Control Charts Based on Hierarchical Bayesian Modeling

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Abstract:

Control charts are a representative method of statistical process control. Control charts make it possible to effectively manage the manufacturing process but it assumes the availability of large historical data sets. In the high-mix, low-volume production environment that has become a mainstream in recent years, sufficient samples for estimating process parameters cannot be obtained often. In such a situation, the control chart does not function properly. In addition, in manufacturing processes such as cutting, process averages can change due to deterioration even if the process operates in the in-control state, leading to type I error increases. Therefore, herein, we use hierarchical Bayesian modeling to propose control charts that function appropriately for trendy data sets in high-mix, low-volume production. We then show the usefulness of such an approach by comparing with a conventional method. Since hierarchical Bayesian modeling makes it possible to assume the same distribution for parameters of various types, it is possible to use all the information to estimate parameters comprehensively. This capability makes the new approach effective for high-mix, low-volume production.

Keywords

High-mix low-volume production, Type I error, Time series model, Trendy data sets

1. Introduction

The control chart, as proposed by Shewhart (1931), is a representative method of statistical process control (SPC) used broadly in manufacturing. A control chart includes two phases, Phase I and II, for statistical process management. In Phase I, process parameters used in the control chart are estimated based on the data sets obtained from the process. In Phase II, the control limit line is determined using the estimated parameters and monitoring determines whether the process is in the in-control state.

Parameter estimation in Phase I requires a sufficient number of samples, and Montgomery (2005) recommended at least 25 or 30 samples, each consisting of four or five data points in the case of the $\bar{X}$ control chart. However, Snoussi et al. (2007) stated that the traditional control charts assume that data sets can be obtained independently and noted that this assumption is typically not satisfied in high-mix, low-volume production, which recently has become a common manufacturing environment (Gu et al. 2014). In particular, traditional control charts designed for mass production cannot perform appropriate process control in the modern production environment.

For such cases, Hawkins (1987) proposed a self-starting CUSUM control chart, which converts the obtained samples into statistics that follow a normal distribution and then plots these statistics on the CUSUM control chart. The self-starting CUSUM control chart is designed for use at the start of the process so it does not require a large number of data sets and is useful for low-volume production. Quesenberry (1991) proposed a Q control chart based on the self-starting CUSUM control chart, which converts the obtained samples into Q statistics that follows the standard normal distribution and then manages these statistics. By converting samples to Q statistics, the Q control chart can manage multiple samples that follow different normal distributions, which is useful for high-mix, low-volume production. Kawamura et al. (2013) evaluated the ability to detect sudden anomalies in

the Q control chart using data obtained from semiconductor manufacturing processes that require high-mix, low-volume production. In this context, they pointed out that the ability of the Q control chart to detect persistent abnormalities is poor.

Even when the data sets show a trend, there is a possibility that control charts cannot perform appropriately. For example, in cutting processes, the process average decreases as manufacturing progresses. In such a case, a traditional control chart determines the process to be abnormal despite the in-control state. Kawamura et al. (2013) made a similar finding when measuring values obtained from the semiconductor manufacturing process, as did Cai et al. (2002) for deterioration of print quality due to toner consumption. Laura et al. (1991) used a Shewhart control chart, an EWMA control chart, and a CUSUM control chart for trendy data sets, and then evaluated with average RL (ARL). They showed that ARL decreases as the change in measured value increases, causing an increase in the type I error in traditional control charts for trendy data sets.

To overcome this limitation, Mandel (1969) proposed a regression control chart that manages residuals between the measured and predicted values using a regression model. In addition, Snoussi et al. (2007) proposed and evaluated a method of managing the residuals from the appropriate time series model as an approach to data sets showing autocorrelation. In the case of trendy data, it is common to identify appropriate models and manage their residuals. However, it is difficult to identify appropriate time series models from the small sample sizes in high-mix, low-volume production, making it difficult to properly manage them. For high-mix production, Jensen et al. (2006) noted that the number of samples needed to create control charts increases with an increase in the number of parameters to be estimated. They also stated that the number of samples required is higher than that in the case of independent data obtained for trendy data sets.

This study proposes control charts based on hierarchical Bayesian modeling for trendy data sets in a high-mix, low-volume production. Hierarchical Bayesian modeling makes it possible to assume same distribution for various types of parameters as prior distributions. Therefore, it is possible to use all the information to comprehensively estimate the parameters; this capability is effective for high-mix, low-volume production.

