Expander and Coolant-Bleed Cycles of Methane-Fueled Rocket Engines*

Takeshi KANDA,^{1),2)†} Masaki SATO,¹⁾ Toshiya KIMURA,¹⁾ and Hiroya ASAKAWA³⁾

¹⁾Japan Aerospace Exploration Agency, Kakuda, Miyagi 981–1525, Japan

²⁾Currently, Chubu University, Kasugai, Aichi 487–8501, Japan

³⁾IHI Corporation, Tomioka, Gunma 370–2398, Japan

This paper describes the characteristics of methane-fueled rocket engines and compares these characteristics with those of hydrogen-fueled engines in terms of both expander and coolant-bleed cycles. Methane vaporizes in a cooling jacket under low-pressure operating conditions, whereas it liquefies in the turbine in the coolant-bleed cycle. The thermodynamic property of methane limits the operating range of methane-fueled engines. When the coolant-bleed cycle is used, the specific impulse degradation of methane-fueled engines becomes larger compared to that of hydrogen-fueled engines. This is due to methane having a lower specific heat and temperature after regenerative cooling. Even though the heat absorbing ability of methane is much lower than that of hydrogen, methane-fueled engines can operate with higher chamber pressures using the expander cycle. This is due to the larger density and the higher temperature after the regenerative cooling of liquid methane. Throttling of the methane-fueled engines does not have a great impact on the pump exit pressure in the expander cycle, whereas it increases the bleed ratio and degrades the specific impulse in the coolant bleed cycle of methane-fueled engines.

Key Words: Methane, Rocket Engine, Expander Cycle, Coolant-Bleed Cycle, Liquefaction, Vaporization

Nomenclature

A : parameter in Eq. (4), cross-section
 a : parameter in Eq. (4)
 B : parameter in Eq. (4)
 b : parameter in Eq. (4)
 c_{ex} : parameter defined by Eq. (19)
 c_f : friction coefficient
 c_H : Stanton number
 c_p : specific heat
 d : diameter
 h : enthalpy
 k : constant in Eq. (6)
 M : Mach number
 MW : molecular weight
 $\dot{m}$: mass flow rate
 p : pressure
 Pr : Prandtl number
 Q : volume flow rate
 q : heat flux
 Re : Reynolds number
 $r_{ft,1}$: pressure ratio defined by Eq. (12)
 $r_{ft,2}$: pressure ratio defined by Eq. (13)
 S : parameter in Eq. (4), emissivity in Eq. (6), surface area
 s : mean thickness of radiating gas layer in Eq. (6)
 T : temperature
 W : power

γ : ratio of specific heats

η : efficiency

Λ : Darcy friction factor of turbulent flow

μ : viscosity

ρ : density

Subscripts

a : acceleration

aw : adiabatic wall

CH_4 : methane

c : combustion chamber, compressible flow, coolant

d : diameter-based value

e : exit, edge of boundary layer

f : fuel, friction

g : combustion gas

H_2 : hydrogen

HO : hydrogen-oxygen propellant engines

i : incompressible flow

MO : methane-oxygen propellant engines

p : pump

r : radiation

total: total

tp : turbopump

w : wall

x : x based value

1: entrance

2: exit

1. Introduction

The properties of methane are suitable for rocket engines. As shown in Table 1,¹⁾ methane has a higher density than hydrogen. Therefore, to use methane as a rocket fuel contributes to the miniaturization of fuel tanks. Methane-fueled

engines have a higher specific impulse than other hydrocarbon-fueled engines. Methane is a cryogenic fuel, and presents a higher cooling ability than other hydrocarbon fuels. Furthermore, methane has a relatively high boiling temperature and requires less insulation than hydrogen. Since methane is an attractive propellant for liquid rocket engines, studies on methane-fueled engines have been conducted.^{2–6)} Furthermore, the development of methane-fueled engines has already been launched.^{7,8)} In addition, conceptual studies for methane-fueled engines have been conducted to examine their operating characteristics.^{9–13)}

Pempie and Boccaletto studied the expander cycle of 150 kN thrust upper-stage engines and discussed the operating features.⁹⁾ They compared the operating conditions of the engine with those of a hydrogen-fueled engine. Schuff et al. prepared a conceptual design of an expander-cycle engine with a thrust of 110 kN.¹⁰⁾ They especially focused on the design of the cooling jacket. Crocker and Peery examined the advantages and disadvantages of single/dual rotor turbopumps in the expander-cycle engine based on RL-10.¹¹⁾

To enable engines to have a larger thrust range, whether or not to apply a staged-combustion (SC) cycle or the gas-generator (GG) cycle to methane-fueled engines was studied. Both cycles are now widely used for a large number of hydrogen-fueled engines. Pempie and Gorecki examined the operating characteristics of the SC cycle and the GG cycle engines with about 4000 kN of thrust.¹²⁾ They concluded that about 10 MPa of combustion chamber pressure was suitable from the viewpoint of the specific impulse and engine mass. Boccaletto conducted studies for reusable 2000 kN engines in the SC and the GG cycles.¹³⁾ He investigated the throttled operations ranges from 40% to 125%.

A number of hydrogen-fueled engines have been developed, along with many conceptual studies on these engines.^{14–16)} As a result of these studies, various aspects of hydrogen-fueled engines have been identified. For example, thermodynamic property of methane is different from that of hydrogen, which creates gaps in the operating conditions of engines under various circumstances. Compared to hydrogen, methane has a higher cooling requirement and lower heat exchanging ability due to a low specific heat, as explained in the Appendix. Therefore, the operating range of methane-fueled engines is expected to be lower than that of hydrogen-fueled engines in the heat absorption engine cycles of the expander (EX) cycle. The EX cycle has already been adopted to use in the hydrogen-fueled RL-10 series.

In this study, the operating features of methane-fueled en-

gines using the EX cycle and the coolant-bleed (CB) cycle are discussed and compared with those of hydrogen-fueled engines. The difference in thermodynamic property directly affects the operating conditions of the heat absorption cycle. Even though the CB cycle has not been discussed in regards to methane-fueled engines, this cycle looks attractive for rocket engines. Neither the CB cycle nor the EX cycle has combustion components for driving turbines. While the CB cycle has lower specific impulse than the EX cycle, its pump exit pressure is lower than that of the EX cycle. This will raise the operating range of the engine in terms of the chamber pressure. This CB cycle is used for the hydrogen-fueled engines LE-5B and LE-9. The operating features of this cycle are compared with those of the SC cycle and the GG cycle.

Several studies have been conducted on the thrust control ability of liquid rocket engines, most of which targeted hydrogen-fueled engines and bipropellant engines.^{17–21)} For example, it was found that the throttled operation of a rocket engine can degrade excessive acceleration during launch while reducing the thrust required when landing a reusable vehicle. Boccaletto conducted studies on the throttling operation of methane-fueled engines ranging from 40% to 125% when using the SC cycle and the GG cycle.¹³⁾ Since the throttling engine is designed to operate under certain operating conditions, it may face a large pressure drop, both in the regenerative cooling jacket and in the liquid injector pressure under other operating conditions. This large pressure drop may degrade the specific impulse and the operating range of the engines. Herein, the characteristics of the throttling operation of methane-fueled booster engines in the coolant bleed cycle and the expander cycle are also studied.

