Optimization of Flat Z-stiffened Panel Subjected to Compression*

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This paper presents a method to optimize the dimensions of z-stiffened panels under a compressive load. Buckling coefficient curves for local buckling and stiffener lateral buckling are developed. The effects of transverse shear, coupling of buckling modes, and stiffeners on one side of the skin are considered in the column buckling stress estimation. An optimization tool is developed using MS-Excel. Examples of the design charts are presented and a design guideline is derived.

Key Words: Structural Optimization, Z-stiffened Panel, Local Buckling, Stiffener Lateral Buckling, Column Buckling

Nomenclature

\[
\begin{align*}
A & : \text{total cross-sectional area of z-stiffened panel per stiffener pitch} \\
A_{ST} & : \text{cross-sectional area of stiffener} \\
b_x & : \text{width of attached flange} \\
b_f \text{ or } d & : \text{width of free flange} \\
b_y & : \text{width of stiffener web} \\
E & : \text{Young’s modulus} \\
E_c & : \text{tangent modulus} \\
F & : \text{Farrar’s structural efficiency} \\
F_{b,c} & : \text{buckling stress} \\
F_{c,y} & : \text{compression yield stress} \\
F_f & : \text{failure stress} \\
G & : \text{shear modulus} \\
h & : \text{height of stiffener free flange from skin thickness centerline} \\
i & : \text{cross-sectional moment of inertia of z-stiffened panel per stiffener pitch} \\
k & : \text{correction factor of transverse shear deformation for column buckling} \\
k' & : \text{correction factor for column buckling (all effects included)} \\
k_{c,y} & : \text{buckling coefficient} \\
L & : \text{support span of z-stiffened panel} \\
n & : \text{shape parameter of Ramberg-Osgood} \\
N & : \text{compression load per width} \\
N_s & : \text{skin thickness} \\
N_y & : \text{stiffener thickness} \\
\beta & : \text{reduction factor to } k \\
\nu & : \text{Poisson’s ratio} \\
\rho & : \text{radius of gyration of area} \\
\end{align*}
\]

E: Euler
sym: symmetry

1. Introduction

Z-stiffened panels are efficient structural elements for compressive load, and have been used for the wing skins of aircraft.1 Other important merits of z-stiffened panels include ease of assembly and good accessibility for inspection. Although panels with closed section stiffeners such as Y-shaped and hat-shaped stiffeners are slightly more efficient than z-stiffened panels, they have the demerit of being difficult to inspect. A cross-section of a flat z-stiffened panel is shown in Fig. 1.

In 1949, Farrar published a paper on the optimization of z-stiffened panels under a compressive load.2 He showed a contour chart of structural efficiency in the paper. The contour chart is still used in the aircraft industry.1,3 The local buckling, stiffener torsional buckling, and column buckling are considered to be failure modes. The buckling coefficients for local buckling and stiffener torsional buckling are based on ESDU 02.01.25.4 The stiffener lateral buckling mode, which is one of the critical buckling modes, was not accounted for by Farrar.

In 1976, Williams and Stein developed a simple (approximated) buckling analysis method for open-section stiffened compression panels including a z-stiffened panel made of orthotropic material, they called it “J-stiffened.”5 They studied minimum weight panels with hat-stiffeners, blade-stiffeners and J-stiffeners made of both aluminum alloy and composite material. Rivet connection between the skin and stiffeners is not considered during buckling analysis. Consideration of the rivet connection in buckling models is important because the connection defines the constraints of the stiffeners. Design charts were not presented in the paper.

More recently, in 1998, Bushnell conducted an optimization study of riveted z-stiffened panels using the PANDA2 optimization tool.5 The dimensional imperfection of panels and post-buckling behavior were considered during the analysis, however, plasticity was not considered. Some opti-
In this study, the author developed a tool to analyze the compression buckling of z-stiffened panels, and investigated buckling and failure modes of the panels utilizing various dimensional parameters from a previous paper. It is shown that local buckling, stiffener lateral buckling and column buckling are critical buckling modes. All of the buckling modes of z-stiffened panels are shown in Fig. 2.

The purpose of this paper is to develop a tool to optimize the dimensions of a z-stiffened panel under a compressive load in order to update Farrar’s work. Typical contour charts of structural efficiency are presented.

