the EKF.

The absence of prior information for both approaches was implemented by taking a big-magnitude matrix (e.g., $10^{10} \cdot \mathbf{I}$) as an initial covariance for the prior PDF and the EKF. For

other prior information cases, the same area ($1250\pi \text{ m}^2$) of the 1- σ confidence region but with different prior shapes ($\eta = 0.04, 0.2, 1, 5, 25$) was used as the initial condition.

In the early phase of observation, the cases with prior information outperforms the case without prior information, and the performance difference decreases as the observation progresses. It is also found that the effect of prior information remains longer in cases with abundant prior information.

6. Conclusion

This paper addressed the effect of *a priori* information about the target location (expressed as a PDF) on the trajectories of multiple cooperative UAVs for optimal observation performance using Bayesian information-based guidance. The determinant of the Bayesian Cramér-Rao Lower Bound was employed as the performance index and minimized.

The optimal UAV initial deployment was affected by the prior information balance between the x -axis and y -axis. When the support of the prior PDF is circular, the optimal relative bearing angle is 90 deg for the two-UAV case. However, when the prior support is elliptical, the optimal relative bearing angle is different from 90 deg.

Bayesian information-based guidance was also developed based on the optimal control theory, and the posterior PDF was recursively used as a prior PDF in the next stage to calculate the Bayesian Fisher Information Matrix. In order to avoid the computational load due to the Bayesian approach, we employed the Bayesian Monte Carlo integration method to calculate the guidance online. The effect of the prior information was strong when the amount of the prior information was large.

For a two-UAV case, the performance of the current approach was numerically compared with the classical CRLB-EKF approach, yielding the exact same result. The formal proof of this observation is an issue of the future research. Although this study only used bivariate Gaussian distribution, any arbitrary distribution satisfying the regularity condition can be applied using special techniques such as

the Gaussian mixture, and it can be extended to the estimation problems in three-dimensional situations.

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Appendices

A. Derivation of FIM for a bearing-only sensor

From Eqs. (6) and (10),

$$\begin{aligned}
 F_{D,rs} &= -E_{z|\theta} \left[\frac{\partial^2 \ln p(z|\theta)}{\partial \theta_r \partial \theta_s} \right] \\
 &= -E_{z|\theta} \left[\frac{\partial^2 \left\{ \ln \prod_{k=1}^M \prod_{i=1}^N p(z_i[k]|\theta) \right\}}{\partial \theta_r \partial \theta_s} \right] \quad (41) \\
 &= -\sum_{k=1}^M \sum_{i=1}^N E_{z|\theta} \left[\frac{\partial^2 \{ \ln p(z_i[k]|\theta) \}}{\partial \theta_r \partial \theta_s} \right].
 \end{aligned}$$

Let $F_{D,rs}, i[k] \triangleq E_{z|\theta} [\partial^2 \ln p(z_i[k]|\theta) / \partial \theta_r \partial \theta_s]$ which denotes the (r, s) element of the FIM for the k -th measurement

of the i -th UAV. The second-order partial derivative terms of $\ln p(z_i[k]|\theta)$ are obtained

$$\frac{\partial^2 \{ \ln p(z_i[k]|\theta) \}}{\partial \theta_x^2} = \frac{1}{\sigma_w^2} \left\{ -\left(\frac{\partial h}{\partial \theta_x} \right)^2 + (z_i[k] - h) \frac{\partial^2 h}{\partial \theta_x^2} \right\}, \quad (42)$$

$$\frac{\partial^2 \{ \ln p(z_i[k]|\theta) \}}{\partial \theta_y^2} = \frac{1}{\sigma_w^2} \left\{ -\left(\frac{\partial h}{\partial \theta_y} \right)^2 + (z_i[k] - h) \frac{\partial^2 h}{\partial \theta_y^2} \right\}, \quad (43)$$

$$\frac{\partial^2 \{ \ln p(z_i[k]|\theta) \}}{\partial \theta_x \partial \theta_y} = \frac{1}{\sigma_w^2} \left\{ -\left(\frac{\partial h}{\partial \theta_x} \right) \left(\frac{\partial h}{\partial \theta_y} \right) + (z_i[k] - h) \frac{\partial^2 h}{\partial \theta_x \partial \theta_y} \right\}. \quad (44)$$

The terms containing $(z_i[k] - h)$ in Eqs. (42)–(44) become zeros by the expectation using $p(z_i[k]|\theta)$. Then, Eqs. (17)–(19) are derived from

$$\begin{aligned}
 \frac{\partial h}{\partial \theta_x} &= \frac{\partial}{\partial \theta_x} \left\{ \tan^{-1} \left(\frac{\theta_y - y_i[k]}{\theta_x - x_i[k]} \right) \right\} \\
 &= -\frac{\theta_y - y_i[k]}{(\theta_x - x_i[k])^2 + (\theta_y - y_i[k])^2}, \quad (45)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial h}{\partial \theta_y} &= \frac{\partial}{\partial \theta_y} \left\{ \tan^{-1} \left(\frac{\theta_y - y_i[k]}{\theta_x - x_i[k]} \right) \right\} \\
 &= \frac{\theta_x - x_i[k]}{(\theta_x - x_i[k])^2 + (\theta_y - y_i[k])^2}. \quad (46)
 \end{aligned}$$