This paper is organized as follows. Section 2 describes data set generation for simulation and design of control charts based on hierarchical Bayesian modeling. Then, we analyze the performance and properties of the proposed method with several evaluation index. Section 4 discusses the analysis results, and we conclude the paper in Section 5.

2 Control charts based on hierarchical Bayesian modeling

2.1 Data set generation

In this paper, we assume that the measured values show autocorrelation and we then generate data sets using IMA (1,1), which is a time series model. Data sets are generated as follows:

$$y_{(i+1)j} = \mu_j + \varepsilon_{(i+1)j} + \theta_j \varepsilon_{ij} + y_{ij} \quad (i = 1, 2, \dots, N, j = 1, 2, \dots, V) \quad (1)$$

$$y_{1j} = \mu_j + \varepsilon_{1j} \quad (2)$$

where μ_j are constant terms and θ_j are moving average parameters. Also, i is the order in which the measured values are obtained and j is the product type. Note that ε_{ij} may be expressed as

$$\varepsilon_{ij} \sim N(0, \sigma_j). \quad (3)$$

In this case, $\{y_i\}_{i=1}^N = \{y_1, y_2, y_3, \dots, y_N\}$ is an unsteady data set showing autocorrelation. Therefore, the difference between a certain measured value y_{ij} and the next measured value $y_{(i+1)j}$ is calculated from $\{w_{ij}\}_{i=1}^{N-1} = \{w_{1j}, w_{2j}, w_{3j}, \dots, w_{(N-1)j}\}$ as

$$w_{ij} = y_{(i+1)j} - y_{ij}. \quad (4)$$

Then the time series model of (1) is

$$w_i = \mu_j + \varepsilon_{(i+1)j} + \theta_j \varepsilon_{ij} \quad (i = 1, 2, \dots, N-1, j = 1, 2, \dots, V). \quad (5)$$

For evaluation by simulation, we next consider the following two scenarios:

1. For all product types, the values of μ_j and θ_j are constant.
2. The values of μ_j and θ_j are different depending on the product types.

Scenario 1 is intended for the case in which similar measured values can be obtained among different product types, whereas scenario 2 is intended for the case in which there are differences.

2.1.1 Parameter design of scenario 1

In scenario 1, the parameters are designed as follows:

$$\mu_j = -0.1, \quad (6)$$

$$\theta_j = -0.1, \text{ and} \quad (7)$$

$$\sigma_j = 1, \quad (8)$$

where $j = 1, 2, \dots, V$ represents the product type, μ_j is the constant term for product type j , and θ_j is the moving average parameter for product type j .

2.1.2 Parameter design of scenario 2

In scenario 2, the parameters are designed as follows:

$$\mu_j = -0.1 - \frac{R}{V-1}(i-1), \quad (9)$$

$$\theta_j = -0.1 - \frac{R}{V-1}(i-1), \text{ and} \quad (10)$$

$$\sigma_j = 1, \quad (11)$$

where V is the number of product types and R is the range of the parameters. For example, when $R = 0.2$ and $V = 3$, $\mu_j = \{-0.1, -0.2, -0.3\}$.

2.2 Hierarchical Bayesian modeling

In the hierarchical Bayesian model, the same distribution can be assumed for parameters of multiple product types. As a result, improved parameter estimation accuracy is expected, especially in high-mix production.

We propose a method for structuring a model using hierarchical Bayesian modeling, as follows:

Level 1:

$$w_{ij} = y_{(i+1)j} - y_{ij}, \quad (12)$$

$$w_{ij} \sim N(\mu_j + \theta_j \varepsilon_{ij}, \sigma_j^2), \text{ and} \quad (13)$$

$$\varepsilon_{(i+1)j} = y_{ij} - (\mu_j + \theta_j \varepsilon_{ij}). \quad (14)$$

Level 2:

$$\mu_j \sim N(\mu_\mu, \sigma_\mu^2) \text{ and} \quad (15)$$

$$\theta_j \sim N(\mu_\theta, \sigma_\theta^2). \quad (16)$$

Level 3:

$$\mu_\mu \sim N(0, 1000^2), \quad (17)$$

$$\sigma_\mu \sim N(0, 1000^2), \sigma_\mu \geq 0, \quad (18)$$

$$\mu_\theta \sim N(0, 1000^2), \quad (19)$$

$$\sigma_\theta \sim N(0, 1000^2), \sigma_\theta \geq 0, \text{ and} \quad (20)$$

$$\sigma_j \sim N(0, 1000^2), \sigma_j \geq 0. \quad (21)$$

Level 1 generates the obtained measurement value from the time series model. Equation (12) is the setting for removing the tendency that the measured value increases or decreases, and the moving average model is applied to steady data sets w_{ij} according to (13).

