Evaluation on alias relations between main effect and interaction in Plackett Burman design (N=20) and its application to guide assignments

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Abstract:

Plackett Burman designs, that is one of the fractional factorial designs, have a property that the columns to be assigned the main effects are mutually orthogonal. Plackett Burman designs have been used in screening experiments for identifying active main effects out of candidate factors frequently. The screened results are not always correct due to the experimental errors and the alias relations between main effects and two-factor interactions. While several previous researches have been discussed the alias relations in Plackett Burman designs, not many researches have been focusing on the alias relations between main effect and interaction in N=20 as a structured manner. In fact, although Hamada and Wu (1992) enumerated large combinations of correlations, this study has organized the structure manner into a clear and unique table as table4. We demonstrated the alias relations in a systematic manner as a form of 0.6-triplet, indicating that the correlation coefficient between estimates of main effect and two-factor interaction is equal to 0.6. Furthermore, a new rule to describe appearance of 0.6 triplet is demonstrated as an application of BIBD. We also propose a new assignment method in N=20 Plackett Burman designs that avoids 0.6-triplets. The assignment method is confirmed that it has better property comparing to the conventional approach.

Keywords

0.6-triplets, two-factor interactions, correlation of columns, BIBD, allocation method

1. Introduction

It is important to know what procedure to conduct experiments when doing experiments, where the techniques of designs of experiments provide the specific procedure for this purpose. All experiments, especially in the field of physic and chemistry, must be conducted carefully based on the three principles given by Fisher, R. A, such that replication, local control and randomization with taking into consideration of variation of experimental conditions. After guaranteeing that the experimental conditions can be kept carefully based on the three principles, there are many problems to be overcome for designing experiments. One of the problems that appears frequently is to adapt increase the number of factors to investigate, because the number of experimental runs also increases explosively, especially when trying to cover all combinations of factor levels, called as factorial design. In many cases, it is impossible in terms of time and economy to investigate the all combinations of factor levels by factorial design. In order to meet the requirements to reduce the number of experimental runs, fractional factorial designs are widely applied for many factors. Fractional factorial designs examine of some of the combinations factor levels, not the all combinations, therefore fractional is added to the factorial designs. Two-level fractional factorial designs are widely applied in many cases for screening purpose, that screens out inert factors out of many candidate factors because the number of experimental runs can be reduced efficiently with keeping the performance for correct screening. One of the well-known two-level fractional factorial design is the class of $N = 2^p$ fractional factorial designs, consisting of $N = 4, 8, 16, 32, \dots$ experimental runs. Another class of well-known two-level fractional factorial designs is $N = 4k$ fractional factorial designs, where Plackett and Burman (1946) has shown the construction of the design by Hadamard matrix. These designs are referred as PB design hereafter.

The both 2^p fractional factorial designs and PB designs are orthogonal design for main effects when the factors allocated to any two columns in the designs since these designs consist of mutually orthogonal columns. The alias relation between the main effect and two-factor interaction is different. Specifically, two factor interaction in 2^p fractional factorial designs completely aliased with one column in the design. Therefore, if a factor is assigned the column that is completely aliased with the two-factor interaction, the main effect and the two-factor interactions aliased completely. On the other hand, alias relations between main effect and two-factor interactions in PB designs are different from 2^p fractional factorial designs, see the standard textbook of design of experiments, such as Montgomery (2001), Wu and Hamada (2009) and Yamada (2004). Some papers have examined the aliased relations in $N = 12$ PB designs, while aliased relations in other PB design are not examined in detail. In addition, not many researches derive the alias relations in a structured manner. In this paper, we demonstrated the alias relations in a systematic manner as a form of 0.6-triplets, indicating that the correlation coefficient between estimates of main effect and two-factor interaction is equal to 0.6. Furthermore, a new rule to describe appearance of 0.6 triplets is demonstrated as an application of BIBD. We also propose a new assignment method in $N=20$ PB designs that avoids 0.6-triplets. The assignment method is verified that it has better property comparing to the conventional approach. Table 1 shows Plackett Burman design that we mainly deal with in our research.

As regard the Hadamard matrix, Hall (1965) has obtained that there are three classes, say class P, class Q and class N. Wang and Wu (1995) has obtained that PB design is categorized in class Q. In this paper, we show the result based on PB design categorized in class Q while the other two classes have similar properties shown in this paper except Table 7 description.