B. Derivation of the optimal guidance law

In order to derive the current guidance law based on Bryson,¹¹⁾ consider the system described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t); \quad \mathbf{x}(t_0) \text{ given}, \quad t_0 \leq t \leq t_f. \quad (47)$$

Then, the augmented performance index constrained by Eq. (47) is used as

$$\begin{aligned}
 J &= \phi(\mathbf{x}(t_f), t_f) \\
 &+ \int_{t_0}^{t_f} [L(\mathbf{x}(t), \mathbf{u}(t), t) + \ell^T(t)(\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) - \dot{\mathbf{x}})] dt. \quad (48)
 \end{aligned}$$

The solution of minimizing J is given by Bryson¹¹⁾ as follows

$$\dot{\ell}^T = -\partial H / \partial \mathbf{x} = -\partial L / \partial \mathbf{x} - \ell^T \partial \mathbf{f} / \partial \mathbf{x}, \quad (49)$$

$$\ell^T(t_f) = \partial \phi / \partial \mathbf{x}|_{t_f}, \quad (50)$$

$$\partial H / \partial \mathbf{u} = 0, \quad (51)$$

where

$$H(\mathbf{x}(t), \mathbf{u}(t), \ell(t), t) = L(\mathbf{x}(t), \mathbf{u}(t), t) + \ell^T(t) \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t). \quad (52)$$

The situation of this study corresponds to $L = 0, \partial \mathbf{f} / \partial \mathbf{x} = 0, \phi = -\det \text{BFIM}, H = \sum_i (\ell_{x_i} v \cos u_i + \ell_{y_i} v \sin u_i)$. Plugging these conditions into Eqs. (49)–(51) yields Eqs. (29)–(31).

C. Bayesian Monte-Carlo integration method¹²⁾

The Bayesian Monte-Carlo (BMC) integration method is

appropriate for the following integral form

$$f = \int h(\boldsymbol{\theta})\lambda(\boldsymbol{\theta})d\boldsymbol{\theta}, \quad (53)$$

where $\boldsymbol{\theta}$ is a $m \times 1$ parameter vector and $h(\boldsymbol{\theta})$ is a general function to be evaluated by taking expectation on PDF $\lambda(\boldsymbol{\theta})$. The features of the method are that particles are generated according to the PDF and the weight of each particle is calculated based on the Kernel. The Kernel is a function describing the relation of each particle and can be modeled like uniform, quadratic, Gaussian distribution. The integration of Eq. (53) is calculated using the following five steps:

Step 1. Particle generation: Generate n particles,

$$\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n] \quad (54)$$

according to the PDF $\lambda(\boldsymbol{\theta})$.

Step 2. Kernel matrix calculation: Calculate the Kernel matrix by plugging the generated particles into the selected Kernel function

$$\boldsymbol{Q} = \boldsymbol{K} + \sigma_c^2 \boldsymbol{I}, \quad (55)$$

where

$$K_{ij} = w_0 \exp \left[-\frac{1}{2}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)^T \boldsymbol{D}^{-1}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j) \right], \quad (56)$$

($i, j = 1, \dots, n$).

Here σ_c^2 is a small value for the numerical stability, w_0 is a scaling factor, and $\boldsymbol{D}$ is the hyper-parameter matrix expressed by $\text{diag}(w_1^2, \dots, w_m^2)$.

Step 3. Weights calculation of particles: Calculate the weight of each particle. The meaning of the weight is the probability density of the Gaussian process of each particle.

$$\begin{aligned} \boldsymbol{a} &= [a_1, \dots, a_n] \\ a_l &= \frac{w_0}{\sqrt{\det(\boldsymbol{D}^{-1}\boldsymbol{C} + \boldsymbol{I})}} \exp \left[-\frac{1}{2}(\boldsymbol{\theta}_l - \bar{\boldsymbol{\theta}})^T (\boldsymbol{D} + \boldsymbol{C})^{-1} (\boldsymbol{\theta}_l - \bar{\boldsymbol{\theta}}) \right], \\ &\quad (l = 1, \dots, n), \end{aligned} \quad (57)$$

where $\boldsymbol{C}$ is covariance of the PDF $\lambda(\boldsymbol{\theta})$.

Step 4. Object function value calculation at particles: Calculate the object function values at each particle,

$$\boldsymbol{b} = [h(\boldsymbol{\theta}_1), \dots, h(\boldsymbol{\theta}_n)]^T. \quad (58)$$

Step 5. Integral value calculation: Calculate the integral value using results from Step 2–Step 4.

$$f = \boldsymbol{a}\boldsymbol{Q}^{-1}\boldsymbol{b}. \quad (59)$$

Although this method has an additional step for the weight calculation of particles, a fewer number of particles can yield good performance compared to Monte Carlo integration. Especially, this method is more efficient with larger dimension parameters. For more information see Ghahramani and Rasmussen.¹²⁾

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