Level 2 generates the constant terms μ_j from the average over all product types and the variation around that average. Moving average parameters θ_j are generated analogously. These settings aim to comprehensively use information by assuming the same prior probability distribution for parameters of different product types.

At level 3, an appropriate distribution is assumed independently for each parameter generated at level 1 or 2 using available information about that parameter. However, in this paper, to avoid setting an arbitrary prior

probability distribution, a sufficiently wide normal distribution is set as an uninformative prior probability distribution for each parameter. These settings indicate the minimum performance of the proposed method, and further performance improvement can be expected by setting the appropriate prior distribution based on the correct prior information of parameters.

Using this model, Markov chain Monte Carlo (MCMC) sampling is performed by the No-U-Turn sampler method proposed by Hoffman et al. (2014). In MCMC, the user must specify two parameters, namely the number of generated random numbers and the burn-in period. We set the number of random numbers to 2500 and the burn-in period to 500. $\theta^{(t)} (t = 1, 2, \dots, T)$ is defined as a random number sequence excluding the burn-in period obtained by MCMC sampling. Then, the estimated value $\hat{\theta}_{eap}$ of the parameter $\theta^{(t)}$ is obtained as follows:

$$\hat{\theta}_{eap} = \frac{1}{T} \sum_{t=1}^T \theta^{(t)}, \quad (22)$$

where T is the number of random numbers by MCMC.

2.3 Estimation of control limits

(20) is used to estimate μ_j, θ_j . When the estimated values are $\hat{\mu}_j, \hat{\theta}_j$, the predicted value $\hat{w}_{ij}$ is

$$\hat{w}_{(i+1)j} = \hat{\mu}_j + \hat{\theta}_j(w_{ij} - \hat{w}_{ij}). \quad (23)$$

The residuals r_{ij} are calculated as

$$r_{ij} = w_{ij} - \hat{w}_{ij}. \quad (24)$$

Next, the average and standard deviation of the residuals are calculated, respectively, as follows:

$$\bar{r}_{i\cdot} = \frac{1}{n} \sum_{j=1}^n r_{ij} \quad \text{and} \quad (25)$$

$$S_{i\cdot} = \sqrt{\frac{1}{n-2} \sum_{j=1}^n (r_{ij} - \bar{r}_{i\cdot})^2}. \quad (26)$$

Then, the control limits of hierarchical Bayesian control charts are as follows:

$$\text{UCL} = \bar{r}_{i\cdot} + 3S_{i\cdot}. \quad (27)$$

$$\text{CL} = \bar{r}_{i\cdot}. \quad (28)$$

$$\text{LCL} = \bar{r}_{i\cdot} - 3S_{i\cdot}. \quad (29)$$

3 Evaluation of hierarchical Bayesian control charts

3.1 Evaluation index

To evaluate the performance of the method, RL is obtained by simulation. RL is the number of points existing on the control chart in one trial prior to exceeding the control limits. Two evaluation indices, ARL and SDRL, are used. ARL is the average of RL, while SDRL is the standard deviation of RL. When RL obtained by $z = \{1, 2, \dots, L\}$ trials is defined as RL_z , ARL is

$$\text{ARL} = \frac{\sum_{z=1}^L RL_z}{L} \quad (30)$$

and SDRL is

$$\text{SDRL} = \sqrt{\frac{1}{L} \sum_{z=1}^L (RL_z - \text{ARL})^2}. \quad (31)$$

When $RL = (RL_1, RL_2, \dots, RL_z)$ is obtained, the set of RL values is rearranged in order of decreasing value; members of this rearranged set RL' are defined as $RL' = (RL'_1, RL'_2, \dots, RL'_z)$.

3.2 Simulation mechanism

It is assumed that $n = \{5, 10, 20, 50\}$ data sets have been obtained for the process with the number of product types $V = \{1, 5, 10\}$. Next, the control limits are calculated based on the estimated parameters. Suppose that new measured values are obtained successively, and the residuals of the measured values and the predicted values of the $(n + 1)$ th and subsequent ones are plotted on control charts. When plotted points exceed the control limits, one trial is completed and RL is counted. RL starts counting from the $(n + 1)$ th measured value. This trial is repeated 5000 times ($L = 5000$) under the same conditions. Trials with an RL value of 100000 or more are removed as outliers.

