Capturability Analysis of TPN Guidance Law for Circular Orbital Pursuit-Evasion*

Dateng YU,1) Hua WANG,1)† Jianping ZHOU,1,2) and Kebo LI1)

1)College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China
2)China Manned Space Engineering Office, Beijing 100094, China

This work analyzes the capturability of the true proportional navigation (TPN) guidance law applied in circular orbital pursuit-evasion, which is a new view compared to the prior method used in orbital pursuit-evasion. The saddle point is a rather difficult problem to resolve when the orbital pursuit-evasion problem is formulated as a zero-sum differential game. The research in this paper focuses on analyzing the capture capability of the pursuer and the escape capability of the evader. The relative motion equations for the line-of-sight (LOS) coordinate and the modified polar coordinate (MPC) system are united. Then, nonlinear transformation is used in the dynamic equations so as to acquire the relative motion equation. The optimal strategies of both satellites are determined and expression of the analytical capture-escape equation is proposed. The capture-escape region of an orbital pursuit-evasion game is defined. The simulation clearly shows the capture region of the pursuer, and the results are in agreement with the analysis. The results of the analytic capturability method proposed in this paper are visual and understandable, which means that it will be very convenient to use in analyzing capturability in orbital pursuit-evasion problems.

Key Words: Pursuit-Evasion, Capturability, Capture Region, True Proportional Navigation (TPN), Evasion Maneuver

1. Introduction

Pursuit-evasion problems, in which opposing decision-makers interact within a complicated dynamic environment, are challenging aporia to solve. They usually have two solutions: one is to formulate the pursuit-evasion problem as a zero-sum differential game, and the other is to treat the problem as an optimization problem of guidance law design and focus on analyzing the capability of pursuer to capture the evader and the capability of the evader to escape.

When the pursuer and evader each choose the optimal strategy, the pursuit-evasion game becomes a two-player zero-sum differential game. Isaacs1,2) initially proposed this problem, and then Berkovitz3) and Friedman5) proposed the saddle point existence theorem of the differential game and proved the necessary conditions. The differential game method has been widely used to solve the pursuit-evasion problem of aircraft,5) spacecraft,6–8) and missiles.9–11) This kind of differential game problem is usually converted into a two-point boundary-value problem, after which the respective method can be used. Raivio and Ehtamo12) solved a pursuit-evasion problem for a visually identified target using a direct iterative method. Horie and Conway13,14) solved the problem by converting the continuous optimization problem into a nonlinear programming (NLP) problem. Kelley et al.15) studied the coplanar pursuit-evasion problem when both satellites use impulse maneuvers. Zhang et al.16) obtained a result that a saddle point for the differential game exists if, and only if, the two optimal control problems have the same optimal value. Pontani and Conway18) developed a direct numerical method and found some saddle-point equilibrium solutions.

The emphasis and aporia of the research above is to establish the objective function $J$, and find the saddle point. Although scholars often conduct research under many simplified assumptions, the saddle point is still a rather difficult problem to solve and the result is usually hard to understand. The other solution for the pursuit-evasion problem has also been studied for a rather long time. The problem is frequently modeled with a given duration, bounded controls, and fixed players’ dynamics. Shinar and Shima17–19) proposed many well-known solutions to solve the planar linear pursuit-evasion problem. Glizer and Turetsky20–22) treated the pursuit-evasion problem as an optimal control problem and a robust control problem when target behavior can and cannot be predicted, respectively. Recently, pursuit-evasion problem formulations have been extended to consider hybrid dynamics.23–25) Because of its visual and understandable results, the analytic capturability method has been used widely for the pursuit-evasion problems between two missiles. Dhar and Ghose26) researched the quality of true proportional navigation (TPN) and obtained the TPN capture region. Alkaher and Moshiaov27) proposed a method to calculate the dynamic escape zone for avoiding a coasting missile. Shinar et al.23) considered a maneuverable target using a linearized kinematical model with first-order acceleration and derived an analytical description of the capture region. Grossman28) used modified polar coordinates (MPCs) to express the relative state, and verified the MPC state equation. Tyan29) expanded two-dimensional proportional navigation (PN) to three-dimensional and gained the capture region. Li et al.30–32) compared the capture capability of ideal proportional naviga-
tion with that of TPN using the line-of-sight (LOS) rotation plane. All of the literature listed above have researched the pursuit-evasion problem of two missiles by analyzing capture capability or capture region. However, analogous research of the orbital pursuit-evasion problem is puruous, as the relative dynamics are much more complex than those of missiles. Furthermore, the acceleration of both the pursuer and evader is rather small, and the expected terminal relative velocity of the pursuer approaches zero (for capture objective). These conditions are all different from missile combat. The existing method will be unsuitable if considering the new demands for orbital combat. The objective of this paper is to analyze the relative motion equation in an orbital pursuit-evasion problem and derive an analytical expression of capturability.