2. Calculation Methods

2.1. Cycles and configurations of engines

Figure 1(a) and (b) show the schematics of the CB cycle and EX cycle investigated in this study. The oxidizer is oxygen. In the CB cycle, some of the fuel is used to cool the combustion chamber and nozzle while heated coolant fuel is used to drive turbines. The rest of the coolant fuel not used for driving the turbines is injected into the combustion chamber along with the fuel not used for cooling. Since the pressure ratio in the turbine is high, the mass flow rate for driving the turbine becomes low.

In the EX cycle, all of the fuel is used for cooling and driving the turbines. However, in the EX cycle, most of the fuel is used for running the turbines, which results in a lower pressure ratio in the turbine.

For comparison, the features of the GG cycle and the SC cycle are investigated as shown in Fig. 1(c) and (d). In the GG cycle, most of the fuel is used for regenerative cooling, whereas only a small amount of fuel and oxygen is bled and injected into the gas generator. The combustion gas from the gas generator drives the turbines. In the SC cycle, some oxygen is further pumped up and injected into the pre-burner. The operating conditions for the turbine are similar

Table 1. Properties of methane, oxygen and hydrogen.

	Methane	Oxygen	Para-hydrogen
Critical pressure, MPa	4.627	5.042	1.293
Critical temperature, K	190.8	154.6	32.98
Saturated liquid density at 1 atm, kg·m ⁻³	432.7	1141	70.79
Boiling temperature at 1 atm, K	111.6	90.18	20.27
Cp of saturated liquid at 1 atm, kJ·kg ⁻¹ ·K ⁻¹	3.39	1.76	9.64

Fig. 1. Schematic of engine cycles: (a) CB cycle, (b) EX cycle, (c) GG cycle, and (d) SC cycle.

to those of the EX cycle.

The mixture ratio of the combustion chamber is set to 3.4 for methane-fueled engines, while it is set to 6 for hydrogen-fueled engines. The mixture ratio of the pre-burner in the SC cycle and the mixture ratio of the gas generator in the GG cycle is 0.28 for methane-fueled engines. This suppresses the combustion gas temperature below 1000 K. In this study, the effect of soot formation when the engine is operating is not discussed.

Table 2 lists the dimensions of the engine investigated in this study. Engine-S is set as an upper-stage engine with dimensions equivalent to those of LE-5B. The operating characteristics of Engine-S are investigated with combustion chamber pressures ranging from 3 MPa to 10 MPa. Engine-L is set as a booster engine that has a size equivalent to LE-7A. The combustion chamber pressure of Engine-L varies from 5 MPa to 15 MPa. The effect of engine size on the cooling requirement is explained in the Appendix.

The nozzle geometry is designed using the parabolic approximation method proposed by Rao.²²⁾ The nozzle area ratio of Engine-S is set to 100. Regenerative cooling is applied to the nozzle of Engine-S until the area ratio reaches 39. Radiation cooling is also applied to the nozzle extension, which is downstream of the regenerative cooling part.

In Engine-L, the nozzle area ratio is designed based on a no-flow separation condition with the chamber pressure designed to be below sea level. In Engine-L, the nozzle length varies depending on the chamber pressure, thus there is no

Table 2. Engine dimensions.

	Engine-S	Engine-L
d , chamber, m	0.27	0.4
d , throat, m	0.16	0.24
Injector-throat length, m	0.3	0.5
d , regenerative cooling nozzle, m	1	—
d , radiative cooling nozzle, m	1.6	—
Cooled nozzle length, m	1.25	1.73
Total length, m	1.55	—
Nozzle area ratio	100	—
No. of coolant passages	251	376
Height of cooling passages at throat, mm	20	20
Width of coolant passages at throat, mm	1	1

description of total length in Table 2. For example, the length of Engine-L is 2.2 m at a chamber pressure of 10 MPa. The estimation method of the flow separation is given in Section 2.7.

In the examination of throttled operation, the nozzle area ratio is designed to be maximum throttle for the chamber pressure condition at the separation limit below sea level. The nozzle geometry is fixed during throttled operation.

In the regenerative cooling analysis, the local wall temperature distribution of the copper alloy liner is determined by the iterative calculation of local heat balance, where the amount of heat transferred from the combustion gas to the wall equates to the amount of heat transferred from the wall to the coolant. Some of the wall temperature distributions

Table 3. Efficiencies of turbopumps.

	η_{fp}	$\eta_{second-fp}$	η_{op}	$\eta_{split-op}$	η_{ft}	η_{ot}
CB cycle	0.7	0.7	0.7	—	0.5	0.5
GG cycle	0.7	0.7	0.7	—	0.5	0.5
EX cycle	0.7	—	0.7	—	0.7	0.7
SC cycle	0.7	—	0.7	0.7	0.7	0.7

calculated are described in Chapter 3. In the calculation, the combustion gas-side wall temperature is presumed to be equal to that of the coolant side, and the thickness of the combustion chamber wall between the combustion gas and coolant is not a design parameter here. In this study, the maximum temperature of the liner is set to 900 K.^{2,23)}

Core vacuum thrust and a vacuum-specific impulse of Engine-S are 410 kN and 3780 m·s⁻¹ at the combustion chamber pressure of 10 MPa, while those of Engine-L are 880 kN and 3580 m·s⁻¹ at 10 MPa, respectively. Even though the chamber pressure decreases, the vacuum-specific impulse of Engine-S remains the same due to the same area ratio of the nozzle. In Engine-L, however, when the chamber pressure decreases, the nozzle area ratio also decreases so that the gas does not separate at the nozzle exit. Therefore, the core thrust drops to 420 kN, and a vacuum-specific impulse is 3450 m·s⁻¹ at a chamber pressure of 5 MPa.

In the hydrogen-fueled engines, the core vacuum thrust and vacuum-specific impulse of Engine-S are 410 kN and 4720 m·s⁻¹, respectively, at a combustion chamber pressure of 10 MPa, while those of Engine-L are 880 kN and 4480 m·s⁻¹ at 10 MPa, respectively.

The efficiency of the pump and turbine are assigned in the calculation with reference to the efficiency of the actual engines,^{14,24,25)} which are listed in Table 3. With a high pressure ratio, the turbine operates with low efficiency in the bleed cycle; that is, the ratio of blade-tip speed to the turbine gas spouting speed is low.

In the bleed cycle, partial admission is frequently adopted due to the small volume flow rate of the turbine gas. It also degrades the turbine efficiency.²³⁾ Since partial admission is not adopted in this study, efficiency degradation is not investigated. In the EX cycle or the SC cycle, the ratio of blade-tip speed to the spouting velocity is high, which drives turbine efficiency higher.

2.2. Fuel and oxidizer properties

The properties of methane, oxygen and hydrogen are calculated using the property calculation code.¹⁾ The properties of the combustion gas are calculated using a different code.²⁶⁾ The combustion gas is frozen in the combustion chamber and nozzle.