To compare with the proposed method, residual control charts using IMA(1,1) by maximum likelihood estimation (MLE) are shown.

3.3 Properties and performance of hierarchical Bayesian control charts

We indicate the performance of the hierarchical Bayesian control chart when the parameters are changed. In scenario 2, the parameters differ depending on the product type. We consider two cases of parameter range $R=0.18$ and 0.36 . These settings are intended to give a range of 2 to 3 times the initial value of parameters μ_j and θ_j .

Table 1 shows ARL and SDRL for the hierarchical Bayesian control charts. Similarly, Table 2 shows ARL and SDRL for the control charts based on MLE.

Table 1: ARL (SDRL) for hierarchical Bayesian control charts

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	4496.3	5438.9	5174.7	5450.0	5525.3	5520.6	5460.6
	(13670.1)	(14975.3)	(14488.5)	(15004.0)	(15063.2)	(14925.1)	(14917.2)
10	4486.8	4497.2	4315.3	4598.5	4495.5	4487.9	4438.9
	(12721.2)	(12837.5)	(12564.8)	(12981.6)	(12791.1)	(13049.2)	(12868.2)
20	2820.4	2770.7	2519.8	2963.3	2749.1	3052.3	2868.6
	(8710.4)	(9096.5)	(8455.7)	(9522.5)	(9023.5)	(9888.9)	(9281.5)
50	1037.8	924.8	847.7	1089.0	1024.2	1279.5	1211.2
	(3129.0)	(2922.3)	(2436.7)	(3736.6)	(3425.3)	(4466.5)	(4256.0)

Table 2: ARL (SDRL) for MLE control charts

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	239.9	329.3	292.8	295.7	286.1	333.7	343.9
	(3068.4)	(3815.8)	(3223.7)	(3220.4)	(3260.3)	(3667.1)	(4021.0)
10	1663.9	1665.3	1642.9	1354.0	1404.1	1031.7	2855.4
	(7927.1)	(7816.6)	(7777.8)	(7126.6)	(7127.8)	(6034.1)	(10269.9)
20	2436.5	2329.3	2331.4	2261.2	2204.8	1977.2	1907.4
	(8333.9)	(8302.5)	(8129.0)	(8206.1)	(8115.5)	(7903.8)	(7682.8)
50	1136.3	1028.5	1042.3	1303.5	1252.5	1397.3	1391.3
	(367.8)	(3292.9)	(3322.3)	(4905.4)	(4351.7)	(5089.4)	(5115.8)

In Tables 1 and 2, ARL is shown at the top of the cell and SDRL at the bottom of the cell. From Table 1, it is indicated that the ARL becomes smaller as the measured value or number of product types increases in the hierarchical Bayesian control charts. At the same time, SDRL is getting smaller. On the other hand, from Table 2, it can be seen that in the control chart based on the MLE, the ARL does not become small even if the number of product types increases.

Residual control charts need to estimate two parameters, θ_j and μ_j . In order to consider the accuracy of parameter estimation by hierarchical Bayesian modeling, mean squared errors of the two parameters are defined as follows:

$$MSE_{\theta} = \frac{1}{LV} \sum_{k=1}^L \sum_{j=1}^V (\theta_j - \hat{\theta}_{jk})^2 \quad (32)$$

$$MSE_{\mu} = \frac{1}{LV} \sum_{k=1}^L \sum_{j=1}^V (\mu_j - \hat{\mu}_{jk})^2 \quad (33)$$

where $\hat{\theta}_{jk}$ and $\hat{\mu}_{jk}$ are estimates of θ_j and μ_j at the k th trial. Tables 3 and 4 show MSE_{θ} for hierarchical Bayesian modeling and MLE. Similarly, Tables 5 and 6 show MSE_{μ} for hierarchical Bayesian modeling and MLE. If the accuracy of parameter estimation is good, these values will be small.