In this paper, it should be noted that the pursuer is approaching for the purpose of capturing, not colliding. In order to simplify the orbital pursuit-evasion problem and improve the clarity of the analytic problem, we study the circular orbital combat for short range (e.g., ≤ 10 km). The remainder of this paper is organized as follows. In Section 2, the relative motion equations of the LOS coordinate and MPC system are united. In Section 3, the pursuit-evasion strategies of the pursuer and evader are analyzed, and an analytical expression for the capture region is derived and the definition of capture and escape in MPC space is given. In Section 4, two examples of orbital pursuit-evasion simulation are performed. Conclusions are presented in Section 5.

It should be noted that this study is only applicable for a pursuer satellite with a circular/near-circular orbit. This paper offers a solution to analyze the capture capability of a pursuer and solve the orbital pursuit-evasion game in an easily accessible way.

2. Relative Dynamic Equation

In a circular/near-circular orbital pursuit-evasion game, the pursuer tries to capture the evader using guidance laws and the evader tries to escape from the pursuer by choosing an optimal strategy. It should be noted that this paper studies orbital combat for a rather short range and all the following assumptions are based on this. Furthermore, the motion of pursuer is assumed to be restricted to near the orbital plane of the evader. The orbit coordinate system is described in Fig. 1: the oy axis is along the direction from Earth to O, the ox axis is perpendicular to oy and the z axis is obtained by the cross product of ox × oy.

The relative dynamic equation can be derived as a typical Clohessy-Wiltshire (CW) equation, which is derived from the linearization of the Keplerian orbital:

\[
\begin{align*}
\dot{x} + 2\omega \dot{y} &= f_x, \\
\dot{y} - 2\omega x - 3\omega^2 y &= f_y, \\
\dot{z} + \omega^2 z &= f_z
\end{align*}
\]

where \( f_x, f_y, f_z \) are the relative control quantity of pursuer and evader along each axis,

\[\mu \text{ is the constant of earth gravitation, } r_E \text{ is the geocentric distance of the pursuer, } \omega \text{ is the angular rate of the evader orbit } \omega_E.\]

Usually, the measurement information needed for orbit control is relative position and relative velocity. In this study, it is assumed that the evader knows the relative motion measurement, such as relative position, relative velocity, angle of sight and angle velocity of sight. To go a step further, we use the LOS coordinate to analyze the relative motions of the evader and pursuer. The LOS coordinate system \( o_{xL}y_Lz_L \) is shown in Fig. 2. The \( o_{xL}x_L \) axis of this coordinate system is LOS, \( o_{yL}y_L \) is perpendicular to \( o_{xL}x_L \) and the \( z_L \) axis is obtained by the cross product of \( o_{xL}x_L \times o_{yL}y_L \). \( \rho \) is the relative distance and \( q \) is the angle of sight.

The transformation between \( o_{xyz} \) and \( o_{xL}y_Lz_L \) is as follows:

\[x = \rho \cos q, \quad y = \rho \sin q, \quad \rho = \sqrt{x^2 + y^2} \quad (2)\]

