A Precise Connection Method for Surface Shape Data Measured by the Grating Projection Method

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A highly precise connection method for two sets of three-dimensional surface shape data measured by the grating projection method is proposed with the aim of capturing the full-field surface shape of a large space structure with high accuracy and spatial resolution. The coordinate transformation matrix that serves as an interface between the two sets of surface shape data is precisely calculated by feature-point matching of the virtual targets using the singular value decomposition algorithm, and the two sets of the surface shape data are connected by the coordinate transformation. In the verification experiment, the virtual targets are created from preliminary surface shape measurement of dedicated concave models using the grating projection method. The results show that the two sets of surface shape data can be connected with $40.2 \times 10^{-6}$ mRMS accuracy. This connection accuracy is roughly the same as that achieved with photogrammetry using the grating projection method, thus confirming the effectiveness of the connection method proposed.

Key Words: Connection Method, Shape Measurement, High Accuracy and Resolution, Large Space Structure, Grating Projection Method

1. Introduction

An astronomical observation satellite with a precise and large-scale antenna reflector is needed to accomplish future advanced space science missions. However, the realization of this antenna reflector requires other technical developments to be achieved first. Initially, a ground test to verify the mirror accuracy of the antenna reflector developed is necessary. Then, a short-time measurement method capable of capturing the full-field surface shapes of large space structures with high accuracy and high spatial resolution is required.

The photogrammetric measurement method has been investigated in the last two decades as a possible short-time measurement method for this application. However, because photogrammetry degrades measurement accuracy and spatial resolution as the area measured increases, it is difficult to apply to capture the full-field surface shape of large space structures with high accuracy and spatial resolution.

To solve this problem, our past studies proposed a surface shape measurement method that combined photogrammetry using a grating projection method with a laser tracker measurement system. Figure 1 illustrates this method. It was intended to first measure the partial surface shape of a large space structure using photogrammetry (Fig. 1(a)), and then reconstruct the full-field surface shape of the structure by connecting the captured partial surface shape data with a coordinate transformation matrix. The matrix was to be estimated using the point measurement of several targets that were fixed in the measurement space by a laser tracker measurement system (Fig. 1(b)). However, experimental results showed that the connection accuracy of the partial surface shape data was an accuracy of approximately $357 \times 10^{-6}$ mRMS, which was much less accurate compared to the measurement accuracy of the photogrammetry using the grating projection method, $50 \times 10^{-6}$ mRMS. Accordingly, an alternative method to precisely connect partial surface shape data is required to reconstruct the full-field surface shapes of large space structures with high accuracy and spatial resolution.

Thus, in this paper, a high-precision connection method is proposed to connect the partial surface shape data measured using a grating projection method. This paper describes the connection method proposed and use of virtual targets, and presents the results of the experiments performed to verify the effectiveness of the connection method proposed.

2. Precise Connection Method using Virtual Targets

2.1. Technical issues in connecting shape measurement data using a grating projection method

Photogrammetry using a grating projection method is an active measurement method that captures the three-dimensional (3D) position coordinates of an object surface on a pixel-to-pixel basis. The measurement system of the grating projection method is composed of a digital camera and a projector. When surface shape measurement is conducted, a fringe pattern is projected onto the object surface by the projector, and image data of the object surface with a fringe pattern is taken by the digital camera. Then, by analyzing the phase information of the fringe pattern on the object surface, the 3D position coordinate of the object surface...
is calculated. Accordingly, since the grating projection method does not measure the 3D position coordinates of a target embedded on the object surface, it does not result in clear identification of corresponding points that can be connected to partial surface shape data using the feature-point matching algorithm that is generally used to connect measurements to surface shape data.

One method of connecting these 3D surface shape data that has been widely explored is a registration method using the iterative closest point (ICP) algorithm. In the ICP algorithm, two sets of 3D surface shape data are connected so that one surface shape data set coincide with another in the connected region. However, several problems, listed below, have been reported with its use.9)

(a) The final solution depends on the initial value used in the convergence calculation.
(b) Computational effort is high as the number of 3D points increases.
(c) A global minimum solution is not always given.

Accordingly, when the partial surface shape data of large space structures measured by the grating projection method are connected by the ICP algorithm, the following technical problems must be considered.

(a) Since an enormous number of 3D points of the surface shape data have to be treated, computational effort is high.
(b) Since the surface shape data of the grating projection method generally include random errors, it is difficult to consistently obtain the global minimum solution.
(c) Since the initial positions of the partial surface shape data to be connected are widely separated from each other, it is difficult to decide on an appropriate initial value to use in the convergence calculation.

Thus, in this study, to avoid the above technical problems with the ICP algorithm, we propose a connection method based on the feature-point matching algorithm using virtual targets.