2. Strength of Z-stiffened Panels Subjected to Compression

2.1. Structural efficiency

Farrar derived a structural efficiency equation for z-stiffened panels.\(^2\)

\[
F_f = F \sqrt{\frac{E_t N}{L}} \tag{1}
\]

Here, \(F_f\) is failure stress, \(N\) is load per width, \(E_t\) is tangent modulus, \(L\) is support span, \(F\) is Farrar’s structural efficiency factor, and \(N/L\) is structural index.

2.2. Critical failure modes and strength

An example of the buckling curves of z-stiffened panels is shown in Fig. 3. Local buckling or stiffener lateral buckling, whichever is smaller, determines the strength of a z-stiffened panel in the short column region (lower slenderness ratio). Column buckling determines the strength in the long column region (higher slenderness ratio). A transition region exists between the short column region and long column region in the test data.

The test data of NACA TN-1829\(^7\) and NACA TN-1978\(^8\) show that the buckling loads and failure loads are almost equal for the panels at a high load level, \(N/L > 1.6 \text{N/mm}^2\). This means that the buckling load can be considered as the failure load.

3. Optimization Tool

3.1. Specifications of optimization tool

Table 1 shows the specifications of the optimization tool.


Table 1. Specifications of optimization tool.

<table>
<thead>
<tr>
<th>Description</th>
<th>Considered</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local buckling</td>
<td>Considered</td>
<td>Buckling coefficient curves are developed. (Figs. 5, 6)</td>
</tr>
<tr>
<td>Stiffener lateral buckling</td>
<td>Considered</td>
<td>Buckling coefficient curves are developed. (Figs. 7–9)</td>
</tr>
<tr>
<td>Stiffener torsional buckling</td>
<td>Neglected</td>
<td>Not critical</td>
</tr>
<tr>
<td>Column buckling</td>
<td>Considered</td>
<td>Euler buckling load with the following corrections. (Eqs. (5'), (6))</td>
</tr>
<tr>
<td>Transverse shear</td>
<td>Considered</td>
<td>Engesser’s equation is used for the correction of Euler buckling.</td>
</tr>
<tr>
<td>Stiffeners on one side of skin</td>
<td>Considered</td>
<td>These effects are included in the correction factor of transverse shear effect. (Eq. (8))</td>
</tr>
<tr>
<td>Coupling of stiffener lateral buckling and column buckling</td>
<td>Considered</td>
<td>Tangent modulus theory is applied. (Eq. (9))</td>
</tr>
<tr>
<td>Wrinkling buckling</td>
<td>Neglected</td>
<td>Wrinkling bucking can be avoided with design consideration.</td>
</tr>
<tr>
<td>Inter-rivet buckling</td>
<td>Neglected</td>
<td>Inter-rivet bucking can be avoided with design consideration.</td>
</tr>
<tr>
<td>Transition from short column region to long column region</td>
<td>Neglected</td>
<td>This effect cannot be considered by analysis. Reduction factor should be considered in design.</td>
</tr>
<tr>
<td>Plasticity</td>
<td>Considered</td>
<td>This effect can be considered by analysis, but not included in the tool. Reduction factor should be considered in design.</td>
</tr>
<tr>
<td>Geometrical imperfection</td>
<td>Neglected</td>
<td>Same material is used for the skin and the stiffeners.</td>
</tr>
<tr>
<td>Materials</td>
<td>—</td>
<td>Same material is used for the skin and the stiffeners.</td>
</tr>
<tr>
<td>Width of free flange</td>
<td>—</td>
<td>$d/h = 0.4$</td>
</tr>
</tbody>
</table>

$d/h = 0.4$ is used instead of 0.3, which was used by Farrar, because the smaller $d/h$ reduces the stiffener lateral buckling stress. The tangent modulus theory is used for correcting the plasticity of all of the buckling modes, and it is a conservative assumption.

“Solver,” of MS-Excel, is used to optimize the dimensions of the z-stiffened panels.

It is necessary to calculate the buckling stresses of z-stiffened panels with a wide variety of dimensions. Design charts of the local buckling and stiffener lateral buckling coefficients are prepared and the charts are approximated applying equations to be used in the optimization tool. The approximate equations are the functions of three variables; $t_w/t_s$, $b_s/t_s$ and $h/b_s$. The artificial neural network theory is used to develop the approximate equations with good accuracy.