2.3. Power of turbopump

The power generated by the turbine and used by the pump is calculated using the following equations.

$$W_t = \eta_t \cdot \dot{m}_t \{h_t(T_1, p_1) - h_t(T_2, p_2)\} \quad (1)$$

$$W_p = \dot{m}_p \cdot \frac{p_{p,2} - p_{p,1}}{\eta_p \cdot \rho_{p,1}} \quad (2)$$

In this study, the pressure at the oxidizer turbine exit is set

to 300 kPa for the CB cycle and the GG cycle. According to the preliminary calculation, the pressure value had only a slight effect on the power balance of the turbopump and the turbine-driving gas conditions.

2.4. Heat transfer from combustion gas

Convective heat transfer from the combustion gas to the wall is calculated using the Reynolds analogy.

$$c_H \approx \frac{1}{2} c_f \quad (3)$$

Friction coefficient is calculated using the formula for a flat plate flow, as in the previous study.²⁷⁾ The formula for a compressible flow on the flat plate derived by White²⁸⁾ is adopted with the adiabatic wall condition.

$$c_{f,c} \approx \frac{0.455}{S^2 \ln^2 \left(\frac{0.06}{S} Re_x \frac{\mu_e}{\mu_w} \sqrt{\frac{T_e}{T_w}} \right)}$$

where

$$S = \frac{(T_{aw}/T_e - 1)^{1/2}}{\sin^{-1} A + \sin^{-1} B} \quad (4)$$

$$A = \frac{2a^2 - b}{(b^2 + 4a^2)^{1/2}} \quad \text{and} \quad B = \frac{b}{(b^2 + 4a^2)^{1/2}}$$

$$a = \left(\frac{\gamma - 1}{2} M_e^2 \frac{T_e}{T_w} \right)^{1/2} \quad \text{and} \quad b = \left(\frac{T_{aw}}{T_w} - 1 \right)$$

With adiabatic wall conditions, the formula for the compressible flow of Eq. (4) is connected smoothly to the formula of the incompressible flow of Eq. (5).²⁸⁾

$$c_{f,i} \approx \frac{0.455}{\ln^2 0.06 Re_x} \quad (5)$$

The effect of using the adiabatic wall condition for the calculation result is mentioned later in Fig. 2.

In the previous estimation procedure,²⁷⁾ the friction coefficient of the compressible flow was calculated using the wall temperature based on Eq. (4). Therefore, the coefficient was not connected smoothly to the friction coefficient of the incompressible flow. Friction coefficient in the convergent section was calculated using interpolation of the coefficients of the combustion chamber and at the throat.

Radiative heat flux is calculated using the formula derived by Schack.²⁹⁾

$$q_{r,H_2O} = 0.0446 \times 10^{-6} \cdot S \cdot (p_{H_2O} \cdot s)^{0.6} (T_g^3 - T_w^3)$$

$$q_{r,CO_2} = 0.8729 \times 10^{-9} \cdot S^3 \cdot \sqrt[3]{p_{CO_2} \cdot s} \cdot (T_g^{3.5} - T_w^{3.5})$$

where

$$s = k \cdot d \quad (6)$$

k is 0.95 for the gas body shape of the infinitely long cylinder and S is 0.65 for the surface of oxidized copper.

The present calculation procedure with the adiabatic wall condition may cause a deviation due to incorrect wall conditions. Figure 2 shows the comparison of heat fluxes calculated using the present method and those measured experimentally in the cylindrical section and at the throat of the

Fig. 2. Heat fluxes of combustion chamber at (a) cylindrical section and (b) throat.

Table 4. Summary of firing test conditions in past experiments.²⁷⁾

	Fuel	Diameter of cylindrical section, mm	Diameter at throat, mm	Chamber length, mm	Combustor pressure, MPa	Mixture ratio, O/F
LE-5 ³⁰⁾	H ₂	240	136	309	3.5	6.1
Kumakawa et al. ²⁾	H ₂	66, 67	23	157, 207	7.12–9.50	4.77–6.86
Niino et al. ³¹⁾	H ₂	66	28	236	2.78–3.52	4.27–6.46
Elam ³²⁾	H ₂	144	90.9	379, 481	10.2–16.4	5.24–6.90
Azuma et al. ³³⁾	C ₂ H ₆ O	66	25, 33	330	4.0, 7.8	1.6
Kumakawa et al. ²⁾	CH ₄	66, 67	23	157, 207	3.5–9.6	2.71–5.06
Kim and Ju ⁴⁾	CH ₄	176	101	342	6.4	3.0

combustion chamber.^{2,4,30–33)} Table 4 shows the summary of firing test conditions for the experiments. The heat flux calculated is the sum of the convective and radiative heat fluxes. As these firing tests of methane- and ethanol-fueled combustors suggest, it can be assumed that there is no radiation from solid carbon. The heat fluxes calculated indicate a reasonable agreement with the heat fluxes measured.

2.5. Heat transfer from the wall to coolant, and coolant pressure drop

The heat transferred from the wall of the combustion chamber or from the nozzle to coolant methane is calculated with the Stanton number connected to the friction factor by the following relationship.²⁸⁾

$$c_H \approx \frac{\Lambda/8}{1 + 12.8(Pr^{0.68} - 1)\sqrt{\Lambda/8}} \quad (7)$$

The friction factor is calculated using the formula of Blasius.²⁸⁾

$$\Lambda \approx 0.3164 Re_d^{-1/4} \quad (8)$$

The pressure drop in the coolant is calculated using the following equations. Acceleration by heating decreases the pressure of the coolant under conservation of momentum. Dynamic pressure is considered to be the pressure drop in the cooling passages.

$$\Delta p_f = \frac{\Lambda}{4} \cdot \frac{\Delta l}{d} \left(\frac{\dot{m}_c}{A_c} \right)^2 \left(\frac{1}{\rho_{c,2}} + \frac{1}{\rho_{c,1}} \right) \quad (9)$$

$$\Delta p_a = \left(\frac{\dot{m}_c}{A_c} \right)^2 \left(\frac{1}{\rho_{c,2}} - \frac{1}{\rho_{c,1}} \right) \quad (10)$$

2.6. Propellant injector pressure drop

Pressure drop between the fuel/oxidizer manifold and the combustion chamber is generally designed to be a value from 10% to 30% of the combustion chamber pressure for a stable propellant injection. Leudiere and Supie used 0.22.⁵⁾ Yatsuyanagi used 0.15 for methane and 0.4 for oxygen.³⁾ In LE-7, 0.31 is applied for oxygen and 0.09 for hydrogen.²⁵⁾ According to the ethanol/oxygen injection engine tests, Azuma et al. reported that a ratio of over 0.05 is necessary for steady combustion.³³⁾ In this study, 15% is used for methane and oxygen.

In the studies of throttling engines, the engines are presumed to have a 15% pressure drop in the injector under the condition of maximum throttle. The drop in injector pressure varies depending on propellant conditions. When methane gas or hydrogen gas is directly injected into the combustion chamber as fuel, the injector pressure will drop in proportion to the mass flow rate. When liquid oxygen is injected, the pressure will drop in proportion to the square of the propellant mass flow rate; that is, the square of the cham-

ber pressure.^{17,21)} Under the condition of maximum throttle, when a pressure drop in the oxygen injector remains at 15%, it will drop further as throttling is reduced. This large pressure drop in injection affects the necessary pump power and operating range of the engines.