### 2.2. Proposed connection method

Figure 2 illustrates the connection method proposed. In this paper, two measurement spaces are treated, and the connection of two partial surface shape data are considered. As shown in the figure, after calibration of the grating projection method is completed for two measurement spaces, a number of the small concave models whose surfaces are precisely processed to be a part of an ideal sphere are installed in the joint area of the two measurement regions. Then, the surface shapes of the concave models in both regions are measured using the grating projection method. The ideal spherical surface is fitted to the measured surface of the concave models, and the center of the ideal sphere is calculated. Since the calculated center position of the ideal sphere is fixed in space, it can be defined as a virtual target used in feature-point matching. Thus, the coordinate transformation matrix between the two measurement regions (1 and 2) is calculated by feature-point matching the virtual targets. In this study, the singular value decomposition (SVD) method is applied to calculate the coordinate transformation matrix.

The advantages of the method proposed are as follows. Since the method calculates the coordinate transformation matrix by feature-point matching, the computational effort is low, regardless of the number of surface shape data points, and thus the final solution is obtained without fail, regardless of the distance between the initial positions of the two sets of surface shape data.

### 2.3. Practical procedures of the method proposed

A practical procedure to capture the full-field surface shape of large space structures using the connection method proposed is summarized below.

(a) The size of one measurement region is determined so that the measurement accuracy required is achieved using photogrammetry.
(b) The number of measurement regions is decided so that the full-field surface shape of the target structure is covered. This procedure determines the number of digital cameras needed for the measurement system.
(c) The measurement spaces are created by conducting photogrammetry calibration. The size of the joint region of each measurement space is determined so that the jig dedicated, which has multiple concave models, is installed.
(d) A preliminary surface shape measurement is conducted for the multiple concave models installed in the joint region in order to create the virtual targets. This measurement must be performed for all joint regions.
(e) Based on the results, a coordinate transformation matrix that serves as an interface between two measurement spaces is calculated using feature-point matching of
the virtual targets with the singular value decomposition method.

(f) The dedicated jig is removed, and the target structure, such as a space antenna reflector, is installed in the measurement space. Alternatively, it is also acceptable to move the measurement system calibrated in front of the target structure.

(g) A surface shape measurement for the target structure is performed. Then, the full-field surface shape of the target structure is reconstructed by transforming the spatial coordinate system of the measured data onto the reference frame.

3. Verification Experiment Method

3.1. Photogrammetry using the grating projection method

Figure 3 illustrates photogrammetry using the grating projection method. In this study, the grating projection method, which uses two reference planes in the calibration, is applied.\(^{10,12,13}\) This method captures the object surface shape without lens aberration effects, obtaining the object surface shape with high accuracy and spatial resolution. The measurement software used in the experiment is Analyzer II, which was developed in the Fujigaki Laboratory at Wакayama University. A phase-shifting method using three different fringe pitches is applied as the phase analysis method. The phase-shifting method with three different fringe pitches provides high accuracy compared to other phase analysis methods such as the sampling moire method or the phase-analysis method.

3.2. Measurement system

Figure 4 shows a detailed layout of the photogrammetric measurement system used in the experiment. Table 1 lists the specifications of the measurement equipment. Two digital cameras and one projector are installed on the measurement jig. The digital cameras are placed parallel on the jig. The distance between the digital cameras and the projector is about 0.95 m. The measurement jig is installed on a single-axis spindle stage, and can be moved back and forth with a positioning accuracy of ±25 × 10\(^{-6}\) m. Due to this movement, calibration is carried out with one reference plane. Here, since the positional repeatability of the single-axis spindle stage is 6 × 10\(^{-6}\) m, the data measured in the measurement system constructed includes an uncertainty of 6 × 10\(^{-6}\) m. A reference plane with a size of 0.5 m × 0.7 m is installed on the vibration isolation table at a distance of 1.25 m from the measurement jig.

In the grating projection method using the reference plane, the x and y axes of the measurement space are defined on the reference plane surface, and the z axis is perpendicular to the reference plane. The size of the measurement space is set to 0.5 m × 0.7 m × 0.01 m. The in-plane area of the measurement space, 0.5 m × 0.7 m, is decided so that the objects measured can be installed within the measurement space. The depth of the measurement space, 0.01 m, is determined so that the object surface shape can be measured within 50 × 10\(^{-6}\) mRMS accuracy. Since the measurement errors are affected by the laboratory environment, such as the darkness in the room as well as the performance of the measurement equipment, precise prediction of the measurement errors is difficult. However, from our past studies, it is empirically given that the magnitude of the measurement error appearing in the z-axis direction is roughly two-hundredth of the depth of the measurement space when the phase-shifting method using a single fringe pitch is applied.\(^{5,10}\)