The relative velocity of the evader and pursuer in the LOS coordinate system is

\[V = v_\rho e_\rho + v_q e_q = \dot{\rho} e_\rho + \rho \dot{q} e_q \quad (3)\]

where \( e_\rho \) is the unit vector of the relative velocity along LOS, \( e_q \) is the unit vector of the relative velocity perpendicular to LOS. \( e_i \) is the cross product of \( e_\rho \times e_q \), and \( e_\rho, e_q \) and \( e_i \) form a right-handed coordinate system. Consider that \( \rho \) is rather small compared to the orbit altitudes of evader and pursuer, thus the angle velocity of the evader and pursuer can be regarded as equal, namely, \( \omega_E = \omega_p \).
Substituting Eq. (2) into Eq. (1), we have
\[
\dot{\rho} - [\dot{q} - \omega_p^2] - (1 - 3 \sin^2 q) \omega_p^2 \rho = f_{\rho}
\]
\[
\dot{\rho} q + 2 \dot{\rho} q + (-2 \rho \omega_p - 1.5 \omega_p^2 \rho \sin 2q) = f_q
\]
where,
\[
f_{\rho} = f_s \cos q + f_s \sin q
\]
\[
f_q = -f_s \sin q + f_s \cos q
\]
In Eq. (4), the first equation represents the relative motion along LOS, which can be called a vertical relative motion equation; and the second equation represents the relative motion perpendicular to LOS, which can be called a normal relative motion equation. Equation (4) can be collated as
\[
\dot{\rho} = f_{\rho} + \dot{\rho}^2 \rho - 2 \dot{\rho} \omega_p + 3 \sin^2 \omega_p^2 \rho
\]
\[
\dot{q} = \frac{1}{\rho} [f_q - 2 \dot{\rho} q + 2 \dot{\rho} \omega_p + 1.5 \omega_p^2 \rho \sin 2q]
\]
where \( \Omega = \dot{q}, \Omega = \ddot{\rho} \).

Here, the MPC system is introduced to analyze the relative motion a step further. The measurement in MPC, such as tracking a target, has the advantage in that it is linear and can be used in an Extended Kalman Filter (EKF). Therefore, the capture and escape region mentioned in this paper are described in MPC. We use the definition of the state vector of the MPC system reported by Grossman\(^{28}\) as follows:
\[
x_p = \left[ \begin{array}{c} x_{p1} \\ x_{p2} \\ x_{p3} \\ x_{p4} \end{array} \right] = \left[ \begin{array}{c} e_{\rho} \\ \dot{e}_{\rho} \\ \frac{1}{\rho} \frac{d\rho}{dt} \end{array} \right]
\]

Under the definition of Eq. (6), the state dynamics are given by differentiating each component of state vector \( x_p \). Taking \( x_{p4} \) as an example, one can get:
\[
\frac{dx_{p4}}{dt} = -\frac{1}{\rho^2} \left( \frac{d\rho}{dt} \right)^2 + \frac{1}{\rho} \frac{d^2 \rho}{dt^2}
\]
Substituting Eq. (5) into Eq. (7) \( (\ddot{\rho} = d^2 \rho / dt^2) \), we have
\[
\frac{dx_{p4}}{dt} = \Omega^2 - x_{p4}^2 + x_{p3} \left( a_{E_{\rho}} - a_{P_{\rho}} - 2 \Omega \frac{1}{x_{p3}} \omega_p + 3 \omega_p^2 \frac{1}{x_{p3}} \sin^2 q \right)
\]
In a similar way, we can have the following third coupled scalar differential equations:
\[
\frac{dx_{p4}}{dt} = \Omega^2 - x_{p4}^2 + x_{p3} \left( a_{E_{\rho}} - a_{P_{\rho}} - 2 \Omega \frac{1}{x_{p3}} \omega_p + 3 \omega_p^2 \frac{1}{x_{p3}} \sin^2 q \right)
\]
\[
\frac{d\Omega}{dt} = -2 x_{p4} \Omega
\]
\[
\frac{dx_{p3}}{dt} = -x_{p3} x_{p4}
\]

Compared with other variables, the quadratic of \( \omega_p \) is a minor. Finally, Eq. (8) can be simplified as follows:
\[
\frac{dx_{p4}}{dt} = \Omega^2 - x_{p4}^2 + x_{p3} \left( a_{E_{\rho}} - a_{P_{\rho}} - 2 \Omega \frac{1}{x_{p3}} \omega_p \right)
\]
\[
\frac{d\Omega}{dt} = -2 x_{p4} \Omega + x_{p3} \left( a_{E_{\rho}} - a_{P_{\rho}} - 2 \Omega \omega_p \right)
\]
\[
\frac{dx_{p3}}{dt} = -x_{p3} x_{p4}
\]

In Eq. (10), the relative motion equations in the LOS coordinate and MPC system are united. Equation (10) contains all of the necessary state variables, and describes the relative dynamics between pursuer and evader in MPC space. In the following section, the entire pursuit-evasion analysis is based on Eq. (10). It should be noted that a simplified equation will produce errors compared with Eq. (9). The error analysis can be seen in Section 5.