2.7. Separation at nozzle exit

The separation pressure at the exit of the nozzle is calculated using Reshotko-Tucker's criterion.³⁴⁾ When operating at sea-level conditions, combustion exhaust gas is over-expanded.

When throttling is applied to Engine-L, the nozzle geometry of the engine is specified so that combustion gas does not separate under the condition of maximum throttle (i.e. under the lowest chamber pressure condition). It is presumed that an engine that is capable of throttling is able to operate with this small nozzle under different operating conditions.

2.8. Throttling

As mentioned in Section 2.6, in the studies of throttled engines, the pressure drop in the propellant injector is set to 15% of the combustion chamber pressure under the condition of maximum throttle. The engine operates under different throttling conditions using the same injector. The pressure drop in the injector becomes larger when the throttling condition drops below the maximum throttle designed for liquid propellants. On the other hand, for no-throttle, the pressure drop in the injector is 15% under all combustion chamber conditions.

As mentioned in Section 2.7, the nozzle of the throttled engine is designed to accommodate the maximum throttled condition in which the combustion gas flowing from the nozzle does not separate from the nozzle wall. Then the exhaust gas does not expand sufficiently when throttling is lower. On the other hand, no-throttle engines are designed so that no separation occurs in the nozzle at each combustion chamber pressure condition. Then the exhaust gas of no-throttle engines becomes slightly over-expanded at the nozzle.

3. Results and Discussion

3.1. Vaporization and liquefaction of methane

Figure 3(a) shows the temperature-pressure diagram for the coolant methane in Engine-S. Figure 3(b) shows the wall temperature distribution of the CB cycle at a combustion chamber pressure of 6 MPa as an example of the calculation condition. When the pressure in the cooling jacket is low, the coolant methane vaporizes in the jacket in any engine cycle, as reported by Pempie and Boccaletto.⁹⁾ They used the EX cycle and set the combustion chamber pressure to about 6 MPa to avoid two-phase flow. There is no such vaporization in hydrogen-fueled engines.

When the condition of the coolant is in the vicinity of the critical point, the heat-transfer coefficient may decrease, at which time the wall temperature increases.³⁵⁾ In this study, the calculation result of the wall temperature is larger than 1000 K when the coolant methane nears the critical point. Though a combustion chamber pressure of 3 MPa or higher is set for the operating range of Engine-S in this study, meth-

Fig. 3. (a) Temperature-pressure diagrams during lower engine pressure conditions of Engine-S. (b) Wall temperature distribution of CB engine at $p, c = 6$ MPa.

ane-fueled engines can operate at 5 MPa or higher in the CB cycle. In EX cycle engines, the pressure in the cooling jacket is higher than that of the CB cycle, thus the engines can operate at a combustion chamber pressure of 4 MPa or higher. Since methane has a higher critical point than hydrogen, methane-fueled engines operate at higher levels of combustion chamber pressure compared to hydrogen-fueled engines.

Figure 4(a) shows the temperature-pressure diagram of the methane-fueled CB cycle in Engine-L at high chamber pressures and at the maximum wall temperature of 700 K and 900 K. Figure 4(b) shows the wall temperature distribution as an example of the calculation conditions. The driving gas liquefies in the turbine where the combustion chamber pressure is 15 MPa and the maximum combustion chamber/nozzle wall temperature is 700 K. The temperature of the driving gas substantially decreases and can be liquefied by large expansion in the turbine. Liquefaction does not occur when the combustion chamber pressure is below 10 MPa or the maximum wall temperature is over 900 K.

No liquefaction occurs in hydrogen-fueled engines. In the turbine of hydrogen-fueled engines, the temperature is much

higher than the liquefaction temperature. Thus, the operating range of methane-fueled engines is limited to low pressure in the CB cycle. In this cycle, both the lower and higher bound-

daries of the range of operation are narrow in methane-fueled engines.

In the GG cycle and the SC cycle, the temperature of the turbine driving gas is high enough, thus the combustion gas does not liquefy in the turbine.

3.2. Specific impulse

Figure 5 shows the specific impulse of the engines. The specific impulse of the methane-fueled engines is about $1000 \text{ m}\cdot\text{s}^{-1}$ lower than that of the hydrogen-fueled engines. Engine-S operates in a vacuum, and the specific impulse of the EX cycle does not change with the combustion chamber pressure. The specific impulse of EX-cycle Engine-S is $3780 \text{ m}\cdot\text{s}^{-1}$ in the methane-fueled engines, and $4720 \text{ m}\cdot\text{s}^{-1}$ in the hydrogen-fueled engines.

In the hydrogen-fueled engines of EX-cycle Engine-S and Engine-L, the operating chamber pressure is limited to less than 8 MPa . In methane-fueled engines, the engines can operate at a higher chamber pressure. This wide operating range of methane-fueled engines is discussed in Section 3.3.

In the methane-fueled Engine-L, differences in specific impulse between the CB cycle and the EX cycle are greater compared to those in hydrogen-fueled engines. In methane-fueled CB-cycle engines, the product of specific heat and temperature at the turbine entrance is much lower than that of hydrogen-fueled engines. This creates the need for a larger amount of gas for driving the turbine. Figure 6 shows the bleed ratio of the fuel to the total flow rate. When the GG cycle is used, the bleed ratio and the degradation of the specific impulse become low even in methane-fueled engines.

The specific impulse of Engine-L increases in proportion to the increase in the chamber pressure. With a higher chamber pressure, the nozzle area ratio increases under no-separation flow conditions. This increased nozzle area causes the specific impulse to rise. In methane-fueled CB-cycle Engine-L, the amount of increase is small, and is caused by a larger increase in the turbine bleed ratio compared to that of hydrogen-fueled engines.

Fig. 4. (a) Temperature-pressure diagrams during higher engine pressure conditions of the CB cycle in Engine-L. (b) Wall temperature distribution of CB engine at $p, c = 10 \text{ MPa}$.

Fig. 5. Specific impulse: (a) methane-fueled engines, and (b) hydrogen-fueled engines.

Fig. 6. Ratio of turbine driving gas flow rate to the total propellant flow rate of the coolant bleed cycle and the gas generator cycle.

3.3. Power balance of EX cycle

Figure 7 shows the fuel pump exit pressures of the EX cycle engines. The operating conditions for the SC cycle of methane-fueled engines are also plotted for comparison.

The pressure at the fuel pump exit is twice the level of the combustion chamber pressure. In the hydrogen-fueled engines, with a combustion chamber pressure exceeding approximately 5 MPa, the fuel pump exit pressure increases sharply and the engines cannot run at pressures of 8 MPa or above. Sutton described the EX cycle as not practical when the chamber pressure is higher than 7.58 MPa.³⁶⁾ Huzel and Huang described the expander cycle as limited to its relatively low thrust levels.²³⁾ On the other hand, methane-fueled engines can operate at combustion chamber pressures greater than 15 MPa, though the product of specific heat and temperature after the regenerative cooling is lower than those of hydrogen-fueled engines. Herein, the reason of the higher operating ability of methane-fueled engines is discussed.