Column buckling loads are calculated using the Euler buckling equation with a correction factor for the transverse shear effect and the coupling of buckling modes.

3.2. Buckling coefficients

(a) Buckling analysis tool and model

Finite element analysis is not suitable to develop buckling coefficient charts because it is necessary to calculate the buckling stresses for thousands of z-stiffened panels with various dimensional parameters. For local buckling and stiffener lateral buckling, the minimum values of the buckling curves should be calculated, as shown in Fig. 3. The buckling analysis tool for thin-walled structures developed in a previous paper is used. The tool can directly calculate the minimum value of each buckling mode.

The model to analyze local buckling stresses and stiffener lateral buckling stresses is shown in Fig. 4. The width of the flange attached to the skin, $b_A$, is obtained using Eq. (2). This equation is based on the dimensions of the test panels in NACA TN-1829.

$$b_A = \frac{t_w/t_s + 8}{1.2}$$

Buckling analysis was conducted for the z-stiffened panels having the following dimensions. The Poisson’s ratio is assumed to be 0.33.

For local buckling, asymmetric:

- $b_s/t_s = 20, 25, 30, 35, 40$
- $h/b_s = 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0$
- $t_w/t_s = 0.6, 0.65, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3$

For local buckling, symmetric:

- $b_s/t_s = 20, 25, 30, 35, 40$
- $h/b_s = 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0$
- $t_w/t_s = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3$

For stiffener lateral buckling, asymmetric:

- $b_s/t_s = 20, 25, 30, 35, 40$
- $h/b_s = 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0$
- $t_w/t_s = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3$

For stiffener lateral buckling, symmetric:

- $b_s/t_s = 20, 25, 30, 35, 40$
- $h/b_s = 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0$
- $t_w/t_s = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3$

The buckling coefficients are calculated from the buckling stresses using the following equation.
\[
F_{cr} = \frac{k_{cr} \pi^2 E}{12(1 - \nu^2)} \left( \frac{t_S}{b_S} \right)^2
\]  
(3)

where, \(F_{cr}\) is buckling stress, \(k_{cr}\) is buckling coefficient, \(E\) is Young’s modulus, and \(\nu\) is Poisson’s ratio.

(b) Charts of local buckling coefficients

Figures 5 and 6 show a chart of the asymmetrical local buckling coefficient and a chart of the symmetrical local buckling coefficients, respectively. They are more accurate than the charts in NACA TN-1482.9

(c) Charts of stiffener lateral buckling coefficients

Figures 7–9 show the representative charts for the stiffener lateral buckling coefficients.

(d) Approximate equations of buckling coefficient curves

Artificial neural networks are suitable for approximating nonlinear functions with multiple variables.10 Approximate...
equations of the buckling coefficient curves are developed using artificial neural networks. The networks consist of three layers of neurons, as shown in Fig. 10. The inputs are the dimension parameters, \( t_W/t_S, b_S/t_S \), and \( h/b_S \). The output is the buckling coefficient, \( k_{cr} \). The “tanh” function is used as the transfer function for the first and second layers. A linear function is used as the transfer function for the third layer.

A MS-Excel tool was developed to calculate the biases and weights of the networks. A back-propagation algorithm is used to train the networks. The training data used are the buckling coefficients calculated in (a).

Table 2 shows the number of neurons used for the buckling modes and the accuracy of the approximate equations.

The maximum error is defined as follows:
\[
E_{\text{max}} = \max \left( \frac{k_{\text{cr,approximated},i} - k_{\text{cr},i}}{k_{\text{cr},i}} \right) \text{ for } i = 1, \text{number of data}
\]

where \( E_{\text{max}} \) is maximum error, \( k_{\text{cr,approximated},i} \) is buckling coefficient, and \( k_{\text{cr},i} \) is approximated buckling coefficient.

Examples of comparisons between the approximate equations and buckling curves are shown in Figs. 11 and 12. These figures show good accuracy for the approximate equations.

3.3. Column buckling
It is shown that the column buckling loads of z-stiffened panels are always smaller than the Euler buckling loads.6) The reasons for reduction from the Euler buckling loads are as follows:
(a) Transverse shear deformation,4)
(b) Stiffeners are positioned on one side of the panel, and
(c) Coupling of the stiffener lateral buckling and column buckling

Engesser’s equation\(^1\) shown below, is used to account for the effect (a):
\[
F_{\text{column}} = \frac{P_E}{1 + \frac{P_E}{kG A}}
\]

where \( F_{\text{column}} \) is column buckling stress, and \( P_E \) is Euler buckling load.