3. Capture and Escape Region Analysis

3.1. Nonlinear transformation of the relative motion equation

The relative dynamics in Eq. (10) are in a complex form and are hard to use directly. A nonlinear transformation is executed by some new variable replacement so as to analyze the capture/escape regions.

In this paper, the following new modified polar variables (MPVs) are defined as
\[
u = \frac{\rho}{\sqrt{\rho}}, \quad w = \frac{1}{\sqrt{\rho}}
\]
Substituting Eq. (11) into Eq. (10), then
\[
\frac{du}{dr} = w \left( -\frac{1}{2} u^2 + v^2 + a_{E_{\rho}} - a_{P_{\rho}} + 2 \omega_p v \right)
\]
\[
\frac{dv}{dr} = w \left( -\frac{3}{2} v^2 + a_{E_{\rho}} - a_{P_{\rho}} + 2 \omega_p u \right)
\]
With the new MPVs, Eq. (10) can be simplified. From Eq. (12), we know that the three differential equations are coupled. The first two expressions in Eq. (12) describe the
variation of relative motion. The last one describes variation of the rotation plane. When the pursuit and evasion strategy, namely the expression of acceleration in Eq. (12), is determined, the capture and escape region will be obtained.

3.2. Strategy of pursuit and evasion

The relative velocity in the final phase for missiles is out of consideration. However, it is a very important final constraint for satellites, since the purpose of a pursuit satellite is to approach and capture. Additionally, the pursuer may collide with the evader, which will make both the pursuer and evader lose effectiveness if the final relative velocity doesn’t approach zero. Assume that both the pursuer and evader use optimal strategies, the expression of acceleration will be determined in the following.

3.2.1. Pursuer acceleration determination

In order to approximate actual conditions, it is assumed that the pursuer uses a PN-type guidance law to approach the evader, which means that TPN controls the LOS rate. The general expression of TPN is as follows:

\[ a_{pq} = -N\rho \dot{\Omega} \]  \hspace{1cm} (13)

where \( N \) is the proportional coefficient. Substituting Eq. (11) into Eq. (13), then

\[ a_{pq} = -Nu \nu \]  \hspace{1cm} (14)

For the direction of sight, one of the necessary conditions for the pursuer capturing the evader is that both the final relative distance and velocity approach zero. Then, we have

\[ \begin{cases} \frac{1}{2} a_{p\rho} \rho \dot{\rho}^2 = \rho \\ a_{p\rho} \dot{\rho} = |\dot{\rho}| \end{cases} \]  \hspace{1cm} (15)

According to Eq. (15), we can obtain the expression of \( a_{p\rho} \) as

\[ a_{p\rho} = -\frac{|\dot{\rho}|^2}{2\rho} \]  \hspace{1cm} (16)

Substituting Eq. (11) into Eq. (16), then

\[ a_{p\rho} = -\frac{1}{2} \nu \nu^2 \]  \hspace{1cm} (17)

Taking the ability of orbital engines into account, we can have

\[ \text{sat}(a_{\text{real}}) = \begin{cases} a_{\text{max}} & a_{\text{real}} > a_{\text{max}} \\ a_{\text{real}} & -a_{\text{max}} \leq a_{\text{real}} \leq a_{\text{max}} \\ -a_{\text{max}} & a_{\text{real}} < -a_{\text{max}} \end{cases} \]  \hspace{1cm} (18)

where \( a_{\text{real}} \) is the required acceleration defined as Eqs. (14) and (17), and \( a_{\text{max}} \) is the maximum acceleration the engines of the satellite can deliver.