 Table 3: MSE_{θ} for hierarchical Bayesian modeling

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	0.0962	0.0458	0.0342	0.0608	0.0352	0.0799	0.0459
10	0.0867	0.0458	0.0251	0.0465	0.0259	0.0519	0.0320
20	0.0626	0.0268	0.0131	0.0281	0.0148	0.0321	0.0203
50	0.0241	0.0092	0.0047	0.0109	0.0067	0.0153	0.0082

 Table 4: MSE_{θ} for MLE modeling

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	0.8105	0.8108	0.8098	0.7103	0.7104	0.6203	0.6225
10	0.5204	0.5294	0.5282	0.4839	0.4845	0.4347	0.3230
20	0.1855	0.1877	0.1873	0.1904	0.1911	0.1839	0.1890
50	0.0317	0.0305	0.0305	0.0317	0.0316	0.0330	0.0332

 Table 5: MSE_{μ} for hierarchical Bayesian modeling

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	0.8426	0.1131	0.0609	0.0980	0.0511	0.0865	0.0480
10	0.1095	0.0441	0.0236	0.0382	0.0215	0.0355	0.0231
20	0.0445	0.0195	0.0102	0.0176	0.0104	0.0192	0.0139
50	0.0171	0.0072	0.0036	0.0076	0.0049	0.0095	0.0058

 Table 6: MSE_{μ} for MLE modeling

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	0.2241	0.2229	0.2240	0.1941	0.1943	0.1695	0.1655
10	0.0957	0.0973	0.0959	0.0820	0.0819	0.0683	0.1141
20	0.0440	0.0439	0.0444	0.0355	0.0363	0.0297	0.0298
50	0.0168	0.0168	0.0168	0.0137	0.0137	0.0111	0.0111

As shown in Table 3, hierarchical Bayesian modeling shows that as sample size or number of product types increases, MSE_{θ} decreases. In addition, the results listed in Table 3 are better than those of the MLE. Tables 5 and 6 show similar results for MSE_{μ} .

These findings suggest that hierarchical Bayesian modeling may be better able to estimate the appropriate

control limit compared to MLE. To check whether appropriate control limits are set, the average and standard deviation of UCL and LCL in 5000 simulation iterations are shown. Table 7 shows the average values of UCL and LCL in the hierarchical Bayesian control chart. The upper value in each cell shows the average of UCL, and the lower shows the average of LCL. Similarly, average values of UCL and LCL in MLE are shown in Table 8.

Table 7: Average UCL and (LCL) for hierarchical Bayesian control chart

<i>n</i>	<i>R</i> = 0			<i>R</i> = 0.18		<i>R</i> = 0.36	
	<i>V</i> = 1	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10
5	2.9338	3.0383	3.0626	3.1347	3.2029	3.2906	3.3604
	(-2.9746)	(-3.0358)	(-3.0626)	(-3.1348)	(-3.2001)	(-3.2756)	(-3.3507)
10	3.3318	3.2792	3.2268	3.4500	3.3995	3.6625	3.6190
	(-3.3286)	(-3.2836)	(-3.2286)	(-3.4450)	(-3.3954)	(-3.6500)	(-3.6069)
20	3.3447	3.2475	3.1977	3.4223	3.3699	3.7015	3.6347
	(-3.3449)	(-3.2484)	(-3.1982)	(-3.4168)	(-3.3663)	(-3.6832)	(-3.6193)
50	3.1894	3.1405	3.1188	3.3202	3.2955	3.6390	3.4012
	(-3.1891)	(-3.1404)	(-3.1186)	(-3.3150)	(-3.2915)	(-3.6233)	(-3.3986)

Table 8: Average UCL and (LCL) for MLE control chart

<i>n</i>	<i>R</i> = 0			<i>R</i> = 0.18		<i>R</i> = 0.36	
	<i>V</i> = 1	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10
5	5.7667	5.8506	5.8314	6.1879	6.1891	6.5578	6.5620
	(-5.7633)	(-5.8486)	(-5.8265)	(-6.1914)	(-6.1889)	(-6.5575)	(-6.5312)
10	5.7302	5.7934	5.7892	6.3926	6.4180	7.0795	4.6586
	(-5.7303)	(-5.7953)	(-5.7868)	(-6.3975)	(-6.4159)	(-7.0828)	(-5.0274)
20	4.4540	4.4388	4.4371	5.1364	5.1267	6.2305	6.2317
	(-4.4534)	(-4.4390)	(-4.4347)	(-5.1362)	(-5.1277)	(-6.2291)	(-6.2312)
50	3.2555	3.2383	3.2325	3.5237	3.5038	4.0896	4.0618
	(-3.2549)	(-3.2385)	(-3.2324)	(-3.5241)	(-3.5040)	(-4.0895)	(-4.0616)

If prediction accuracy in the time series model is good, residuals follow $N(0,1)$. Therefore, it is expected that $UCL = 3$ and $LCL = -3$. Tables 7 and 8 show that hierarchical Bayesian control charts can better set appropriate control limits compared with MLE. Tables 9 and 10 show the standard deviation of UCL and LCL of the hierarchical Bayesian chart and the control chart by MLE, respectively. The upper value in each cell shows the standard deviation of UCL, and the lower shows the standard deviation of LCL.