3.3. Measurement object

In this study, two objects are measured; one is a small concave model, and the other is a spherical mirror model. Figure 5 shows the small concave model. This model is used to create the virtual targets. The concave model is made of epoxy resin (Araldite B, HT901). The diameter of the model is 0.05 m, and its thickness is 0.02 m. The surface of the concave model is processed with specifically given that the magnitude of the measurement error appearing in the z-axis direction is roughly two-hundredth of the depth of the measurement space when the phase-shifting method using a single fringe pitch is applied.\(^{5,10}\)

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<th>Table 1. Specifications of measurement equipment.</th>
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<td><strong>Equipment</strong></td>
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<td>Digital camera</td>
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<td>Projector</td>
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<td>Spindle stage</td>
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that occurs in photogrammetric measurement when using the grating projection method. Ten concave models were constructed. Figure 6 represents the concave models installed on the dedicated jig. Since the method proposed connects the 3D shape data sets by feature-point matching with virtual targets, it is believed that the connection performance of the proposed method is affected by the configuration of the virtual targets. Thus, ten concave models are installed tilting away from each other so that the virtual targets created are spread out.

Figure 7 shows the spherical mirror model. This model is used to verify the effectiveness of the connection method proposed. The spherical mirror model is made of CREARCERAM-Z by OHARA Inc., a glass-ceramic with an ultra-low thermal expansion coefficient. The diameter of the mirror model is 0.3 m, and the surface of the model is processed to be a part of an ideal sphere with a radius of 0.1 m. The processing accuracy is 0.001 m. The spherical mirror model is painted without any spray, and its surface shape is measured directly using photogrammetry with the grating projection method.

3.4. Experimental procedures

First, the grating projection method is calibrated, and two measurement spaces are created. Figure 8 shows the layout of the calibration process. Measurement region 2 is created by laterally moving the reference plane from measurement region 1. The width of the joint area of the two measurement regions is about 0.2 m. Since the effect of the joint-area width on connection performance is considered to be small compared to the position of the virtual targets, the width of the joint area is determined so that the ten concave models are installed within the area.

Second, ten small concave models are placed in the joint area of the two measurement regions as shown in Fig. 2, and surface shape measurement of the concave models is performed using the grating projection method for each measurement region. The measurement is conducted five successive times in each region, and random errors appearing on the data measured are reduced by calculating the average. Then, ten virtual targets are created by fitting the ideal sphere to the data measured, and the coordinate transformation matrix is estimated via feature-point matching of the virtual targets created using the SVD algorithm.

Third, the ten concave models are removed and the spherical mirror model is installed in the joint area of the two measurement regions. Surface shape measurement of the spherical mirror model is carried out for each measurement region, and the two partial surface shape data are obtained. The measurement is performed five successive times for each measurement region to reduce random errors appearing in the data measured.

Finally, using the estimated coordinate transformation matrix, the spatial coordinate system of the partial surface shape data of the spherical mirror model in measurement region 1 is transformed onto that of measurement region 2, and the connection accuracy of the proposed method is investigated.

4. Results and Discussion

4.1. Shape measurement of concave model

Figures 9 and 10 show the overall views of the measurement of the small concave models in regions 1 and 2, respectively. These figures represent the average of the five successive measurement results. The x and y axes indicate the pixel value, and the contour color indicates the position coordinates in the direction of the z axis. The results show the small concave models installed in the joint area tilting away from each other.
The measurement errors of the photogrammetry using the grating projection method for each concave model are listed in Table 2. The measurement errors are the root mean square (RMS) values of the differences between the measurement results and the ideal spherical surface fitted to the measurement results. From the table, it can be seen that photogrammetry using the grating projection method can measure the concave model surface with an accuracy of approximately $15 \times 10^{-6} \text{mRMS}$ in our laboratory environment.

Figure 11 indicates the error distribution mode appearing on the measurement results of the bottom concave model in region 2. The error distribution is given by the differences between the measurement results and the ideal spherical surface fitted to the measurement results. From the figure, horizontal-stripe patterns are observed along the $y$ direction. Since the random error in the measurement result is reduced by the average calculation, this error mode indicates a systematic error in the photogrammetry using the grating projection method. From our past study, it turns out that the error mode with a horizontal-stripe pattern occurs due to the effects of the projected grating patterns, and can be reduced by an elimination method using a two-dimensional FFT algorithm. However, the magnitude of the error mode in this measurement is roughly $15 \times 10^{-6} \text{mRMS}$, and is smaller than the processing accuracy of the concave model. Accordingly, the elimination method for the systematic error was not applied in this study, and the measurement results as shown in Figs. 9 and 10 are used to create the virtual targets.