Table 1. Plackett Burman design $N=20$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
2	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1
3	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1
4	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1
5	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1
6	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1
7	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1
8	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	-1
9	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1
10	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1
11	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1	1
12	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1	-1
13	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1
14	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	1
15	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1
16	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	1
17	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1
18	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1
19	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1
20	1	-1	-1	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1

2. Problem presentation and previous research

2.1 Problem presentation

In this section, we introduce problems because of alias relation in $N = 20$ PB design by an application of simple simulator of radio-controlled car (RC simulation (2018)). The factors included in the case study is summarized in Table 2. Since $N = 12$ and $N = 20$ PB designs are constructed by sequential addition of by a row vector with circulation of one element at once, it is not deeply considered the assignment of factors to the columns comparing to 2^p fractional factorial designs. In the case study of $N = 12$ PB design, 8 factors are randomly assigned to 8 out of 11 columns in the design. We analyze in order to screen out the inert factors and to find active factors based on p value. This trial has been repeated 5 times. The result is summarized in Table 3. We also examine the similar trial in $N = 20$ PB designs, where 14 factors are randomly assigned to 14 out of 19

columns in the design. The result is also shown in Table 3. The factors are listed in ascending order of P value, and the factors at a significance level of 1% are colored as active factors. This table indicate that the screening result in $N = 12$ PB designs are stable for all 5 trials, such as almost same value of adjusted R-squares value R^{2*} , a factor identified as active and so forth. On the other hand, the results in $N = 20$ PB designs are not stable. For example, variation of R^{2*} is large comparing to the results in $N = 12$ PB designs, factors identified as active are varying through the 5 trials.

Table 2. Factors of radio-controlled car simulator

No.	Factors	Abbreviations	Min	Max
1	Vehicle weight	SZ	1.2	1.8
2	Grip of the tire	TG	1.28	1.92
3	The drive wheel load ratio	KH	0.8	1.2
4	The drive wheel diameter	KC	56	84
5	Gear ratios	GR	2	6
6	Gear efficiency	GK	0.68	1.02
7	Rolling resistance coefficient	KT	0.0528	0.0792
8	Rotation equivalent weight	KS	0.18	0.27
9	Float <u>ing</u> of brake brakes	SU	0.56	0.84
10	Rear wheel load	KK	0.4	0.6
11	Drag coefficient	CD	0.294	0.788
12	Front projected area	ZT	0.0191	0.0224
13	Front wheel downforce factor	ZD	-0.032	0.186
14	Rear wheel downforce factor	KD	0.082	0.84

The difference of results would be able to be explained based on the alias structure between main effect and two-factor interactions. Many authors, such as Wu (1993) and Iida (1994) have introduced this alias relation in $N = 12$ PB designs. Specifically, the main effects are either partly or no aliased with two-factor interaction. Specifically, correlation coefficient r between a column of main effect and a column of two-factor interaction is equal to 0 or ± 0.33 . Let $\beta_i, \hat{\beta}_i$ and β_{jk} the main effect of factor assigned at i -th column, its estimator and effect of two-factor interaction of a factor assigned at j -th column and a factor assigned at k -th column. The no and partly alias relation bring the bias of the estimator such that

$$\begin{aligned} E(\hat{\beta}_i) &= \beta_i \text{ for } i = j \text{ or } i = k. \\ E(\hat{\beta}_i) &= \beta_i \pm 0.33\beta_{jk} \text{ for } i \neq j, i \neq k \end{aligned}$$

On the other hand, the alias relations between a column of main effect and a column of two-factor interaction in $N = 20$ PB design had not been obtained in a structured manner. Wu (1993) has obtained the alias relation such that the correlation coefficient r of i -th column and a column of two-factor interaction of j -th column and k th column is

$$\begin{aligned} r &= 0 \text{ for } i = j \text{ or } i = k \\ r &= \pm 0.2 \text{ or } 0.6 \text{ for } i \neq j, i \neq k \end{aligned} \quad (1)$$

through numerical calculation. The instability discussed in the above would be brought by the highly aliased relation between a main effect of two-factor interaction. It is suspected that an interaction β_{jk} is large. The alias relation brought the over estimation of the effect of main factor assigned at i th column such as the relation of

$$E(\hat{\beta}_i) = \beta_i + 0.6\beta_{jk}.$$

The above equation indicates the bias due to the alias relation that would be the cause of incorrectness of data analysis in $N = 20$ PB designs.

