On-board Downwash Speed Estimation of Drone Rotor-Based State Observer*

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Drone dynamics are affected by flight condition uncertainties, such as the surrounding structure, gust disturbance, and rotor damage, resulting in the deterioration of flight performance and safety. Real-time modeling of drone dynamics plays an important role in flight safety. In this study, the on-board measurement of downwash speed, which is an important parameter in drone dynamics, was performed using ultrasonic passing-time information. In this method, ultrasonic transmitters and receivers were arranged below the rotor, and the passing time in the downwash was estimated using an extended Kalman filter, which was designed for the constructed ultrasonic propagation dynamics model. It was validated that the proposed method is sufficiently robust to accurately estimate the downwash speed in a disturbed flow under a rotating rotor.

Key Words: Downwash Speed, Drone, State Observer, Time Delay Estimation

1. Introduction

Drone dynamics vary according to the surrounding conditions, such as walls, ground,1,2 and gust disturbance. Rotor damage also has a fatal effect on drone dynamics. Real-time modeling of drone dynamics enhances control capabilities in unexpected conditions. The fault detection of drones has been attempted to ensure flight safety.3 Usually, responses of drone dynamics, which consist of an actuator, aerodynamics of airframe, and sensory system, are measured, and variations from nominal responses are detected using model-based or data-driven methods.4,5 In Liu et al.6 and Rangel-Magdaleno et al.,7 audio data were measured and applied to detect damage to and imbalance in the rotor blade, respectively. Flight safety will be improved using more detailed information on varied dynamics.

In a previous study, the authors constructed a theoretical aerodynamic model of a drone and tested its application.8 The aerodynamic force and moment of the drone are generated mainly by rotors. Downwash speed is an important parameter of rotor thrust in momentum theory. Fukuda et al.9 showed that gust disturbance affects the downwash speed of the drone rotor. Rotor damage directly affects rotor downwash. Estimating the rotor downwash speed is essential for analyzing drone dynamics.

The wave equation of sound indicates that the waveform propagates at the actual sound speed, which is the sum of the sound speed in the static medium and the tangential component of the medium speed along the path. Therefore, the downwash speed can be estimated using the amount of time it takes for sound to pass along the path below the rotor. Ultrasonic anemometers perform the non-contact measurement of wind speed by measuring the amount of time it takes for ultrasonic waves to pass using a threshold value or waveform information. However, because the downwash below the rotor contains a large disturbance, which causes an irregular variation in the waveform, it is difficult to detect wind speed using the conventional ultrasonic method.

If the propagation model formulating the relation between the input to the ultrasonic transmitter and ultrasonic receiver output is constructed using measured data, the medium speed can be determined from the time delay in the propagation model, which corresponds to the amount of time that is required for ultrasonic waves to pass. State-space realization using the Hankel matrix of Markov parameters can be performed using the impulse response.10 The time delay is represented in the Hankel matrix as elements close to zero. In Lima and Barros,11 this method was modified to estimate time delay using a combination of pulse signal inputs. Chen and Zhang12 determined the time delay in the Auto-Regressive Moving Average (ARMA) model to minimize the cost function. In Zhao and Sagara,13 the time delay in the ARMA model was determined so that the cost function was minimized using Newton’s method. Fukuda et al.19 reported a method for estimating the time delay in the measured ultrasonic receiver outputs using cross-correlation. For the on-board estimation of drone downwash speed, a recursive algorithm with a lower calculating load is desired.

In this study, a non-contact estimation method for downwash speed using ultrasonic waves is proposed. The propagation dynamics of ultrasonic waves are modeled experimentally in a simple form, and then applied to the newly developed time delay estimation algorithm based on the state observer. The downwash speed is calculated using the estimated time delay. The downwash speed estimation was performed and the level of precision was evaluated.

