Feasibility Study on Real-time Observation of Flow Velocity Field using Sparse Processing Particle Image Velocimetry*

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1. Introduction

Active flow control such as the use of a plasma actuator has been gathering much attention. Its effectiveness in flow separation control has been investigated experimentally and numerically.¹⁾ However, the capability during high-speed airflow is limited due to the lack of the flow control effect. Hence, feedback control utilizing the real-time measurement of the flow state is expected to improve applicability.²⁾ Because of the complexity and nonlinearity of the flow phenomena, feedback control based on not the local flow information, but the full-state or global flow information clearly appears to be better for the future feedback control of flows.

The use of particle image velocimetry (PIV), which provides the instantaneous velocity field in laboratory measurement can be used for nearly full-state observation. Therefore, real-time PIV measurement of the flow field seems to be a powerful tool for flow control. The velocity field is calculated from the cross-correlation coefficient for each interrogation window of the particle images during the PIV measurement, but the number of windows that can be processed in a short duration is limited. This is because the PIV computational time is too long when real-time PIV measurement is applied to aerodynamic flow-control experiments, which have a shorter time scale than hydrodynamic experiments.

In this study, reduced-order modeling is employed and reducing the calculation time is considered. The authors proposed sparse processing PIV (SPPIV) as a method to achieve the real-time nearly full-state estimation. The PIV measurement of the flow field around a NACA0015 airfoil model was conducted and the flow field obtained using SPPIV and the processing time were evaluated.

2. SPPIV

The method of performing PIV using particle images at a limited number of observation points and estimating the flow field from the sparse velocity vectors obtained is called SPPIV. This is one of the applications of data-driven field reconstruction using the sparse sensors of previous studies,^{3–5)} but the application to PIV is an advanced point in the present study. Figure 1 shows a schematic diagram of creating training-

Fig. 1. Schematic diagram of creating training data for SPPIV.

Fig. 2. Flowchart of SPPIV.

ing data used for SPPIV, and Fig. 2 illustrates a flowchart of SPPIV. The data decomposition method is described in Section 2.1, the sparse sensor selection method used in SPPIV is described in Section 2.2, and the flow field estimation method is described in Section 2.3.

2.1. Proper orthogonal decomposition

Proper orthogonal decomposition (POD) is a method to find means to expand data most efficiently. Using this method, low-dimensional components can be extracted from multidimensional data. The means obtained using POD is called the POD mode. When POD can be applied to fluid analysis, it is possible to reduce the dimensions of flow field data obtained using PIV measurement and to extract important flow field phenomena.⁶⁾

We consider the data matrix of training data $X \in \mathbb{R}^{n \times m}$ ($n > m$), which consists of $X_1, \dots, X_s$, where n , m and s are the number of spatial dimension, the number of temporal dimension and the number of components of the measurement vector, respectively. Because SPPIV is applied to the velocity field which has u and v components ($s = 2$), the data matrix becomes $X = [X_1^T \ X_2^T]^T$, where $X_1 \in \mathbb{R}^{\frac{n}{2} \times m}$ and $X_2 \in \mathbb{R}^{\frac{n}{2} \times m}$ are data matrices of the u and v components, respectively. The data matrix X can be decomposed using POD in the following equation:

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \approx \mathbf{U}_r\mathbf{\Sigma}_r\mathbf{V}_r^T = \mathbf{U}_r\mathbf{Z}. \quad (1)$$

Here, columns of $\mathbf{U} \in \mathbb{R}^{n \times m}$ and $\mathbf{V} \in \mathbb{R}^{m \times m}$ are the spatial and temporal POD modes, respectively, and diagonal entries of $\mathbf{\Sigma} \in \mathbb{R}^{m \times m}$ are the POD mode amplitudes. In addition, $\mathbf{U}_r \in \mathbb{R}^{n \times r}$, $\mathbf{\Sigma}_r \in \mathbb{R}^{r \times r}$ and $\mathbf{V}_r \in \mathbb{R}^{m \times r}$ are the truncated $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$, and $\mathbf{Z}$ is the POD mode amplitude matrix ($\mathbf{Z} = \mathbf{\Sigma}_r\mathbf{V}_r^T$), where r is the number of truncated POD modes.