<table>
<thead>
<tr>
<th>Number of neurons</th>
<th>First layer</th>
<th>Second layer</th>
<th>Third layer</th>
<th>Maximum error of approximate equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local buckling, asymmetric</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>2.9%</td>
</tr>
<tr>
<td>Local buckling, symmetric</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>3.3%</td>
</tr>
<tr>
<td>Stiffener lateral buckling, asymmetric</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>1.3%</td>
</tr>
<tr>
<td>Stiffener lateral buckling, symmetric</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

Fig. 9. Stiffener lateral buckling coefficients, \( b_S/t_S = 40 \).

Fig. 10. Artificial neural network for approximate equations of buckling coefficients.
The following equation of $k$ is derived using the energy method.

$$
\frac{1}{k} = \frac{t_w A}{E I} \left[ \begin{array}{l}
(h - z_{CG})^2 b_F^2 h \\
+ b_F (h - z_{CG}) \left\{ \frac{1}{3} (h - z_{CG})^3 + \frac{1}{3} z_{CG}^3 \right\} \\
+ \frac{1}{4} \left\{ (h - z_{CG})^4 h - \frac{7}{15} (h - z_{CG})^5 \right\} \\
+ \frac{2}{3} (h - z_{CG})^3 z_{CG}^3 + \frac{1}{5} z_{CG}^5 \end{array} \right]
$$

(7)

$z_{CG}$ : height of neutral axis from center of skin thickness.

The equation developed by Bijlaard(12) can be used to account for effect (b), but the effect is found to be very small. Effect (c) has the same magnitude as effect (a) and no equations are available to account for effect (c). An easy and useful method to account for effect (c) is to modify $k$ in Eq. (5). This can also include effect (b). The modified correction factor $k'$ is expressed as follows using reduction factor $\beta$.

$$
k' = \beta k
$$

(8)

$$
F_{\text{column}} = \frac{P_E}{1 + \frac{P_E}{k' G A}} A
$$

(5')

Buckling curves for some representative panels (34 panels) were produced to find $P_E$ and $F_{\text{column}}$ shown in Fig. 13. Then, $k'$ for the representative panels is calculated from Eq. (5’). It is found that the range of $\beta$ is 0.3 to 0.5 for all the representative panels. $\beta = 0.4$ is used in the optimization tool.
3.4. Correction of plasticity

The tangent modulus theory is used to account for the plasticity of the material. Tangent modulus $E_t$ is used in Eqs. (3) and (6) instead of $E$. $E_t$ is calculated from Eq. (9):

$$E_t = \left( \frac{3}{0.002 \times 7E} \right)^\frac{1}{n-1} F_{cy}\frac{a}{n-1}$$  \hspace{1cm} (9)

where $E_t$ is tangent modulus, $n$ is shape parameter of Ramberg-Osgood, and $F_{cy}$ is compressive yield stress.

3.5. Optimization of panel dimensions

The optimization tool is a MS-Excel worksheet. When a load is applied per unit width, a support span and material properties are given, and the tool finds a set of panel dimensions that minimizes the weight of the panel. “Solver” is used for optimization. The flowchart of the tool is shown in Fig. 14.

4. Design Charts for Optimized Panels

Design charts of z-stiffened panels were developed as shown in Figs. 15–17. The charts are contour maps of the structural efficiency, with $F$ in the $h/b_s-t_{W}/t_{S}$ plane. The material and its properties are assumed as follows.
Material: 7075-T6 aluminum alloy
Young’s modulus: \( E = 72400 \) MPa
Poisson’s ratio: \( v = 0.33 \)
Compressive yield stress: \( F_{cy} = 500 \) MPa
Ramberg-Osgood shape parameter: \( n = 25 \)

The following features are observed in Figs. 15–17.