3.2.2. Evader acceleration determination

To escape from a maneuver-approaching pursuer, the evader should use maximum maneuverability to evade in order to preserve its safety, namely

\[ a_{E\rho} = a_{E,max}, \quad a_{E\nu} = a_{E,max} \]  \hspace{1cm} (19)

We use zero-effort miss (ZEM) to analyze the optimal evasive direction of the evader. According to the definition of ZEM, one can get

\[ \text{ZEM} = -\frac{\dot{\rho}^2}{\dot{\rho}} \left( \dot{\rho} < 0 \right) \]  \hspace{1cm} (20)

Differentiating Eq. (20),

\[ \frac{d\text{ZEM}}{dt} = \frac{\dot{\rho}^3}{\dot{\rho}} \left( \dot{\rho}^2 - \dot{\rho} \ddot{\rho} \right) - 2\dot{\rho} \dot{\Omega} \]  \hspace{1cm} (21)

The expression of relative motion differential equations is proposed in the LOS coordinate system:

\[ \begin{cases} \dot{\rho} = a_{E\rho} - a_{P\rho} + \rho \dot{\Omega}^2 \\ \dot{\Omega} = a_{P\rho} - a_{E\rho} - 2\dot{\rho} \dot{\Omega} \end{cases} \]  \hspace{1cm} (22)

Substituting Eq. (23) into Eq. (21), then

\[ \frac{d\text{ZEM}}{dt} = \frac{\dot{\rho}^3}{\dot{\rho}} \left( \dot{\rho}^2 - \dot{\rho} \ddot{\rho} \right) - 2\dot{\rho} \dot{\Omega} \]  \hspace{1cm} (24)

The nominal time is defined as

\[ t_{go} = -\frac{\rho}{\dot{\rho}} \]  \hspace{1cm} (25)

Substituting Eq. (25) into Eq. (24), then

\[ \frac{d\text{ZEM}}{dt} = \rho \dot{\Omega}^2 t_{go} + t_{go} \left[ \dot{\Omega} t_{go} \left( a_{E\rho} - a_{P\rho} \right) + \left( a_{Eq} - a_{Pq} \right) \right] \]  \hspace{1cm} (26)

where \( \dot{\Omega} \) is negative throughout the entire approach phase. It is indicated from Eq. (26) that both \( a_{E\rho} \) and \( a_{Eq} \) can deduce ZEM, and the weighting has a relation with \( \dot{\Omega} t_{go} \).

It is known from Eq. (25) that

\[ \dot{\Omega} t_{go} = -\frac{\rho \dot{\rho}}{\dot{\rho}} \left( v_q - v_p \right) \]  \hspace{1cm} (27)

According to Eqs. (26) and (27), it is clarified that: Case 1, \( v_q > v_p \), the weighting of \( a_{E\rho} \) is bigger than that of \( a_{Eq} \) since \( \dot{\Omega} t_{go} > 1 \), therefore the optimal maneuver direction should be \( a_{E\rho} \); and Case 2, \( v_q < v_p \), the weighting of \( a_{Eq} \) is bigger than that of \( a_{E\rho} \) since \( \dot{\Omega} t_{go} > 1 \), therefore the optimal maneuver direction should be \( a_{Eq} \). Generally, it is easy to find that the following inequality can be given during the pursuit-evasion of two satellites:

\[ v_q < v_p \]

Therefore, we obtain

\[ a_{E\rho} = 0, \quad a_{Eq} = a_{Eq,max} \]  \hspace{1cm} (28)

3.3. Definition and analysis of capture and escape regions

When capture occurs, the final state variables become
\[
\rho_t \to 0, \quad \dot{\rho}_t \to 0
\]  
(29)

where \( f \) denotes the final state.