2.2. Greedy sensor selection method

In this research, the greedy sensor selection method in the vector field proposed by Saito et al.^{4,5)} was utilized, and the sensor position was determined. The idea here is expressed using the following equation:

$$\mathbf{y} = \mathbf{H}_s\mathbf{U}_r\mathbf{z} = \mathbf{C}\mathbf{z}. \quad (2)$$

Here, p , $\mathbf{y} \in \mathbb{R}^{sp}$, $\mathbf{H}_s \in \mathbb{R}^{sp \times n}$, $\mathbf{C} \in \mathbb{R}^{sp \times r}$ and $\mathbf{z} \in \mathbb{R}^r$ are the number of sensors, the observation vector, the sparse sensor location matrix for the vector measurement, the measurement matrix ($\mathbf{C} = \mathbf{H}_s\mathbf{U}_r$) and the temporal POD mode amplitude vector ($\mathbf{Z} = [\mathbf{z}_1 \ \mathbf{z}_2 \ \cdots \ \mathbf{z}_N]$), respectively. The element corresponding to the sensor location is unity and the others are 0 in each row of $\mathbf{H}_s$. This method selects the sensor location highly sensitive to the POD modes that are reconstructed in this study. First, we conduct the PIV measurement as the training data. Then, the POD modes of the training data are calculated and the optimized sensor locations are determined using the greedy vector-sensor selection method. Refer to Saito et al.^{4,5)} for a detailed explanation.

2.3. Kalman filter

The Kalman filter is the method used to calculate the state vector of a linear stochastic system based on observed data sequentially when system and observation equations are assumed to be linear. This method allows us to obtain the least square estimate from the noisy data. It consists of a correction step and a prediction step. In this study, the system matrix is determined using Eq. (4)⁷⁾

$$\mathbf{Z}_{m-1} = [\mathbf{z}_1 \ \mathbf{z}_2 \ \cdots \ \mathbf{z}_{m-2} \ \mathbf{z}_{m-1}], \quad (3)$$

$$\mathbf{Z}_m = [\mathbf{z}_2 \ \mathbf{z}_3 \ \cdots \ \mathbf{z}_{m-1} \ \mathbf{z}_m], \quad (4)$$

$$\mathbf{F} = \mathbf{Z}_m(\mathbf{Z}_{m-1})^+. \quad (5)$$

Here, A^+ represents the Moore-Penrose pseudoinverse of A . The estimated and observed variables are expressed as Eqs. (5) and (6), respectively.

$$\mathbf{z}_{k+1|k} = \mathbf{F}\mathbf{z}_{k|k} + \mathbf{v}_k, \quad (6)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{z}_k + \mathbf{w}_k, \quad (7)$$

where $\mathbf{F} \in \mathbb{R}^{r \times r}$, $\mathbf{v} \in \mathbb{R}^r$ and $\mathbf{w} \in \mathbb{R}^{sp}$ are the system matrix, the system noise and observed noise, respectively. Here, $A_{k|k}$ represents the estimation of A at the k th step obtained using the information up to k th step, and $A_{k+1|k}$ represents the estimation of A at the $(k+1)$ th step obtained using the information up to k th step, where A is an arbitrary variable. Then, the prediction step is expressed as the following equations:

$$\hat{\mathbf{z}}_{k+1|k} = \mathbf{F}\hat{\mathbf{z}}_{k|k}, \quad (8)$$

$$\mathbf{P}_{k+1|k} = \mathbf{F}\mathbf{P}_{k|k}\mathbf{F}^T + \mathbf{Q},$$

where $\mathbf{P} \in \mathbb{R}^{r \times r}$ and $\mathbf{Q} \in \mathbb{R}^{r \times r}$ are the error covariance ma-

trix and the noise covariance matrix, respectively. The correction step can be expressed using the following equations:

$$\mathbf{K}_n = \mathbf{P}_{k|k-1}\mathbf{C}_n^T(\mathbf{C}_k\mathbf{P}_{k|k-1}\mathbf{C}_k^T + \mathbf{R})^{-1}, \quad (9)$$

$$\hat{\mathbf{z}}_{k|k} = \hat{\mathbf{z}}_{k|k-1} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{C}_k\hat{\mathbf{z}}_{k|k-1}), \quad (10)$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{C}_k\mathbf{P}_{k|k-1}, \quad (11)$$

where $\mathbf{K} \in \mathbb{R}^{r \times sp}$ and $\mathbf{R} \in \mathbb{R}^{sp \times sp}$ are the Kalman gain and the noise covariance matrix, respectively. Assuming that the estimated and observed variables are $\mathbf{z}$ and $\mathbf{y}$, respectively, the noise covariance matrices are calculated using Eq. (12) from the noise at each step obtained from the training data using Eqs. (5) and (6).