2.2. Previous research on partly aliased relation

In order to obtain the correct screening result by application of PB designs, data analysis and assignment of factors would be options to be considered. As regard the data analysis, Hamada and Wu (1992) have discussed the data analysis of $N = 12$ PB designs based on the partly aliased structure. In this approach, it is utilized the alias relation of a column of main effect and a column of two-factor interaction such that the correlation coefficient r is as follows:

$$\begin{aligned} r &= 0 \text{ for } i = j \text{ or } i = k \\ r &= \pm 0.33 \text{ for } i \neq j, i \neq k. \end{aligned}$$

Table 3. The screening results in radio control car simulator

N=12						N=20					
	1	2	3	4	5		1	2	3	4	5
1	TG	TG	TG	TG	TG	1	TG	TG	TG	TG	TG
2	GR	GR	GR	GR	GR	2	GR	GR	GR	GR	GR
3	KC	KC	KC	GK	KC	3	KC	KC	KH	GK	KC
4	KH	KT	KT	KC	CD	4	ZD	GK	KC	KC	KD
5	SU	KH	SU	KT	SU	5	ZT	KT	SZ	KK	GK
6	GK	CD	KH	KH	GK	6	SZ	ZD	CD	SZ	SZ
7	CD	GK	GK	CD	KT	7	CD	KS	ZD	CD	KT
8	KT	SU	CD	SU	KH	8	KK	SZ	KD	ZT	ZD
R^{2*}	0.418	0.492	0.424	0.458	0.554	9	KD	KH	KK	ZD	SU
						10	SU	ZT	GK	KS	KS
						11	KS	CD	SU	SU	KH
						12	KT	KD	ZT	KT	ZT
						13	GK	KK	KS	KH	KK
						14	KH	SU	KT	KD	CD
						R^{2*}	0.956	0.702	0.490	0.870	0.634

In other words, a structural description of alias relation is mandatory to create a data analysis approach. As regard $N = 20$ PB designs, Wu (1993) had obtained the alias relation shown in Equation (1). Specifically, this paper has utilized $N = 20$ PB design for constructing supersaturated design, where supersaturated design refers to a design, in which the number of factors is equal to or larger than the number of experiments. Initial work has been proposed by Satterthwaite (1962) and Booth and Cox (1962) and Lin (1993) was a trigger for further successive researches. As well as Lin (1993) applies PB designs for constructing supersaturated designs, Wu (1993) has utilized 19 columns of $N = 20$ PB design to construct supersaturated designs by adding two-factor interaction columns. $N = 20$ PB designs are obtained by Hadamard matrix and constructed by sequential addition of by a row vector with circulation of one element at once. Therefore, because of circulation property, the first row is used for convenience because it does not affect the further discussion. Assuming that the $N = 20$ PB design to be used for the first time is $I_0 = \{c_i, i = 1, \dots, 19\}$, it is $I_1 = \{c_{1i}, i = 2, \dots, 19\}$, where c_{1i} denotes a column of two-factor interactions between column c_1 and c_i . Therefore, $I_0 + I_1$ consists of 37 columns that can be regarded as supersaturated designs. This paper explores additional columns to keep the correlation as small as possible. The following procedure is continued i -th column, j -th column, k -th column. As regard the correlation coefficient between c_{1i} and c_{jk} , it is obtained in numerical calculation that 48 correlation coefficients are 0.6 and the rest are ± 0.2 out of $\binom{18}{3} = 816$ combination of i, j, k columns. This paper has tried to avoid combinations where this correlation is 0.6 as much as possible for constructing supersaturated designs.

There is a similarity with us in terms of focusing on combinations of columns where the correlation coefficient is equal to 0.6. While Wu (1993) has investigated mainly the correlation between two columns of two-factor interactions, we examine the correlation between a column of main effect and a column of two-factor interaction. In addition, we explore to grasp the alias relations with correlation $r = 0.6$ in a structured manner while the previous research obtained such combination by numerical calculation.