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2. Propagation Model of Ultrasonic Waves

Small lightweight ultrasonic transmitters and ultrasonic receivers are available in the market. These devices are designed such that the resonance frequency of the structure is equal to the desired ultrasonic frequency. Consequently, the amplitudes of the ultrasonic waves generated by the transmitter and receiver output increased. However, delay times exist in the transmitter and receiver dynamics, in addition to the time required for passing through the air. Therefore, the fundamental responses of the ultrasonic receiver to the output from the transmitter were analyzed, and response models were utilized to estimate the time required for the ultrasonic waves to pass. The input to the transmitter is a frequency-sweep square wave with a frequency range of 10 Hz–10 kHz, which is used in the modeling of propagation dynamics and downwash estimation. The ultrasonic transmitter and receiver used were UTI1612MPR and UR1612MPR of SPL (Hong Kong) Limited, respectively. The input to the transmitter and output from the receiver were measured using a NR-600 (Keyence Corporation) with a sampling frequency of 500,000 Hz. The receiver outputs were measured using the variation in the distance between the transmitter and receivers at 0.01, 0.02, ..., and 0.16 m. Figure 1 shows the time histories of the frequency sweep input to the ultrasonic transmitter and output from the ultrasonic receiver. The amplitude of the oscillating ultrasonic receiver output increases after ultrasonic waves arrive at the receiver and then decreases gradually. The receiver outputs at different distances have similar figures. A time delay can be observed between the input to the transmitter and the receiver response. These lags are derived from the time required to pass through the air and the internal dynamics of the transmitter and receiver. The power spectrum densities of the receiver output are shown in Fig. 2. It can be seen that only one peak exists at a frequency of 40 kHz.

The relations between the receiver responses at different distances from the transmitter, which have a similar figure with a single frequency, were modeled as a simple function under some assumptions. Here, the following are defined: the input to the transmitter, \(u(t)\); receiver output with a distance of \(l\) from the transmitter, \(y(l)\); ultrasonic output of the transmitter, \(y(t) = f_T(u)\); the receiver dynamics, \(G_S\); and propagation dynamics of ultrasonic waves in air with a distance of \(l\), \(G_{air}\). It is assumed that the propagation and receiver dynamics can be formulated as a transfer function that satisfies a linear relation. Additionally, the following linear relations are assumed:

\[
G_{air(a+b)} = G_{air(a)}G_{air(b)} \tag{1}
\]

\[
G_SG_{air(l)} = G_{air(l)}G_S \tag{2}
\]

If these relations are satisfied, the relations between the receiver outputs can be formulated as follows:

\[
y(a) = G_{air(a)}G_Sf_T(u_0) = G_{air(a)}G_Sf_T(u_0) \tag{3}
\]

\[
y(b) = G_{air(b)}f_T(u_0) = G_{air(b)}G_Sf_T(u_0) = G_{air(b-a)}G_{air(a)}f_T(u_0) = G_{air(b-a)}y(a) \tag{4}
\]

Equation (4) indicates that the relation of the receiver outputs can be formulated as a transfer function with a parameter of the distance between receivers. This relation, which is derived using some assumptions, was validated using experimental results.

The relation between \(y(l)\) and \(y(b)\) \((i = 2, \ldots, 16, l_1 = 0.01, l_2 = 0.02, \ldots, l_{16} = 0.16\) m) is expressed as follows:

\[
y(l) = G_{air(l)}y(1) \tag{5}
\]

The time delay in the propagation dynamics \(G_{air(l)}\) is equal to the time required to pass a distance \(l - l_1\). Therefore, the propagation dynamics \(G_{air}\) is expressed as a time delay and other terms.

\[
G_{air(l-l_1)} = e^{-\tau/s}G(s) \tag{6}
\]

\(\tau\) is \((l - l_1)/v_s\) and \(v_s\) is sound speed.