### 4.2. Calculation of coordinate transformation matrix

Using the measurement results of the concave models shown in Figs. 9 and 10, the virtual targets are created by fitting the ideal spherical surface onto the data measured. Figure 12 shows a 3D view of the virtual targets calculated. The figures show that the virtual targets are not laid in a straight line, due to the tilting of the concave models installed. By using these virtual targets, the coordinate transformation matrix, (which transfers the spatial coordinate system in region 1 to that in the region 2), is calculated using feature-point matching based on the SVD algorithm.

### 4.3. Shape measurement for spherical mirror model

Figures 13 and 14 show the measurement results of the spherical mirror model in measurement regions 1 and 2, respectively. These figures indicate the average of the five successive measurement results, and represent an enlarged view of the partial surface shape data captured for the spherical mirror model. The $x$ and $y$ axes indicate the actual position, and the contour color represents the position coordinate in the $z$-axis direction.

Table 3 lists the measurement error of the grating projection method. The measurement error is expressed as the RMS values of the differences between the measurement results and the ideal spherical surface fitted to the measurement results. From the table, it can be seen that the grating projection method can measure the spherical mirror surface with an accuracy of approximately $30 \times 10^{-6} \text{mRMS}$. The differences between the measurement errors of the spherical mirror model and that of the concave model occur due to the fact that the optical properties of the two object surfaces measured are different.

Figures 15 and 16 show the error distribution for the measurement results of the spherical mirror model. The error...
distribution is given by the difference between the measurement results and the ideal spherical surface fitted to the measurement results. The origin of this figure is the center of the ideal sphere. From the figure, a horizontal-stripe pattern is clearly observed. Similar to the results of the concave model, this error mode is a systematic error in photogrammetry using the grating projection method.

4.4. Connection of partial surface shape data using the method proposed

Using the coordinate transformation matrix calculated, the two sets of the partial surface shape data of the spherical mirror model are connected, and the effectiveness of the method proposed is discussed.

Figure 17 shows the two partial surface shape data of the spherical mirror model after the connection. Figure 18 indicates the cross-sectional shape along the $y$ axis ($x = -100 \times 10^{-3}$ m) shown in Fig. 17. From the figures, it can be seen that the surface shape data transformed in region 1 is in good agreement with the surface shape data in region 2. The RMS value of the connection accuracy in this area is $40.2 \times 10^{-6}$ mRMS. Considering that this connection accuracy includes the measurement error in region 1, namely $27.5 \times 10^{-6}$ mRMS, we conclude that these two partial surface shape data can be connected with almost the same accuracy as photogrammetry using the grating projection method. Accordingly, the connection method proposed leads to a significant improvement in the connection accuracy of the partial surface shape data, which was achieved in our previous study, and the effectiveness of the method is confirmed.

Figure 19 shows the connection error distribution, which represents the difference between the transformed surface shape data in region 1 and the ideal spherical surface fitted to the surface shape data in region 2. The origin of this figure is the center of the ideal sphere. This result also includes the error distribution for the measurement results of the spherical

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<th>Table 3: Measurement errors on spherical mirror model.</th>
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<td>Region 1</td>
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<td>Region 2</td>
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Fig. 13. Measurement results in region 1.

Fig. 14. Measurement results in region 2.

Fig. 15. Error distribution in region 1.

Fig. 16. Error distribution in region 2.

Fig. 17. Surface shape of spherical mirror model after connection.

Fig. 18. Cross-sectional shape of superimposed surface shape.
mirror model in region 1, which is shown in Fig. 15. Compared to Fig. 15, the connection error distribution is slightly biased towards the left-hand side. This implies that the connection error becomes large as distance from the connection area increases, and further fine-tuning is necessary to precisely reconstruct the full-field surface shape of the large space structure by connecting multiple partial-surface shape data. Establishing a fine-tuning method for connection accuracy is one of our future objectives.

5. Conclusion

A highly precise connection method for two sets of 3D surface shape data measured using the grating projection method was proposed to reconstruct the full-field surface shape of a large space structure with high accuracy and spatial resolution, and its effectiveness was explored in verification experiments. The major findings in this paper are:

(1) To achieve highly precise connection of the two sets of surface shape data measured using the grating projection method, a connection method using virtual targets was proposed. The virtual targets are created as the center points of the ideal sphere, which is fitted to the measured surface of the dedicated concave model using the grating projection method.

(2) The verification experiments indicated that the method proposed can connect the two partial surface shape data sets of the spherical mirror model with an accuracy of $40.2 \times 10^{-6}$ mRMS. This connection accuracy is roughly the same as that achieved with photogrammetry using the grating projection method, confirming the effectiveness of the connection method proposed.

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