$$Q_{ij} = \frac{1}{m-1} \sum_{k=1}^{m-1} v_{k_i} v_{k_j} \delta_{ij}, \quad R_{ij} = \frac{1}{m} \sum_{k=1}^m w_{k_i} w_{k_j} \delta_{ij} \quad (12)$$

where v_{k_i} and w_{k_i} denote the i th components of $\mathbf{v}_k$ and $\mathbf{w}_k$, respectively, and δ_{ij} is the Kronecker delta. In this study, the initial values of $\hat{\mathbf{z}}$ and $\mathbf{P}$ are set to be the zero vector and the identity matrix, respectively.

3. Test Conditions and PIV Measurement

3.1. Wind tunnel

Figure 3 shows a schematic diagram of the experimental equipment used in this study. The wind tunnel testing was conducted in the Tohoku University Basic Aerodynamic Research Wind Tunnel (T-BART) with a closed test section having a cross-section of 300 mm $\times$ 300 mm. The airfoil of the test model had an NACA0015 profile, the chord length and span width of which were 100 mm and 300 mm, respectively. The freestream velocity U_∞ and the angle-of-attack α were set to be 10 m/s and 16 deg, respectively. The chord Reynolds number was 6.6×10^4 . The angle-of-attack of 16 deg was set for the present study because the completely separated flow, which is our target, appears at this angle.

3.2. PIV measurement

Time-resolved PIV measurement was conducted using a double-pulse laser for acquiring time-resolved data. The time between pulses, the sampling rate, the particle image resolution, and the total number of image pairs were 75 μ s, 2000 Hz, 256 $\times$ 512 pixels, and 1000 ($= m$), respectively. The tracer particles were dioctyl sebacate. The PIV measurement system used in this study consists of a high-speed camera (IDP-Express R2000, Photron), a double-pulse Nd: YLF laser (LDY-303PIV, Litron Lasers), and a function generator (WF1948, NF). The laser light source was installed on the

Fig. 3. Experimental setup.

Table 1. Computational environments.

Processor information	Intel(R) Core(TM) i5-6200U CPU@2.30 GHz
Random access memory	8.0 GB
Program code	MATLAB R2013a

starboard side of the wind tunnel test section as shown in Fig. 3, and irradiated the laser sheet in a direction parallel to the floor.

3.3. Analysis condition

Two-thousand particle images acquired in this experiment were used for training data. POD was performed on the flow field obtained using PIV with these images. The sparse sensor location matrix was obtained using the greedy method with the first to tenth modes among the obtained POD modes. After that, the covariance matrix of the noise was estimated using the training data. The flow field was estimated using SPPIV for particle images different from the training data, and compared with the POD mode time-series data obtained using regular PIV. In addition, the time required for sequential processing in each program was measured. Table 1 summarizes the specifications of the computer used in this study.

4. Results and Discussion

4.1. Reconstructed flow field

Figure 4 shows examples of the velocity fields of u -direction obtained using PIV with 2000 particle images different from training data, reconstructed using only 10 POD modes and obtained using SPPIV. The black circles are the observation points determined using the greedy method, and they are adopted for SPPIV. The POD mode of the flow fields obtained using SPPIV is compared with that obtained using regular PIV. Figure 5 shows the time histories of the POD-mode amplitude obtained using regular PIV and SPPIV in the first and second modes as examples of the low-order mode. Here, the time histories of the POD modes of regular PIV are mapped to the spatial mode of training data used by SPPIV giving Eq. (13):

$$\mathbf{Z} = \mathbf{U}_r^T \mathbf{X}. \quad (13)$$

Figure 4 shows that SPPIV can reproduce relatively large fluctuations near the boundary layer between the acceleration region and the recirculation region. On the other hand, in this study, the variation of the recirculation region is much smaller than that of the shear layer in the freestream-direction component of the modes up to the tenth mode, although it is not shown for brevity. This might be because the flow variation is not strong in the recirculation region and the fluctuations in that region are not modeled in the present reduced-order model. The error in the case of $r = 10$ is calculated using Eq. (14).

$$\epsilon = \frac{\sqrt{\sum_{i=1}^r \sum_{j=1}^m \{Z(i, j) - \hat{Z}(i, j)\}^2}}{\sqrt{\sum_{i=1}^r \sum_{j=1}^m Z(i, j)^2}} \quad (14)$$

Here, SPPIV can capture the fluctuation in the first mode ob-

Fig. 4. Velocity fields of u -direction obtained using regular PIV and SPPIV. Open black circles represent sensors.