The power of the pump and the turbine are described using Eqs. (1) and (2). The power produced by the turbine can be described in another form.

$$W_t = \eta_t \cdot \dot{m}_t \cdot C_{p,t} \cdot T_{t,1} \left\{ 1 - \left(\frac{p_{t,2}}{p_{t,1}} \right)^{\frac{\gamma-1}{\gamma}} \right\} \quad (11)$$

The ratio of pressure at the turbine entrance to the pressure at the fuel pump exit is written as

$$r_{ft,1} = p_{ft,1} / p_{fp,2} \quad (12)$$

$r_{ft,1}$ is approximately 0.95 due to the pressure decrease in the cooling jacket. The pressure at the exit of the fuel turbine is higher than the combustion chamber pressure due to the pressure drop at the turbine exit manifold, at the duct from the turbine to the fuel injector, and at the fuel injector. This ratio is defined as

$$r_{ft,2} = p_{ft,2} / p_c \quad (13)$$

$r_{ft,2}$ is approximately 1.3 in the EX-cycle engine. Under these conditions, the following relation is derived from

(a)

(b)

(c)

Fig. 7. Fuel pump exit pressure of the EX and the SC cycle: (a) Engine-S and (b) Engine-L, and (c) wall temperature distribution of Engine-S at $p, c = 7$ MPa.

Eqs. (2) and (11) using Eqs. (12) and (13).

$$\frac{p_c}{p_a} \approx \frac{\eta_{fp} \cdot \rho_{fp,1} \cdot C_{p,t} \cdot T_{ft,1}}{p_a} \frac{p_c}{p_{fp,2}} \left\{ 1 - \left(\frac{r_{ft,2} \cdot p_c}{r_{ft,1} \cdot p_{fp,2}} \right)^{\frac{\gamma-1}{\gamma}} \right\} \quad (14)$$

p_a is reference pressure and is atmosphere here. It is used here for non dimensionnalization of Eq. (14). η_{tp} is defined as

$$\eta_{tp} = \eta_t \cdot \eta_p \quad (15)$$

The power of the oxidizer pump is generally smaller than that of the fuel pump. Herein, the power of the oxidizer pump is neglected for simplicity, and the fundamental relationship between the pump exit pressure and combustion chamber pressure is discussed here.

When (p_c/p_a) is differentiated with $(p_{fp,2}/p_c)$,

$$\frac{d\left(\frac{p_c}{p_a}\right)}{d\left(\frac{p_{fp,2}}{p_c}\right)} \approx -\frac{\left(\frac{\eta_{ftp} \cdot \rho_{fp,1} \cdot C_{p,t} \cdot T_{ft,1}}{p_a}\right)}{\left(\frac{p_{fp,2}}{p_c}\right)^2} \cdot \left\{1 - \frac{2\gamma - 1}{\gamma} \left(\frac{r_{ft,2}/r_{ft,1}}{p_{fp,2}/p_c}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad (16)$$

Using this equation, the maximum chamber pressure ratio is obtained for the pump/chamber pressure ratio as derived below.

$$\left(\frac{p_{fp,2}}{p_c}\right)_{\max} \approx \frac{r_{ft,2}}{r_{ft,1}} \cdot \left(\frac{2\gamma - 1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \quad (17)$$

When $r_{ft,1}$ is 0.95 and $r_{ft,2}$ is 1.3, the maximum fuel pump/chamber pressure ratio of $(p_{fp,2}/p_c)$ is 3.24 for $\gamma = 1.5$ in methane-fueled engines. In the hydrogen-fueled engines, $(p_{fp,2}/p_c)$ is 3.30 for $\gamma = 1.4$. The maximum pressure at the fuel pump exit is approximately three times larger than that of the chamber pressure at the maximum operating combustion chamber pressure.

The maximum chamber pressure ratio is calculated using Eqs. (14) and (17)

$$\left(\frac{p_c}{p_a}\right)_{\max} \approx c_{ex} \cdot \frac{r_{ft,1}}{r_{ft,2}} \cdot \frac{\frac{\gamma - 1}{2\gamma - 1}}{\left(\frac{2\gamma - 1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}} \quad (18)$$

where

$$c_{ex} = \frac{\eta_{ftp} \cdot \rho_{fp,1} \cdot C_{p,t} \cdot T_{ft,1}}{p_a} \quad (19)$$

The combustion chamber pressure ratio for the maximum fuel pump/chamber pressure ratio is a function of c_{ex} , which is a product of turbopump efficiency, density, specific heat, temperature and the inverse of the reference pressure.

Figure 8 shows the maximum chamber pressure ratio for c_{ex} , calculated using Eq. (18). In the present study, c_{ex} is 2560 for methane-fueled EX-cycle engines, whereas it is 1120 for hydrogen-fueled EX-cycle engines. These values correspond to the chamber pressure ratio of 197 and 75 in Eq. (18), respectively. In c_{ex} , c_p is $17 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ for hydrogen and $3.1 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ for methane. The temperature at the turbine entrance is 200 K in hydrogen-fueled engines, whereas it is 400 K in methane-fueled engines. The density of liquid hydrogen is $70 \text{ kg} \cdot \text{m}^{-3}$, and that of liquid methane is $432 \text{ kg} \cdot \text{m}^{-3}$. The difference in c_{ex} is caused by the gaps in these properties.

As shown in Fig. 7, the fuel pump exit pressure rises slowly as the chamber pressure increases since the maximum combustion chamber is high. Since the maximum combustion chamber pressure of methane-fueled engines is higher than that of hydrogen-fueled engines, the pump exit pressure is low, being approximately 7 MPa in the combustion cham-

Fig. 8. Maximum combustion chamber pressure ratio for the product of η_{tp} , density, specific heat, density and inverse pressure.

ber of methane-fueled engines.

The relationship of Eqs. (18) and (19) are derived to show the fundamental relationship between the pressure at the hydrogen pump exit and the combustion chamber. The power balance of the hydrogen pump and turbine is used for simplicity, and the power of the oxidizer pump and turbine is neglected. In SC-cycle engines, the power of the oxidizer pump is smaller than that of the hydrogen pump. Accordingly, the power of the oxidizer turbine is also smaller than that of the hydrogen turbine. Though the mass flow rate of the oxidizer is equivalent to that of hydrogen in the turbines, the relationship of Eqs. (18) and (19) is applied to SC-cycle engines for comparison with the maximum combustion chamber ratio of EX-cycle engines.

In the SC cycle, methane-fueled engines can also operate at a chamber pressure of over 15 MPa, and the pressure at the fuel pump exit is lower than that of the EX cycle. c_{ex} of the SC cycle is about 16000, six times larger than that of the EX cycle. The maximum chamber pressure ratio is about 620. In hydrogen-fueled engines, the operating chamber pressure is larger than 10 MPa, as in LE-7 and SSME. In LE-7 with a design chamber pressure ratio of 125, c_{ex} is 5700 and its corresponding maximum chamber pressure ratio of p_c/p_a is 360. Since the maximum chamber pressure ratio is higher in SC-cycle engines, the fuel pump pressure is lower compared to EX-cycle engines.

