Analysis of eddy-currents interaction with a flaw in a conductive plate

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We calculated the eddy current distribution in an aluminum plate, excited perpendicularly by a small ring coil for quantitative understanding of the interaction of the eddy currents with a flaw in the plate. The calculation is based on the equations developed by Dodd and Deeds for axis-symmetric problems. After confirming the accuracy of the calculated results by comparing them with the measured data, we characterized the profiles of amplitude and phase of eddy currents in the aluminum plate. We confirmed that the phase of the eddy currents depends linearly on the distance between the source point and the observation point, as found in an ideal one-dimensional case, when the angle between the vertical line and the line connecting the source point to the observation point is less than 60 degrees. We extended the equation to express the magnetic field that may be produced out of the aluminum plate by the interaction between injected eddy currents and a small flaw in the sample. On the basis of the equation, we also show how to evaluate the magnetic field from a flaw.

Key words: eddy-current NDT, interaction with flaw, ring excitation coil, steady sinusoidal magnetic field, phase profile, amplitude profile

1. Introduction

Eddy-current NDT is widely used in many sectors of the industry to inspect and evaluate materials, parts and other products in ways that do not adversely affect their serviceability. The eddy-currents NDT is based on analyzing magnetic fields perturbation caused by the interaction of eddy currents and a flaw within the metal. The main operational functions common to most Eddy-currents NDT are: (I) applying magnetic field to a metal part to produce eddy-currents within the plate (input signal), (II) perturbation of eddy-currents distribution caused by presence of a flaw (encoding the signal), (III) sensing the magnetic field produced by the perturbed eddy-currents (secondary field) (IV) analyzing the detected field to extract the information of flaw (Decoding the signal). The above mentioned functions are very important and a lot of studies have been done on each of them.

For the first function we should select a suitable type of magnetic field (transient or pulsed field, steady sinusoidal field, exponential field and so on) and it is possible to utilize various coils with different geometries like ring coil, spiral coil, square coil and current plate as a coil. An easier way of looking at the interaction between eddy currents and flaw is to consider the flaw as imaginary coils and by using superposition principle the final perturbed field would be calculated. This way is considered experimentally but not very deeply, therefore, we investigated it theoretically in this paper.

Third function is related to the type of sensor. As the secondary field signal is very weak, finding a proper sensor is of great importance. A lot of studies on this issue can be found in the literature. The fourth function is about processing of the output signal which was detected by the sensor. Here processing of the output signal means detecting some important physical quantities which can be further analyzed to extract the flaw information. These quantities can be coil impedance, magnetic flux density, amplitude of induced voltage to pickup coil, phase of secondary field.

In this study we have investigated behavior of the two most important quantities related to magnetic fields, amplitude and phase of the secondary field. A ring coil as an exciting coil was used to produce steady sinusoidal perpendicular magnetic fields which were analytically investigated by Dodd and Deeds, and a pickup ring coil for detecting the magnetic fields caused by perturbed eddy-currents was used. We utilized a common idea of replacing a flaw by equivalent imaginary ring currents. By using this idea and numerically calculating the magnetic field produced by the imaginary ring currents, amplitude and phase profiles of secondary field were calculated. These profiles help us to determine the necessary sensitivity of our pickup coil, and moreover we can compare them to measured data to characterize flaws.

2. Confirmation of Numerical Calculation

Analytical formulation of the magnetic field generated by a variety of coils and current distribution above a metal plate has been studied for decades. In our calculation we used a ring coil to produce perpendicular magnetic field with the lift-off of 2 mm, above a flawless Aluminum plate with \( \sigma = 3.5 \times 10^{-7} \) S/m.

The magnetic field produced by the ring coil has been calculated for four regions by using equations developed by Dodd and Deeds for the axis symmetric system. They found four integral formulations for the vector potential, \( A(r, z) \), in each region of 1 through 4 as shown in Fig. 1. These integrals should be evaluated numerically for a given set of parameters. For finding the magnetic flux linked to the pick-up coil at the back side of the plate (region 4), we evaluated the related integral to get values for the magnetic vector potential.
We calculated the phase difference between the produced magnetic flux and the excitation current to the exciting ring coil. Phase of the excitation current was taken as a reference. We also measured the phase difference between the two by using the measurement setup shown in Fig. 1 in order to confirm the validity of numerical results.