To an orbital capture mission, the final relative distance \( \rho_t \) approaches zero (if the pursuer successfully captures the evader), and the expected final relative velocity \( \dot{\rho}_t \) should be approaching zero as well. However, it is usually very difficult for the final relative velocity \( \dot{\rho}_t \) to reach zero, even in a cooperative rendezvous mission. It usually reaches a near-zero value that meets some upper limit in most rendezvous missions. Therefore, it is defined that \( \rho_t \) is approaching zero while the velocity is approaching a near-zero negative value \( \varepsilon \) when the pursuer finally captures the evader. Considering the above, Eq. (29) is revised as

\[
\rho_t \to 0, \quad \dot{\rho}_t \to \varepsilon
\]  
(30)

According to Eqs. (11) and (30), we have \( u_t \to -\infty, \quad v_t \to 0, \quad w_t \to \infty \). Thus, the capturing conditions are obtained. Based on those capturing conditions, the capture region is defined as follows:

**Definition 1** The capture region is the region in the \((u, v, w)\) space where the state trajectories from initial states \((u_0, v_0, w_0)\) will lead to \((u_t, v_t, w_t) = (-\infty, 0, \infty)\) and the escape region is the rest of the \((u, v, w)\) space.

Substituting Eqs. (14), (17) and (28) into Eq. (12), then

\[
\frac{du}{dt} = -\frac{1}{2} u^2 w + v^2 w - \left( -\frac{1}{2} u^2 w \right) + 2\omega_P v
\]

\[
\frac{dv}{dt} = \left( N - \frac{3}{2} \right) u w + a_{E_{\text{max}}} w + 2\omega_P w
\]  
(31)

\[
\frac{dw}{dt} = -\frac{1}{2} w^2 u
\]

Equation (30) is the analytical expression of the capture region when the pursuer uses TPN guidance law. The space manifold of capture and escape region is shown in Fig. 3, when \( N = 4 \), \( a_{P_{\text{max}}} = a_{P_{\text{max}}} = 8 \text{const.} \), \( a_{E_{\text{max}}} = a_{E_{\text{max}}} = 4 \text{const.} \), and const. = 0.01 m/s\(^2\). It can be seen from the space manifold in Fig. 3 and Fig. 5 that different \( w \) will generate a different space manifold when \((u, v)\) is the same. Here we just study the situation of \( w = 0.0102 \), namely, the initial relative distance is about 10 km.

The capture space of TPN with an optimal maneuvering target can be seen clearly in \( uv \) plane. The capture region is bounded by two projections of the bold space manifolds in the plane of \( w = 0.0102 \) in Fig. 4. Both of the bold space manifolds start from near \((-0.223, 0.219, 0.0102)\) and approach to \((-\infty, 0, \infty)\). The other region in the plane of \( w = 0.0102 \) doesn’t satisfy the capture conditions and is the escape region. From Fig. 5, we can see that the tendency of the bold curve is pointing to \( w \to +\infty \), which means \( \rho \to 0 \).

Though the maneuverability of the pursuer is twofold that of the evader, it can be seen in Fig. 4 that the capture region of limited TPN with an optimal maneuvering evader is not very large in the \( uv \) plane when \( w = 0.0102 \). One reason is that the evader executes an optimal maneuver, and the other reason is that the orbital capture has rather stricter conditions than collision.

**4. Simulation**

Three-dimensional pursuit-evasion simulations of two satellites are presented in this section to prove the correctness of the analysis above. TPN is used to guide the pursuer and the evader uses the optimal escape strategy mentioned in 3.2. Both pursuer and evader use two body orbital dynamics, which means the oblateness of the Earth is not considered.

It should be noted that the motion of each satellite is simulated using the massless two-body problem in the Earth inertial coordinate (EIC) of the gravitational field with accelera-
4.1. Initial relative states satisfy the escape region conditions

Assume the initial relative states of the pursuer and evader satisfy the escape region conditions, the relative position is \( [x, y, z] = [9501.99 - 800 480] \) m, the relative velocity is \( [v_x, v_y, v_z] = [-40.7 10 6] \) m/s, and the initial orbit of evader is in a circular orbit with a radius of 8380.984 km and the incline is 45 deg.

According to the relative states, we have \( a = -0.4332, v = 0.1063 \) and \( w = 0.0102 \). It is easy to find that the point \((-0.4332, 0.1063, 0.0102)\) is in the escape region in Fig. 4. The initial relative distance is 9547.681 m and the relative position is shown in Fig. 6. To create a more obvious observation, the variations of relative acceleration and velocity are shown in Figs. 7 and 8.