The assumed propagation dynamics in Eq. (6) were identified using the measured receiver output data at distances \(l_2, \ldots, l_{16}\), where \(G(s)\) was identified as the third-order strict proper rational transfer function form. Figure 3 shows the
measured and simulated receiver outputs using the identified model at a distance $l_b$. The receiver responses were precisely modeled using the identified model. Notably, $y^{(i)}$ contains only a single frequency component (Fig. 2). Therefore, $G_i(s)$ can be approximated as $|G_i(j\omega_0)|e^{j\omega_0 t}$ and $\Delta \tau_i = \angle G_i(j\omega_0)/i\omega_0$ in the formulation of the responses to the specific input used. Here, $\omega_0 = 2\pi \times 40000$ rad/s is the dominant peak angular frequency of the receiver output. The modeling precision was evaluated using the following cost function.

$$J_i = \sqrt{\frac{1}{T} \int_0^T (y^{(i)} - \hat{y}^{(i)})^2 dt} / \sqrt{\frac{1}{T} \int_0^T y^{(i)} \bar{y}^{(i)} dt}$$  \hspace{1cm} (7)$$

$y^{(i)}$ is the simulated receiver output by the identified model $e^{-\tau_i s}G_i(s)$ or $|G_i(j\omega_0)|e^{-(\tau_i - \Delta \tau_i )s}$. The theoretical time required to pass distance $l_i - l_j$, $\tau_i$, variation of time delay $\Delta \tau_i$, and modeling error $J_i$ are listed in Table 1. The variation in time delay is sufficiently smaller than the theoretical time required to pass, and the precision of both identified models is almost equal. These results indicate that the assumed relation in Eq. (5) is approximately satisfied, and the propagation dynamics $G_i(s)$ can be approximated in the form of $Ke^{-\tau_i}$. In this study, time delay $\tau$ in propagation dynamics $Ke^{-\tau}$ is estimated using an observer-based method.

### 3. Estimation Method for Time Delay

The pure time delay is formulated using a discrete time system as follows:

$$x[k + 1] = A_mx[k] + B_mu[k]$$  \hspace{1cm} (8)$$

$$A_m = \begin{bmatrix} 0 & \cdots & 0 \\ 1 & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{m \times m},$$  \hspace{1cm} (9)$$

$$B_m = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{m \times 1}$$

$$x[k] = [x_1[k] \ x_2[k] \ \cdots \ x_m[k]]^T$$  \hspace{1cm} (10)$$

$$y[k] = x[l], \ 1 \leq l \leq m$$  \hspace{1cm} (11)$$

where output $y[k]$ is the delayed input of the amount of time that has passed, $\Delta \tau \times l$. $\Delta \tau$ is the sampling period. It is difficult to estimate the time delay using the state observer-based method with this state-space model because it is expressed as the structure of the output equation in Eq. (11). Therefore, a new state equation is formulated expressing the time delay as the state variable for applying the state observer.
Here, a window function is introduced to extract the appropriate delayed input.

\[ g(z, a) = \exp\left(\frac{-M(z - a)^2}{a}ight), \quad 0 \leq a \leq 1, \quad M > 0 \]  

(12)

\[ F_m(x[k], C, a) = \sum_{i=1}^{m} C \left\{ \frac{g\left(\frac{i}{m}, a\right)}{x[i]} \right\} \]  

(13)

The window function \( g(z, a) \) becomes the largest value \( g = 1 \) at \( z = a \), and \( g \cong 0 \) for \( z \neq a \) with \( M \ll 1 \). \( a \) is the parameter, which determines the extracted point. The function \( F_m \) is defined by applying the window function \( g \) to the discrete time data with a parameter of gain \( C \). \( F_m \) outputs the weighted state-space vector \( x \), where the weight for \( x_i, i \in am \) is the largest value. Introducing \( a \) and \( C \) into the state-space vector, the delayed time model is formulated as follows:

\[ x[k + 1] = Ax[k] + Bu[a] + Bv[k] \]  

(14)

\[ y[k] = h(x[k]) + w[k] \]  

(15)

\[ h(x[k]) = F_m(x[k], x_{m+2}[k], \phi(x_{m+1}[k])) \]  