Fig. 5. Time histories of mode amplitude.

tained using PIV, as shown in Fig. 5. On the other hand, fluctuations in second or higher modes (the latter of which are not shown for brevity) are not captured well. This might be because the signal obtained using regular PIV is also contaminated by a low signal-to-noise (SN) ratio. Therefore, it is necessary to improve the SN ratio, at least to improve accuracy even for the regular PIV. This point has already been confirmed through ongoing experiments, but it is not shown here for brevity. Figure 6 shows the relationship between the error in Eq. (14), the processing time and the number of sensors in POD mode $r = 10$. The error decreases as the number of sensors increases. Therefore, increasing the number of sensors may lead to an improvement in estimation accuracy.

When applying this method to control, it is necessary to estimate the flow with control or flow around the airfoil at different angles of attack. In that case, the training data of flow under various conditions are required for the estimation, and the estimation of the flow without training data should be conducted based on the training data of similar conditions. This point is beyond the scope of this paper, and investiga-

Fig. 6. Error and processing time vs. number of sensors. Error bars show standard deviations.

tion of the estimation ability of flow without training data and its application to flow control are left for a future study.

4.2. Computational cost

Table 2 summarizes the processing time per step and the number of calculation points for each method, respectively. The processing time is the average of five processes, and the processing time per step is calculated by dividing the processing time of 1000 steps by the number of steps 1000.

Table 2 shows that SPPIV with five sensors can construct a flow field within a processing time that is approximately 1/126 shorter than the regular PIV. In addition, the number of calculation areas for SPPIV is approximately 1/128 of that for PIV in that case. Figure 6 shows the relationship between the processing time and the number of sensors. The processing time increases in proportion to the number of sensors. Note that the intercept of the approximated line of the processing-time y axis in Fig. 6 is considered to be the computational cost of the Kalman filter. Figure 6 indicates that the improvement in estimation accuracy and the increase in calculation cost due to the increase in the number of sensors are in a trade-off relationship; therefore, the number of sensors needs to be appropriately determined according to the requirements. Fluctuations up to the Strouhal number St of 10 are targeted for future flow control using SPPIV in the present setup. Thus, the flow field should be estimated at a sampling rate of the Strouhal number St of approximately 20 to 100, which is a sampling rate that is twice to 10 times higher than targeted fluctuations. Here, $St = 1$ in the present setup corresponds to 100 Hz considering $U = 10$ m/s and a chord length of 0.1 m. When the sampling rate of SPPIV is 2000 Hz, the Strouhal number of the sampling rate is $St = 20$. This indicates that real-time control might be possible under the same conditions as those in the present study, but a further increase in speed is also required for more resolved real-time observation. When real-time measurement is performed under the same conditions as the test conditions, one pair of particle images can be obtained every 1/2000 s since the imaging speed is 4000 fps. Therefore, it is necessary to complete the processes of one step within a duration that is one-fourth that in the current implementation when the number of sensors is five in this study. In addition, processes such as data transfer are required for real-time measurement of the flow field. Because the calculation program in this research was coded in MATLAB without parallelization for simplicity of the feasibility study, the faster real-time implementation

Table 2. Computational cost.

	Processing time [ms/step]	The number of interrogation windows
Regular PIV	259	641
SPPIV	$p = 5$	5
	$p = 8$	8
	$p = 10$	10
	$p = 13$	13
	$p = 15$	15

discussed above can be realized by employing a faster programming language such as C++, parallelization, etc.

5. Conclusions

In this study, a method for the real-time SPPIV measurement of flow fields was proposed, and the reproducibility of a flow field and the processing speed of SPPIV were evaluated. Regarding the reproducibility of the flow field estimated using SPPIV obtained in this study, the fluctuation can be reproduced in the low-dimensional mode, but not in the higher-order mode. Improving the SN ratio of the velocity field, which is the result of PIV, is considered to be necessary to increase reproducibility. Meanwhile, SPPIV reduced the processing time to 1/126 that of regular PIV, whereas the reduction ratio in processing time is approximately proportional to that in the number of calculation areas. It is necessary to increase the processing speed by at least four times to enable online measurement under the same test conditions as in this study.

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