As shown in Fig. 6, the pursuer guided by TPN failed to capture the evader and the final distance is 235.49 m, and the relative velocity is already approaching zero in Fig. 8; namely, the evader successfully evaded the attempt to catch it.

It should be mentioned that the analytical method is derived applying a coplanar hypothesis, and describes the capturability of the pursuer in different initial relative states. However, the TPN guidance law has the ability to modify the motion in an out-of-plane direction. In order to prove this, a minor initial distance for \( z \) is added in the relative states. In addition, the relationship between the motion in the X–Y plane and out-of-plane direction and limitation of the initial distance in the out-of-plane need to be studied further. These are issues we are interested in and will be in the next step of our research plan.

4.2. Initial relative states satisfy the capture region conditions

Assume the initial relative states of the pursuer and evader satisfy the capture region conditions. The initial relative position is \( [x, y, z] = [9501.99 - 1000 480] \) m, the relative velocity is \( [v_x, v_y, v_z] = [-40.7 8 2] \) m/s, and the initial orbit of evader is the same as that of 4.1.

According to the relative states, we have \( a = -0.4245, v = 0.0562, \) and \( w = 0.0102 \). It is easy to find that the point \((-0.4245, 0.0562, 0.0102)\) is in the capture region in Fig. 4. The initial relative distance is 9566.515 m, and simulation results are shown in Fig. 9 to Fig. 11.

As seen from the simulation, the final distance is 0.73 m. The variation of relative velocity in Fig. 11 tells us that the relative velocity is finally approaching zero. It should be mentioned that the pursuer usually has a mechanical arm or some other active device to capture the evader, which means the pursuer often has an action distance. Therefore, it can be concluded that the pursuer successfully captured the evader. Although this paper studies the capture region of the TPN guidance law, it should be noted that the method proposed here can also be used to analyze the capture and escape regions of other guidance law with the same transformation proposed in this paper.

Section 4.1 and Section 4.2 prove the analysis of the cap-
ture and escape regions in Sections 2 and 3 is correct. Additionally, the simulation results show that, when the initial states are in the capture region or escape region, the final state will reach capture or escape, respectively. Although the initial states of Section 4.1 and Section 4.2 are very similar, the difference can be clearly seen in the space manifold of relative motion (Figs. 3 and 4). The simulation results show that the analytical expression of capturability derived in Sections 2 and 3 is effective, thus an analytical capturability method is obtained for the orbital pursuit-evasion problem.

5. Discussion and Conclusions

This work studies the capture region of the TPN guidance law for orbital pursuit-evasion (two satellites), and offers a analytical capturability method for the circular orbital pursuit-evasion problem. The relative motion equations in the LOS coordinate and MPC system are united. The pursuit-evasion optimal strategies of both the pursuer and evader are analyzed, and an analytical expression for the capture region is derived. The simulation clearly shows the capture region of the pursuer, and the results are in agreement with the analysis, showing its effectiveness. The method proposed here is easier to solve and understand compared to formulating the pursuit-evasion problem as a zero-sum differential game. It analyzes the capture and escape regions of orbital pursuit-evasion considering the different demand of final states between two missiles and between two satellites.

In the simulation, the orbit period of the evader is 7635.782 s. The error analysis of Eq. (10) is as follows: When the game lasts about 700 s, the error in a single direction is no more than 4%. However, if the game lasts several thousand seconds, the error will become very large, meaning that it is not suitable to use Eq. (10) to solve the problem. Therefore, the method proposed here is for orbital combat of a rather short range, as mentioned in Paragraph 1, Section 2.

Although this paper proposes a new method for analyzing the capture region of the TPN guidance law, it should be noted that the method proposed here is also suitable for analyzing the capture and escape regions when the pursuit uses other guidance laws (e.g., RTPN, APN and so on) with the same transformation used in this paper. The results of the analytical capturability method proposed in this paper are visual and understandable, which means that it can be conveniently used for analyzing capturability in orbital pursuit-evasion problems.

Acknowledgments

This study was co-supported by the National Natural Science Foundation of China (No. 11572345). The authors would like to thank the editor and the anonymous reviewer for their constructive comments.

References