Wakamatsu discussed the characteristics of the EX cycle and the SC cycle used by hydrogen-fueled and kerosene-fueled engines.³⁷⁾ He also derived a relationship between the combustion chamber pressure and the pump exit pressure from the viewpoint of the power balance of the turbopump. He showed the engine operating limit at the maximum pump exit pressure. If he had discussed methane-fueled engines and presented the operating range of the combustion chamber pressure, he would have reported results similar to this study, that the operable combustion chamber pressure of methane-fueled engines is higher compared to hydrogen-fueled engines using the EX cycle.

In the study conducted by Pempie and Boccaletto, the

pressure at the methane pump exit was approximately 17 MPa at a combustion chamber pressure of approximately 6 MPa, and the pressure at the hydrogen pump exit was almost the same as that of methane-fueled engines at the same chamber pressure.⁹⁾ In Fig. 7, high pressure is required when the combustion chamber condition is near maximum pressure in hydrogen-fueled engines, whereas the pressure at the pump exit shows a small difference in low combustion chamber pressure conditions. Pempie and Boccaletto investigated the engines at combustion chamber pressures that are much lower than the maximum chamber pressure, thus there would be no large difference in pressure.

In the study by Crocker and Peery, the pressure at the fuel pump exit was 21 MPa at a chamber pressure of approximately 6 MPa¹¹⁾ for a methane-fueled EX-cycle engine. The pressure was much higher than that of the present study. The engine was based on RL-10, thus the high pressure at the pump exit might be caused by a difference in the turbopump efficiency or a loss of pressure in the specified cooling jacket.

The temperature of methane is around 350 K at the cooling jacket exit under the combustion chamber pressure of 5 to 6 MPa in the present study. The temperature was approximately 500 K in the study of Schuff et al.¹⁰⁾ and the temperature was also approximately 500 K in the study of Crocker and Peery.¹¹⁾ The temperature may increase depending on the geometry of the combustion chamber or the design of the cooling jacket. Schuff et al. used Bartz's formula to calculate the heat transfer coefficient of the combustion gas. This formula uses a pipe flow model and calculates a larger heat transfer coefficient for the smaller combustion chamber.²⁷⁾ Though Schuff et al. validated the calculated results with the test data, the modification coefficient is not effective for other engines.

3.4. Throttling

In a throttling engine, the ratio of the fuel-injector pressure drop to the combustion chamber pressure is set to 0.15 under the condition of maximum throttle. The ratio of the oxygen injector pressure drop to the chamber pressure is also set to 0.15. We compared an engine with a throttling capability of 30% to one without it for both CB and EX cycles. The throttling range was between 30% and 100%, almost the same range (40% to 125%) as reported in the research by Boccaletto.¹³⁾ The designed operating chamber pressure is set to 10 MPa for throttling engines. In a throttling engine, the nozzle area ratio is specified so that the combustion gas does not separate at the nozzle exit even during maximum throttling. The specific impulse with the nozzle is therefore lower due to the smaller-area ratio.

Figure 9 compares throttling engines and no-throttle engines in terms of specific impulse and pump exit pressure for Engine-L using EX-cycle. "30% throttling" means that the engine can decrease its combustion chamber pressure up to 30% of the reference chamber pressure of 10 MPa. Given this, the engine is designed for a combustion chamber pressure of 3 MPa. For calculating the no-throttle engine, the engine and nozzle are designed for each combustion chamber pressure. The pressure-drop ratio of the injectors is 15% for

Fig. 9. Specific impulse and fuel pump exit pressure of Engine-L using the EX cycle for throttling and no-throttle engines.

all operating conditions.

The pump exit pressure of throttling engines is higher than that of no-throttle engines. This was also mentioned by Boccaletto.¹³⁾ However it does not affect the engine operating range.

There is a large difference in the specific impulse between throttling engines and no-throttle engines. The specific impulse of throttling engines is high at the sea-level conditions, whereas it becomes low when in a vacuum. As for no-throttle engines operating at sea-level conditions, the pressure at the nozzle exit is lower than the ambient pressure even when there is no separation; that is, the nozzle exhaust gas is over-expanded in the nozzle. As mentioned in Betts and Frederick,¹⁷⁾ this caused drag, and the specific impulse became smaller than that of throttling engines. In throttling engines, the nozzle exhaust gas is under-expanded or optimally expanded at higher combustion chamber pressures than for a designed throttled chamber pressure of 3 MPa. The specific impulse is not degraded by a low-pressure region around the nozzle exit. When in a vacuum, there is no drag area around the nozzle exit, and the specific impulse becomes higher for no-throttle engines with a big nozzle.

Figure 10 shows a comparison of the specific impulse and turbine bleed ratio to the total propellant flow rate of Engine-L using a CB cycle. The turbine flow ratio is larger in the throttling engine due to the larger power required by the oxygen pump and larger pressure drop in the cooling jacket. This larger bleed ratio and smaller nozzle ratio degrade the specific impulse when the throttling engine is operating in a vacuum.

Under sea-level conditions, the difference in specific impulse is small. In throttling engines, the bleed ratio of the turbine-driving gas becomes larger faster than that of the no-throttle engine as the necessary oxygen pump power increases sharply. The low specific impulse of the bleed gas degrades the specific impulse of the engine, and the

Fig. 10. Specific impulse and turbine flow rate ratio to total propellant flow rate of Engine-L using the CB cycle for throttling and no-throttle engines.

advantages of throttling engines over no-throttle engines are reduced in terms of specific impulse under sea-level conditions.

4. Conclusion

In this study, the characteristics of methane-fueled engines were investigated for the EX and CB cycles. The engines equipped with two different configurations, Engine-S for an upper-stage engine and Engine-L for a booster engine, were used. Through this study, the following characteristics of methane-fueled engines were clarified.

(1) Methane vaporizes in the cooling jacket at low operating pressures and liquefies in the turbine of the CB-cycle engine at high combustion chamber pressures. This is caused by high critical pressures and the high liquefaction temperature of methane.

(2) In methane-fueled engines, the degradation of the specific impulse in the CB cycle is larger compared to that of hydrogen-fueled engines. This degradation is caused by methane's low specific heat and temperature after regenerative cooling.

(3) In EX-cycle engines, the operating range of methane-fueled engines is larger than that of hydrogen-fueled engines.

(4) Throttling does not have a significant effect on pump exit pressure for the EX cycle, whereas it increases the bleed ratio and degrades the specific impulse for the CB cycle in methane-fueled engines.

Acknowledgments

The authors thank Dr. James H. Morehart for information on research and development activities on methane-fueled rocket engines.