Using an Aluminum plate of 4mm thick, and 18 mm diameter exciting and pickup ring coil and an integrator to measure the flux, we characterized phase differences between input current and the magnetic flux measured for different frequencies. The off axial distance, d, was 35mm and the exciting coil lift-off was 2 mm. The theoretical and the experimental result are compared in Fig. 2. These results are well matched, so the validity of the numerical calculation is confirmed.

3. Behavior of Injected Eddy-Current

We investigated the phase behavior within the Aluminum plate by solving the vector potential integral for third region. In this calculation we considered a 2mm diameter exciting ring coil with a sinusoidal current. The test frequency was 500 Hz and the other parameters were like previous one. The calculation was done along two lines: on the axis of symmetry and off the axis at an angle of 45 degrees as shown in Fig. 1. Results are shown in Fig. 3; from which the relationship between traveling distances and phase differences is found linear, but theoretical calculations shows the linearity can not be held for angles more than 60 degrees. At this step we extended the calculation to find the profile of amplitude and phase. We used a 2mm diameter ring coil with 0.5 mm lift-off and one ampere sinusoidal 500 Hz current and considered a lot of grid points within the flawless 5 mm thick Aluminum plate in every 1 mm spacing. Then we calculated the phase and amplitude of eddy-currents for each point. These values can be shown by two matrixes; Eddy-current phase matrix, $M_{ph}$, related to Fig. 4 and Eddy-current density matrix related to Fig. 5 which we call it $M_f$. In these two figures the radial distance from the exciting coil center is shown by “r”.

4. Secondary Magnetic Field

A flaw in the plate alters the flow path of eddy-currents as shown in Fig 6 (a), which can be interpreted as “the injected signal is affected by the presence of a flaw”. Fig. 7 Linear relationship between the traveling distance and phase.
Fig. 5. 3D plot of the eddy-current density, \( J_{ED} (A/m^2) \) and the related matrix \( M_J \), consisting of value at each points.

\[
M_J = \begin{pmatrix}
2719.64 & 4623.23 & 3256.97 & 2000.78 \\
810.299 & 1749.85 & 1702.36 & 1304.58 \\
316.032 & 770.859 & 896.008 & 806.794
\end{pmatrix} (A/m^2)
\]

Fig. 6 (a) Eddy current path change due to the presence of a flaw. (b) The redistribution of eddy-current can be expressed by imaginary currents.

Fig. 7 Calculation of vector potential of a ring coil which is placed within the plate.

As can be seen in Fig. 6 (b), we can use an important idea in that a flaw can be replaced by equivalent imaginary loop currents. Then we should solve the problem of a current source within the plate and find the vector potential of the plate caused by this current source. To solve the problem of ring coil current within the plate for finding the produced field out of the plate, we obtained analytical solutions for vector potential in region 1 regarding to Fig. 7. We supposed a ring coil carrying current \( I \) and radius \( r_0 \) is placed at \( z = -l \) within a plate with thickness \( c \) and conductivity \( \sigma_1 \).

Coefficient of Dodd and Deeds equation should be modified to take account of the new boundary conditions. After imposing the related boundary conditions and solving the 6 equations for 6 unknowns, the final result for vector potential out of the plate (region 1) is given by equation (1):

\[
A^{(1)}(r, z) = \int B_i(\alpha) e^{-\alpha r_0} J_1(\alpha r) d\alpha
\]

\[
B_i = \frac{e^{-\gamma r} (e^{2i\alpha} (\alpha_1 - \alpha_2) + e^{2\alpha_2} (\alpha_1 + \alpha_2)) + \alpha_2 (1 + e^{2\alpha_2})(\alpha_1 + (1 + e^{2\alpha_2})\alpha_2)}{(1 + e^{2\alpha_2})\alpha_1 + (1 + e^{2\alpha_2})\alpha_2 + \gamma}
\]

\[
\alpha_1 = (\alpha^2 + i\omega\mu_0\sigma_1)^{1/2}
\]

\[
\gamma = \mu_0 I \alpha J_1(\alpha r_0)
\]