3. Evaluation on alias of interactions

3.1. Correlations of PB design

Figure 1 shows a correlation color map of $N = 12, 20$ PB designs. In the design of $N = 12$, main effects are mutually orthogonal, and correlations between the main effects and two-factor interactions including the factor of main effect are also equal to 0. All other correlations between main effect and two-factor interaction are equal to ± 0.33 . As it is discussed before, it is considered that the experimental result was stabilized in the RC car experiment.

As well as $N = 12$, $N = 20$ PB design provides that main effects are mutually orthogonal and the correlation coefficient between the main effect and two-factor interactions including the main effect is equal to 0. However, the correlation coefficients between other combination of main effects and two-factor interactions are mostly ± 0.2 and some of them are 0.6. When we investigate the cases of 0.6 carefully, a two-factor interaction is equally included anywhere in 19 factors with a correlation of 0.6. In other words, there is a two-factor interaction having

0.6 correlations of nine for each factor. There are 171 in total with this large correlation. Below we discuss the combination with correlation of 0.6 and its appearance pattern.

Figure 1. Correlation color map indicating alias relations

3.2 Appearance pattern of high correlation

We investigate combinations of a column of main effect and a column of two-factor interaction whose correlation coefficient is equal to 0.6. The combination is summarized in Table 4. The value of this table is to indicate the structure clearly. This table indicates the highly aliased interaction with correlation coefficient 0.6 to a main factor, where the two-factor interaction a factor in vertical axis and a factor in horizontal axis. For example, when looking at X_2 in the vertical axis and X_{13} in the horizontal axis, 1, instead of X_1 , is shown in the responded cell. It indicates that the column of two-factor interaction $X_2 \times X_{13}$ has a correlation coefficient 0.6 to the column of main effect X_1 . There are 171 combinations of two-factor interactions, and the numbers from 1 to 19 all appear nine times with a certain rule. This is because of the fact that PB design has a circulating structure.

It can be also indicated in Table 4 that the combination having the correlation of 0.6 has a relationship of the triplet such that a main effect correlates with the two-factor interaction of the remaining two factors whose correlation coefficient is 0.6. Hereafter, it is simply described 0.6-triplet of the relation. More specifically, 0.6-triplet of factors (i, j, k) indicates that 0.6 times of two-factor interaction between i and j is aliased with the main effect of factor k . In the same manner, two-factor interaction between i and k is aliased with the main effect of factor j as well as two-factor interaction of j, k . By this reason, it can be said a manner of three-way deadlock.

Table 4. Appearance pattern of 0.6-triplets

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}
X_1		13	17	6	11	4	16	9	8	14	5	19	2	10	18	7	3	15	12
X_2			14	18	7	12	5	17	10	9	15	6	1	3	11	19	8	4	16
X_3				15	19	8	13	6	18	11	10	16	7	2	4	12	1	9	5
X_4					16	1	9	14	7	19	12	11	17	8	3	5	13	2	10
X_5						17	2	10	15	8	1	13	12	18	9	4	6	14	3
X_6							18	3	11	16	9	2	14	13	19	10	5	7	15
X_7								19	4	12	17	10	3	15	14	1	11	6	8
X_8									1	5	13	18	11	4	16	15	2	12	7
X_9										2	6	14	19	12	5	17	16	3	13
X_{10}											3	7	15	1	13	6	18	17	4
X_{11}												4	8	16	2	14	7	19	18
X_{12}													5	9	17	3	15	8	1
X_{13}														6	10	18	4	16	9
X_{14}															7	11	19	5	17
X_{15}																8	12	1	6
X_{16}																	9	13	2
X_{17}																		10	14
X_{18}																			11
X_{19}																			

3.3 Rule of combination of high correlations

Since PB design are derived by Hadamard matrices, the design has a circulating structure. Relationships in 0.6-triplets can be classified into three types depending on mod 19 (remainder when divided by 19). Table 5 shows the results of classification for each factor number. Using integers i, j, k the relationship of 0.6-triplets is as $(i, i + 3, i + 5), (j, j + 7, j + 8), (k, k + 4, k + 10)$, classifying into three groups. Using Table 5, it is found that which factor includes two-factor interaction having a correlation of 0.6 for each number of factors to be allocated. When assigning factors, the relation described by 0.6-triplet would be used to avoid 0.6-triplets, *i.e.* such interactions would not to be included in alias relations.