References

- Hendricks, R. C., Baron, A. K., and Peller, I. C.: GASP-A Computer Code for Calculating the Thermodynamic and Transport Properties for Ten Fluids: Parahydrogen, Helium, Neon, Methane, Nitrogen, Carbon Monoxide, Oxygen, Fluorine, Argon, and Carbon Dioxide, NASA TN D-7808, 1975.
- Kumakawa, A., Sasaki, M., Sato, K., Tamura, H., Ono, F., Sakamoto, H., and Yatsuyanagi, N.: Hot Gas Side Heat Transfer Characteristics of LOX/H₂ and LOX/HC Type Propellants, National Aerospace Laboratory, Tokyo, Technical Report NAL TR-1062T, 1990.
- Yatsuyanagi, N.: Comprehensive Design Method for LOX/Liquid-Methane Regenerative Cooling Combustor with Coaxial Injector, *Trans. Jpn. Soc. Aeronaut. Space Sci.*, **52** (2009), pp. 180–187.
- Kim, K.-H. and Ju, D.-S.: Development of 'CHASE-10' Liquid Rocket Engine Having 10 tf Thrust Using LOX & LNG (Methane), *AIAA Paper* 2006-4907, 2006.
- Leudiere, V. and Supie, P.: KVD-1 Engine in LOX/CH₄, *AIAA Paper* 2007-5446, 2007.
- Asakawa, H., Shinohara, R., Sakaguchi, H., Ishikawa, Y., Ishihara, S., Nanri, H., and Masuda, I.: Study on Combustion Characteristics of LOX/LNG (Methane) Co-axial Type Injector under High Pressure Condition, *AIAA Paper* 2016-5078, 2016.
- BE-4 Rocket Engine, <https://www.blueorigin.com/be4>, (accessed Nov. 11, 2016).
- Raptor Rocket Engine, <https://www.nasaspacesflight.com/2016/10/its-propulsion-evolution-raptor-engine/>, (accessed Nov. 11, 2016).
- Pempie, P. and Boccaletto, L.: LOX/CH₄ Expander Upper Stage Engine, Proceedings of 55th International Aeronautical Congress, International Astronautical Federation, *Paper IAC-04-S.1.03*, 2004.
- Schuff, R., Maier, M., Sindiy, O., Ulrich, C., and Fugger, S.: Integrated Modeling and Analysis for a LOX/Methane Expander Cycle Engine: Focusing on Regenerative Cooling Jacket Design, *AIAA Paper* 2006-4534, 2006.
- Crocker, A. M. and Peery, S. D.: System Sensitivity Studies of a LOX/Methane Expander Cycle Rocket Engine, *AIAA Paper* 1998-3674, 1998.
- Pempie, P. and Gorecki, H.: High Thrust LOX-CH₄ Engine, 22nd ISTS Paper, 2000-A-01V, Morioka, 2000.
- Boccaletto, L.: A Comparison between Two Possible Thermodynamic Schemes for Reusable LOX/LCH₄ Engines, *AIAA Paper* 2004-3356, 2004.
- Wakamatsu, Y., Kanmuri, A., and Toki, K.: Partial Bleed Expander Cycles for Low Thrust LOX/LH₂ Rocket Engine, National Aerospace Laboratory Technical Report, NAL TR-837T, 1984.
- Kanmuri, A., Kanda, T., Wakamatsu, Y., Torii, Y., Kagawa, E., and Hasegawa, K.: Transient Analysis of LOX/LH₂ Rocket Engine (LE-7), *AIAA Paper* 89-2736, 1989.
- Sato, M., Nakatani, R., Kimura, T., Hashimoto, T., Tomita, T., Kanda, T., Niu, K., and Iwama, K.: Evaluation of Engine Cycles for Small Engineering Test Rocket Engine, 25th ISTS Paper, 2006-g-16, Kanazawa, Japan, 2006.
- Betts, E. M. and Frederick, R. A., Jr.: A Historical Systems Study of Liquid Rocket Engine Throttling Capabilities, *AIAA Paper* 2010-6541, 2010.
- Wanhainen, J. P., Antl, R. J., Hannum, N. P., and Mansour, A. H.: Throttling Characteristics of a Hydrogen-Oxygen, Regeneratively Cooled, Pump-Fed Rocket Engine, NASA TM X-1043, 1964.
- Dressler, G. A.: Summary of Deep Throttling Rocket Engines with Emphasis on Apollo LMDE, *AIAA Paper* 2006-5220, 2006.
- Casiano, M. J., Hulka, J. R., and Yang, V.: Liquid-Propellant Rocket Engine Throttling: A Comprehensive Review, *J. Propul. Power*, **26** (2010), pp. 897–923.
- Kanda, T.: Conceptual Study of Throttled Operation of Rocket Engine Turbopump, *Trans. Jpn. Soc. Aeronaut. Space Sci.*, **56** (2013), pp. 49–60.
- Rao, G. V. R.: Approximation of Optimum Thrust Nozzle Contour, *ARS J.*, **30** (1960), p. 561.
- Huzel, D. K. and Huang, D. H.: *Modern Engineering for Design of Liquid-Propellant Rocket Engines*, Vol. 147, *Progress in Astronautics*

- and Aeronautics, AIAA, Washington DC, 1992, pp. 34–36, 49–51, 171–173, 199.
- 24) Kamijo, K., Sogame, E., and Okayasu, A.: Development of Liquid Oxygen and Hydrogen Turbopump for the LE-5 Rocket Engine, *J. Spacecraft Rockets*, **19** (1982), pp. 226–231.
 - 25) Torii, Y., Sogame, E., Kamijo, K., Ito, T., and Suzuki, K.: Development Status of LE-7, *Acta Astronautica*, **17** (1988), pp. 331–340.
 - 26) Gordon, S. and McBride, B. J.: Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications, NASA RP-1311, 1994.
 - 27) Kanda, T. and Sato, M.: Radiative Heating in Combustion Chamber of Liquid Propellant Rocket Engines, *Trans. Jpn. Soc. Aeronaut. Space Sci.*, **59** (2016), pp. 332–339.
 - 28) White, F. M.: *Viscous Fluid Flow*, McGraw-Hill, New York, 1974, pp. 484–485, 494–495, 498, 564–566, 632–644.
 - 29) Schack, A.: *Applied Heat Transfer*, translated by Takahashi, Y., Corona, Co. & Ltd., Tokyo, 1965, pp. 143–155, 278–279 (in Japanese), or *Industrial Heat Transfer*, Chapman and Hall Ltd., London, 1965, pp. 221–234, 430.
 - 30) Yatsuyanagi, N.: Establishment of Design Method for Liquid Hydrogen Regenerative Cooling Combustor of LOX/Hydrogen Rocket Engine, *Trans. Jpn. Soc. Aeronaut. Space Sci.*, **51** (2009), pp. 259–266.
 - 31) Niino, M., Kumakawa, A., Yatsuyanagi, N., Gomi, H., Suzuki, A., Sakamoto, H., Sasaki, M., and Yanagawa, K.: A Study on Heat Transfer Characteristics of Water Cooled LO₂/LH₂ Rocket Combustor, National Aerospace Laboratory, Tokyo, Technical Report NAL TR-708, 1982.
 - 32) Elam, S. K.: Subscale LOX/Hydrogen Testing with a Modular Chamber and a Swirl Coaxial Injector, *AIAA Paper* 91-1874, 1991.
 - 33) Azuma, N., Hiraiwa, T., and Tomita, T.: Performance Measurement of LOX/Ethanol Rocket Engine, Proceedings of 58th Conference on Space Sciences and Technologies, Nagasaki, Paper 1J05, Oct. 2014 (in Japanese).
 - 34) Schmucker, R. H.: Status of Flow Separation Prediction in Liquid Propellant Rocket Nozzles, NASA TM-X 64890, 1974.
 - 35) Tanaka, H., Nishiwaki, N., Hirata, M., and Tsuge, A.: Forced Convection Heat Transfer to Fluid near Critical Point Flowing in Circular Tube, *Int. J. Mass Heat Transfer*, **14** (1971), pp. 739–750.
 - 36) Sutton, G. P.: *Rocket Propulsion Elements*, 5th ed., John Wiley & Sons, New York, 1986, p. 156.
 - 37) Wakamatsu, Y.: High Combustion Pressure Liquid Rocket Engines, National Aerospace Laboratory, Tokyo, Technical Memorandum, NAL TM-403, Chapter 9, 1980 (in Japanese).