- The efficiency contours vary in shape with the magnitude of the structural index, \( N/L \). This is because stiffener lateral buckling is heavily dependent on \( b_S/t_S \), while local buckling is basically not dependent on \( b_S/t_S \). This is the major difference from Farrar’s contour chart, Fig. 18.
- Maximum efficiency also varies with the structural index, \( F = 0.945 \) to 0.829, corresponding to \( N/L = 1.6 \) to 4.0 N/mm\(^2\). According to Farrar, \( F = 0.95 \).
- The boundaries of the critical buckling modes are also shown in Figs. 15–17. Asymmetrical local buckling is critical in the small \( h/b_S \) region, while stiffener lateral buckling is critical in the large \( h/b_S \) region. The stiffener asymmetrical lateral buckling mode moves to the smaller \( h/b_S \) region as \( N/L \) increases. As \( N/L \) increases, the structural efficiency drops.
- Symmetrical local buckling and stiffener symmetrical lateral buckling are critical in the small \( t_S/t_S \) region.
- Contours of \( b_S/t_S \) are shown in Figs. 15–17. Larger \( b_S/t_S \) is required for a larger structural index, \( N/L \).

Figures 19–21 are another representation of the contour maps in the \( AST = (b_S/t_S) - t_S/t_S \) plane (i.e., \( AST \) is the cross-sectional area of a stiffener). They are compared to the contour chart by Farrar (Fig. 18). The contour chart by Farrar is very similar to Fig. 19 which is the contour for the low structural index (\( N/L = 1.6 \) N/mm\(^2\)). As mentioned earlier, Farrar’s chart does not depend on the structural index because lateral buckling is not considered. According to Farrar, the optimum design point is at \( AST = (b_S/t_S) = 1.5 \), while the present
analysis (Figs. 19–21) shows the optimum design points are much smaller $A_{ST}/(b_{ST}t_{S}) = 1.2$ (smaller stiffener height).

### 4.1. Buckling curves of representative panels

The representative optimized panels shown in Figs. 16 and 17 (points A1 to A5, and B1) are analyzed by the buckling analysis tool in order to check the accuracy of the optimization tool. These points are selected for the following features.

Point A1: Optimum design point, asymmetric local buckling and asymmetric stiffener lateral buckling critical

Point A2: Asymmetric stiffener lateral buckling critical

Point A3: Symmetric stiffener lateral buckling critical

Point A4: Asymmetric local buckling critical

Point A5: Symmetric local buckling critical

Point B1: Optimum design point, Asymmetric stiffener lateral buckling critical

The results are shown in Figs. 22–27. The buckling stresses and modes by the buckling analysis tool and those by the optimization tool are in good agreement. The column buckling curves in the optimization tool (Eq. (5) with $\beta = 0.4$) are good approximations for buckling analysis in the large slenderness ratio.

At the optimum design point for $N/L = 2.5 \text{ N/mm}^2$ (Point A1), the local buckling, stiffener buckling and column
buckling occur simultaneously. At the optimum design point for $N/L = 4.0 \text{N/mm}^2$ (Point B1), the stiffener buckling and column buckling occur simultaneously, but local buckling does not occur because the stiffener lateral buckling coefficients are dependent on $b_S/t_S$ and become very small in small $b_S/t_S$ (see Figs. 7–9).

### 4.2. Design guideline

The design guideline for z-stiffened panels can be derived from the charts as follows.

- Stiffener thickness/skin thickness ratio $t_w/t_S$ should be 0.8 to 1.2 in order to avoid symmetrical local buckling.
- This also precludes wrinkling buckling. Maximum struc-
tural efficiency is obtained at $t_W/t_S \approx 1.0$.

- Structural efficiency increases as stiffener height/stiffener pitch ratio $h/t_S$ increases. $h/t_S$ would be determined by not only the structural efficiency but also design considerations for manufacturing cost, shear tie installation, etc.

5. Conclusions

(1) Buckling coefficient charts for local buckling and stiffener lateral buckling of z-stiffened panels were developed.

(2) Approximation equations based on the artificial neural network theory were developed for the buckling coefficients of local buckling and stiffener lateral buckling. The maximum errors of the equations were less than 3.4%.

(3) An optimization tool for z-stiffened panels was developed. The tool accounts for all critical buckling modes including stiffener lateral buckling.

(4) Design charts for z-stiffened panels were developed and a design guideline was derived.

References

3) Initial Buckling Stress of Flat Panels with Z-section Stringers under Compression, ESDU, 02.01.25, 1947.

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