Table 9: Standard deviation of UCL and (LCL) for hierarchical Bayesian control chart

<i>n</i>	<i>R</i> = 0			<i>R</i> = 0.18		<i>R</i> = 0.36	
	<i>V</i> = 1	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10	<i>V</i> = 5	<i>V</i> = 10
5	2.3582	1.1848	1.1687	1.2207	1.2211	1.2690	1.2964
	(2.4105)	(1.1826)	(1.1667)	(1.2133)	(1.2226)	(1.2848)	(1.3135)
10	0.8710	0.8710	0.8338	0.9708	0.9240	1.1172	1.0517
	(0.8625)	(0.8705)	(0.8344)	(0.9746)	(0.9272)	(1.1334)	(1.0741)
20	0.6850	0.6302	0.5971	0.7462	0.6878	0.9542	0.8448
	(0.6852)	(0.6301)	(0.5975)	(0.7534)	(0.6948)	(0.9778)	(0.8719)
50	0.3998	0.3712	0.3582	0.4685	0.4380	0.7192	0.5781
	(0.4006)	(0.3710)	(0.3581)	(0.4778)	(0.4493)	(0.7466)	(0.5801)

Table 10: Standard deviation of UCL and (LCL) for MLE control chart

n	$R = 0$			$R = 0.18$		$R = 0.36$	
	$V = 1$	$V = 5$	$V = 10$	$V = 5$	$V = 10$	$V = 5$	$V = 10$
5	3.6208	3.6667	3.6805	3.9367	3.9423	4.1932	4.2018
	(3.6348)	(3.6614)	(3.6789)	(3.9228)	(3.9389)	(4.1904)	(4.1993)
10	3.9277	3.9550	3.9530	4.4330	4.4322	4.9116	3.5960
	(3.9247)	(3.9540)	(3.9507)	(4.4354)	(4.4262)	(4.9061)	(3.4394)
20	3.4731	3.4610	3.4644	4.3928	4.3320	5.4378	5.4734
	(3.4722)	(3.4625)	(3.4635)	(4.3955)	(4.3350)	(5.4372)	(5.4716)
50	0.7953	0.7125	0.7137	1.4789	1.3451	2.6779	2.6578
	(0.7920)	(0.7138)	(0.7132)	(1.4824)	(1.3460)	(2.6673)	(2.6557)

The standard deviation of the control limits indicates the stability of the limit line estimation for each trial. From Tables 9 and 10, we conclude that it is possible to set stable control limits for the hierarchical Bayesian control chart.

4 Discussion

When the process is in-control state, ARL is known to assume the following value ARL_α :

$$ARL_\alpha = \frac{1}{\alpha} = 370.4, \quad (34)$$

where α is the probability of type I error and ARL_α is calculated as $\alpha = 0.0027$.

Tables 1 and 2 show that when there are few samples and few product types, both the hierarchical Bayesian and the MLE control charts are away from ARL_α . However, as the number of samples increases, ARL approaches ARL_α . In particular, when the number of product types is large, ARL approaches ARL_α quickly in the hierarchical Bayesian control chart. This is because hierarchical Bayesian modeling improved the estimation accuracy of parameters by assuming the same distribution for parameters of different kinds. Therefore, we do not observe this feature in the MLE control chart. The results are shown in Tables 3, 4, 5, and 6.

When using control charts in low-volume production, the number of observations is an important factor while determining the performance of control charts. When there are few samples and the parameter estimation accuracy is poor, the width of the control limit line is not appropriate, while the variation becomes large, as shown in Tables 7–10. However, in the hierarchical Bayesian control chart, even if sample size is small, it is possible to improve the estimation accuracy and design a better control chart, as long as there are many product types. Therefore, the hierarchical Bayesian control chart method is effective for high-mix, low-volume production.

In this paper, uninformative prior probability distributions were used for the prior probability distributions. We did not consider the setting of prior distributions, but to exploit the advantages of Bayesian statistics, it is desirable to use information present in the process.

5 Conclusion

In this paper, control charts based on hierarchical Bayesian modeling for trendy data sets in high-mix, low-volume production were proposed. The proposed method is shown to be useful through comparison with control charts using MLE. When there are many product types, it is possible to use observations of a variety of product types to effectively estimate the control limits.

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