(16)

\[ x[k] = [x_1[k], x_2[k], \ldots, x_m[k], x_{m+1}[k], x_{m+2}[k]]^T \]  

(17)

\[ \phi(b) = \frac{\phi_{\text{max}} - \phi_{\text{min}}}{2} \tanh b + \frac{\phi_{\text{max}} + \phi_{\text{min}}}{2} \]  

(18)

\[ A = \begin{bmatrix} A_m & 0 \\ 0 & A_d \end{bmatrix}, \quad B_u = \begin{bmatrix} B_m \\ 0 \end{bmatrix}, \quad B_v = \begin{bmatrix} 0 \\ B_d \end{bmatrix} \]  

(19)

\[ A_d = \begin{bmatrix} e^{-\frac{\pi}{2T_1}} & 0 \\ 0 & e^{-\frac{\pi}{2T_2}} \end{bmatrix}, \quad B_d = \begin{bmatrix} K_{d_1}(1 - e^{-\frac{\pi}{2T_1}}) & 0 \\ 0 & K_{d_2}(1 - e^{-\frac{\pi}{2T_2}}) \end{bmatrix} \]  

(20)

The state vector contains the time-shifted input \( x_1[k] = u[k - 1], x_2[k] = u[k - 2], \ldots, x_m[k] = u[k - m] \), and window function parameters \( x_{m+1}[k] \) and \( x_{m+2}[k] \). The window function parameters are modeled as outputs of the discretized first-order system with sampling period \( \Delta T \) from noise input \( v \) with steady gains \( K_{d_1} \) and \( K_{d_2} \), and time constants \( T_1 \) and \( T_2 \). \( \phi \) is a differentiable function that transforms any value into a finite value. Function \( \phi(b) \), which corresponds to parameter \( a \) in Eqs. (12) and (13), has a minimum value \( \phi_{\text{min}} \) at \( b = -\infty \) and a maximum value \( \phi_{\text{max}} \) at \( b = \infty \).

This formulation expresses the delay model \( KE^{-\tau} \) approximately, which corresponds to the response having a single frequency component of ultrasonic propagation dynamics [Eq. (6)]. The output \( y \) gets close to the delayed input as \( M \) becomes large. When the observer estimates the state vector successfully, the time delay in this formulation is approximately defined as follows.

\[ \tau = m\phi(x_{m+1}) \Delta T \]  

(21)

It should be noted that \( \phi(x_{m+1}) \) and estimated \( \tau \) are continuous values. The time delay of \( m\phi_{\text{min}} \Delta T \sim m\phi_{\text{max}} \Delta T \) can be expressed in this formulation. In the present research, \( \phi_{\text{min}} = 0.1 \) and \( \phi_{\text{max}} = 0.9 \) were used. Because time delay is contained in the state variable, it can be estimated using the state observer.

### 4. Application to Rotor Downwash Measurement

The downwash speed of the rotor was estimated from the time delay of ultrasonic waves passing in the downwash. Two sets of ultrasonic transmitters and receivers were arranged below the rotor (Fig. 4). A rotor with a radius of 0.114 m was set at the height of 0.885 m and rotated by a MA2212-1400 KV MARSPOWER brushless motor. The motor was driven by an electronic speed controller (ESC) (BEATLES, ZTW Technology, Co., Ltd.) and the signal input to the ESC was generated by a RX6218 (Renesas Electronics Corporation). The downwash speed \( v_d \) was calculated using the amount of time required for the ultrasonic waves to pass through the upward and downward paths, by eliminating the sound speed as follows:

\[ v_d = \frac{L}{2 \sin \theta} \left( \frac{1}{t_{du}} - \frac{1}{t_{up}} \right) \]  

(22)

Where \( t_{du} \) and \( t_{up} \) are the times required to pass through the downward and upward paths, respectively. \( L \) is the length of the ultrasonic wave path in the downwash. In this formulation, it is assumed that the downwash speed is constant for the ultrasonic path and the vertical component is dominant.