Appendix: Cooling Requirement

In methane-fueled engines, the ratio of coolant to that of propellant is larger compared to that of hydrogen-fueled engines, which is mainly due to the lower specific heat of methane. However, there are other parameters that may affect the ratio. The heat flux to the coolant is calculated as

$$q_c = c_{H,c} \cdot \frac{\dot{m}_c}{A_c} \cdot C_p \cdot T_w \left(1 - \frac{T_c}{T_w} \right) \quad (\text{A.1})$$

Under turbulent conditions, the Stanton number, $c_{H,c}$, is around 0.001. The heat flux from the combustion gas to the wall is expressed in a similar manner.

When the cross-section of the cooling passages is presumed to be the same in methane-fueled and hydrogen-fueled engines, the ratio of heat flux from the wall to the coolant of methane-fueled engines to that of the hydrogen-fueled engines is

$$\frac{q_{\text{CH}_4}}{q_{\text{H}_2}} \approx \frac{\dot{m}_{c,\text{CH}_4}}{\dot{m}_{c,\text{H}_2}} \cdot \frac{C_{p,\text{CH}_4}}{C_{p,\text{H}_2}} \cdot \frac{(1 - T_{\text{CH}_4}/T_w)}{(1 - T_{\text{H}_2}/T_w)} \quad (\text{A.2})$$

The heat flux from the combustion gas to the wall is calculated as

$$q_g = c_{H,g} \cdot \frac{\dot{m}_g}{A_g} \cdot C_p \cdot T_{aw} \left(1 - \frac{T_w}{T_{aw}} \right) \quad (\text{A.3})$$

The adiabatic wall temperature is almost equal to the total temperature of the combustion gas of approximately 3500 K under turbulent flow conditions in both methane-fueled engines and hydrogen-fueled engines. The ratio of the wall temperature to adiabatic wall temperature is low in both engines. When an engine size is specified, the heat flux is approximately proportional to the product of combustion gas flow rate and specific heat. When the specific heat ratio is same, the specific heat is inversely proportional to the molecular weight.

The Stanton number of the combustion gas is approximately 0.0013 for both the engines in the calculation. The heat flux ratio of methane-fueled engines to that of hydrogen-fueled engines is written as below for engines of the same size.

$$\frac{q_{g,\text{MO}}}{q_{g,\text{HO}}} = \frac{\dot{m}_{g,\text{CH}_4}}{\dot{m}_{g,\text{H}_2}} \cdot \frac{1 + (\text{O/F})_{\text{MO}}}{1 + (\text{O/F})_{\text{HO}}} \cdot \frac{MW_{g,\text{HO}}}{MW_{g,\text{MO}}} \quad (\text{A.4})$$

The propellant flow rate is inversely proportional to the specific impulse. Finally, the heat flux of methane-fueled engines is smaller than that of hydrogen-fueled engines.

Under the conditions of thermal equilibrium, Eq. (A.2) is equal to Eq. (A.4). The difference in the ratio of coolant flow rate to fuel flow rate between methane-fueled engines and hydrogen-fueled engines is expressed as

$$\begin{aligned} \frac{\dot{m}_{c,\text{CH}_4}/\dot{m}_{g,\text{CH}_4}}{\dot{m}_{c,\text{H}_2}/\dot{m}_{g,\text{H}_2}} &\approx \frac{MW_{g,\text{HO}}}{MW_{g,\text{MO}}} \cdot \frac{C_{p,\text{H}_2}}{C_{p,\text{CH}_4}} \cdot \frac{(1 - T_{\text{H}_2}/T_w)}{(1 - T_{\text{CH}_4}/T_w)} \\ &\cdot \frac{1 + (\text{O/F})_{\text{MO}}}{1 + (\text{O/F})_{\text{HO}}} \end{aligned} \quad (\text{A.5})$$

The molecular weight of combustion gas of methane-fueled engines is 21.8 g·mol⁻¹ at O/F = 3.4. The molecular weight of hydrogen-fueled engines is 13.5 g·mol⁻¹ at O/F = 6. The wall temperature is set to 900 K. The specific heats and temperatures listed in Table 1 are used for the coolant properties. Then, Eq. (A.5) is written as

$$\frac{\dot{m}_{c,\text{CH}_4}/\dot{m}_{g,\text{CH}_4}}{\dot{m}_{c,\text{H}_2}/\dot{m}_{g,\text{H}_2}} \approx 0.619 \times 2.84 \times 1.12 \times 0.629 = 1.24 \quad (\text{A.6})$$

Methane-fueled engines have a higher coolant flow ratio to the fuel flow ratio when compared to hydrogen-fueled engines.

As the engine size becomes smaller, the required coolant flow rate relative to the fuel flow rate becomes higher. The coolant flow rate is proportional to the heat transferred to the chamber and nozzle wall.

$$\dot{m}_c \propto S_g \cdot q_g \propto d_g^2 \cdot \frac{1}{2} c_{f,g} \frac{\dot{m}_g}{A_g} C_{p,g} (T_{\text{total},g} - T_w) \quad (\text{A.7})$$

Herein, the shape of the chamber and nozzle is presumed to be similar among different engines, and their lengths are pre-

sumed to be proportional to the combustion chamber diameter. When the Reynolds analogy is applied with a skin friction coefficient of the turbulent boundary layer,²⁸⁾ Eq. (A.7) is rewritten as

$$\dot{m}_c \propto d_g^2 \cdot \left(\frac{0.025}{Re_x^{1/7}} \right) \cdot \rho_g \cdot u_g \propto d_g^{13/7} \quad (\text{A.8})$$

The flow rate of fuel is proportional to that of combustion gas.

$$\dot{m}_f \propto \dot{m}_g = A_g \cdot \rho_g \cdot u_g \propto d_g^2 \quad (\text{A.9})$$

Then, the ratio of coolant flow rate to the fuel flow rate is

$$\frac{\dot{m}_c}{\dot{m}_f} \propto \frac{d_g^{13/7}}{d_g^2} = d_g^{-1/7} \quad (\text{A.10})$$

J. R. Hulka
Associate Editor