Using the above relations for each of two ring currents with opposite current direction we obtained the amplitude and phase of secondary magnetic field at the front side of the plate (region 1) by superposing the results. In our calculation for obtaining the phase and amplitude of the secondary magnetic field caused by a flaw first we considered each flaw located at each of the grids of 1 mm spacing within the plate as can be seen in Fig. 8. Then we supposed each flaw as two ring currents with 1 mm diameter and 1 A, sinusoidal 500 Hz current. By using the above formula we calculated the magnetic flux density, \( B \), produced by each pair of ring currents in the front side of the plate at the center of the exciting coil which is shown in Fig. 8. All of these amplitude values can be put in a matrix which we call \( M_{B,\text{raw}} \) that can be seen as follows:

\[
M_{B,\text{raw}} = \begin{pmatrix}
7.5 \times 10^{-5} & 10 \times 10^{-5} & 3.5 \times 10^{-5} & 5.8 \times 10^{-6} \\
1.7 \times 10^{-5} & 3 \times 10^{-5} & 1.9 \times 10^{-5} & 7.6 \times 10^{-6} \\
5 \times 10^{-6} & 1 \times 10^{-5} & 9 \times 10^{-6} & 5.2 \times 10^{-6}
\end{pmatrix}
\]

We used the word “raw” because it was calculated considering a fixed current of 1 A, but we know in a real situation the current related to each flaw on different grids are different in values because of eddy current distribution as we showed in Fig. 5. By using the eddy-current density matrix \( M_J \), we can predict an appropriate current of each imaginary coil if we convert this matrix of current density (A/m^2) to the matrix of filamentary current (A) by considering a cross sectional area posed by the interaction with a flaw. In order to show the process how the secondary magnetic field can be obtained we simply assumed \( 10^{-6} \) as a cross section in this paper. Then we can multiply each element of matrix \( M_J \) to the same element of matrix \( M_{B,\text{raw}} \) to find the “flaw-response matrix” \( M_{FR} \).

\[
M_{FR} = \begin{pmatrix}
2 \times 10^{-7} & 4.6 \times 10^{-7} & 1.1 \times 10^{-7} & 1.1 \times 10^{-8} \\
1.4 \times 10^{-8} & 5.3 \times 10^{-8} & 3.3 \times 10^{-8} & 9.9 \times 10^{-9} \\
1.6 \times 10^{-9} & 8.1 \times 10^{-9} & 8.1 \times 10^{-9} & 4.3 \times 10^{-9}
\end{pmatrix}
\]
This matrix can be used to obtain the profile which is shown in Fig. 9. Each matrix element is related to a flaw located on one of the grids aforementioned.

Before doing any experiments, by looking at this matrix we recognize how deep flaw can be detected by our sensing system with prescribed resolution and then we can decide to utilize a proper sensor. Also Fig. 10 shows the profile for the phase of flux density by which we understand that the phase at the point (11) shown in Fig. 8 has a value of -100 degree.

5. Conclusion

By using an excitation ring coil above an aluminum plate to produce perpendicular ac magnetic field, the behavior of phase, profiles of phase and amplitude of the eddy-current within the plate have been investigated and the linear relationship between the phase shift and the distance has been found. We found the final profiles of amplitude and phase for the secondary field out of the plate by solving the problem of imaginary ring currents situated within the plate and supposing imaginary ring currents as an effect of a flaw on the eddy-currents flow path. An interesting use of the final profile is to determine the situation of a flaw, the size and the depth. Another use of it is the ability for evaluating the necessary resolution for the sensor to be used.

![Exciting coil](image)

**Fig. 8** Each grid point which is shown by \((i,j)\) index related to the same element of \(M_{FR}\). In the calculation, a single flaw was considered at a time.

![Flux density profile](image)

**Fig. 9** Profile of the secondary field amplitude expressed by the matrix \(M_{FR}\).

![Profile for the phase of the flux density](image)

**Fig. 10.** Profile for the phase of the flux density. The phase difference between each contour line is 8.7 degrees and the phase at the point shown by a black point (depth = 1 mm, \(r = 1\) mm) is -100 degrees.

References


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