The input to the ultrasonic transmitter was a frequency sweep input [Fig. 1(a)]. First, the receivers and transmitters were set close, with a distance of 0.01 m, and the receiver responses were measured in static air [Fig. 5(b)]. The response times of the two receivers are almost equal, and individual differences between transmitters and receivers can be neglected. Then, the outputs of the receivers below the rotor were measured at rotating speeds of 0 and 4800 rpm, which is the hovering condition of the supposed flight model [Fig. 5(c) and (d)]. The tangential component of the downwash speed along the ultrasonic path affects the amount to time required to pass through the paths. However, in the downwash below the rotating rotor, it is difficult to distinguish the difference in the time delay in receiver outputs between the upward and downward paths because of the disturbances in the measured signals. The disturbance of the downwash flow appears as oscillations in the receiver responses.

![Fig. 4. Experimental apparatus for downwash speed estimation.](image-url)
with a frequency of approximately 3.3 kHz, which is far from the rotor rotating frequency of 80 Hz. Because of such a large disturbance, it is difficult to apply the usual method using a threshold value or waveform information to estimate the time required for the ultrasonic waves to pass through the paths.

The passing time in the downwash, which corresponds to the time delay in propagation dynamics, was estimated using the extended Kalman filter applied to the delay model in Eqs. (14)–(21). The designed filter estimates the time delay between the reference input $u$ and the measured output of the ultrasonic receiver arranged below the rotor. The reference input $u$ in the delayed model of Eq. (14), which was measured previously, is the output of the ultrasonic receiver located in static air, 0.01 m from the ultrasonic transmitter [Fig. 1(b)].

The estimated time delay histories are shown in Fig. 6. The time delays of the upward and downward paths in the static air were approximately equal [Fig. 6(a)]. The different time delays for the upward and downward paths in the rotating rotor were estimated from the considerably disturbed receiver responses [Fig. 6(b)].

The estimated time delays were applied to Eq. (22), and the downwash speed is calculated. The estimation was performed offline using recorded data. It should be noted that the estimation can be performed online because the proposed method is a sequential algorithm. Figure 7 shows the time histories of the estimated downwash speeds. The time average of the estimated downwash speed from 0.01–0.1 s is
The downwash speed estimation method using ultrasonic waves is proposed. The medium speed was obtained using the time required for the ultrasonic waves to pass through paths below the rotor. The received ultrasonic waveform is disturbed, resulting in the variation in the received ultrasonic waveform. This makes it difficult to detect the time required for the ultrasonic waves to pass using the usual method. However, the proposed method attains a precise estimation with an error of only 2.69%. The time average of estimated downwash speed at a rotating speed of 0 rpm was 0.0946 m/s, which is the nominal error for this method. It can be said that, in terms of time and spatial average, Eq. (22) and estimated time required for the ultrasonic waves to pass are sufficiently validated.

5. Conclusions

A downwash speed estimation method using ultrasonic waves is proposed. The medium speed was obtained using the time required for the ultrasonic waves to pass through paths below the rotor. The received ultrasonic waveform is disturbed, and it is difficult to determine the time required to pass using the traditional method. Hereafter, propagation dynamics were identified in the simple form using fundamental transmit and receive experiments, which contained the delay time corresponding to the time required for the ultrasonic waves to pass. A state-space equation expressing the delay model was newly constructed, and an extended Kalman filter was designed to estimate the delay time. The ultrasonic transmitters and receivers were arranged below the rotating rotor, and the downwash speed was estimated using the designed state-space observer. It was revealed that the estimated downwash speed agreed with the averaged value of the downwash speed distribution measured directly using a hot-wire anemometer. The proposed model-based method enables the detection of delay times in a disturbed flow.

The downwash speed is an important parameter that determines the aerodynamics of a rotating rotor, which is affected by surrounding conditions, wind disturbances, and rotor damage. The proposed method will contribute to advanced flight control for attaining safe flight under such dynamics variations.

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