Table 5. Relationship of 0.6-triplets as $(i, i + 3, i + 5), (j, j + 7, j + 8), (k, k + 4, k + 10)$

# of factors m	# of 0.6-triplets	1st	2nd	3rd
6	1	1 4 6		
7	2	2 5 7		
8	3	3 6 8		
9	5	4 7 9	1 8 9	
10	7	5 8 10	2 9 10	
11	10	6 9 11	3 10 11	1 5 11
12	13	7 10 12	4 11 12	2 6 12
13	17	8 11 13	5 12 13	3 7 13
14	22	9 12 14	6 13 14	4 8 14 1 10 14
15	27	10 13 15	7 14 15	5 9 15 2 11 15
16	33	11 14 16	8 15 16	6 10 16 3 12 16 1 7 16
17	40	12 15 17	9 16 17	7 11 17 4 13 17 2 8 17
18	48	13 16 18	10 17 18	8 12 18 5 14 18 3 9 18
19	57	14 17 19	11 18 19	9 13 19 6 15 19 4 10 19

4. Assignment factors to the column in PD design

4.1 Basic idea

It is confirmed that the relations of 0.6-triplet can be described the form of Balanced Incomplete Block Design (BIBD) through the examination of the 0.6-triplets. By utilizing the form of BIBD, we discuss assignment to the column in $N = 20$ PB designs. Let v, b, k and r denote the number of points in the design, the number of blocks, the size of each block and the number of blocks including any point. Based on this definition, $bk = rv$ holds. Figure 2 presents summary of 0.6-triplets as BIBD, where $v = 19, b = 57, k = 3$ and $r = 9$. Specifically, the BIBD implies 19 columns (points), 57 sets of 0.6-triplet and each column (point) appears 9 times. The bottom line of this table shows how many factors in rows are included 0.6-triplets. We consider the assignment of factors to the column in PB design by deleting the column with the small number of 0.6-triplets. The problem can be described that selection of columns (points) that can include the less number of 0.6-triplets.

In order to explain the basic idea of assigning method to the columns, small size of BIBD is introduced. Let us consider BIBD with $v = 7, b = 3, k = 3$ and $r = 7$. This BIBD implies 7 sets of 0.6-triplet and 7 columns. The 0.6-triplets are summarized in Figure 2 as a graph representation. For example, column 1, column 2 and column 3 forms a 0.6-triplet. When column 1 is deleted, the three triplets (1,2,3), (1,4,5) and (1,6,7) are deleted, therefore 4 sets of 0.6-triplets are remained.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	k
1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3
2	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	3
3	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	3
4	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	3
5	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	3
6	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	3
7	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	3
8	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	3
9	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	3
10	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	3
.						.							.					.		
.						.							.					.		
.						.							.					.		
56	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	3
57	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	3
r	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	

Figure 2. BIBD description of 0.6-triplets

When a set of three columns consisting of 0.6-triplets such as columns 1, 2 and 3, is deleted, it remains no 0.6 triplets. On the other hand, deleting three columns that does not form 0.6-triplet, such as columns 1, 2 and 4, a 0.6-triplet is remained. This example is shown in Figure 3. This result implies that it is beneficial to delete columns forming 0.6-triplet to remain the less number of 0.6-triplets. We apply this idea for assigning factors to columns with deleting columns forming 0.6-triplets. In the specific procedure, a column that is included in many 0.6-triplets is deleted. At this time, if a column is chosen so that the number of 0.6-triplets is the largest among the columns, it is found that 0.6-triplets of the design is becoming small.

Figure 3 Illustrative example of deleting columns based on the graphical representation

4.2 Assigning method of factors to the columns

Let us consider m factors to be assigned $N = 20$ PB design. The column number is denoted with $[]$, for example $[19]$ denotes the 19th column of $N = 20$ PB design. The specific procedure is as follows:

- 1) $m = 18$: Any column can be excluded from the design of $N = 20$ PB design with 19 column. For example, $[19]$ is deleted.
- 2) $m = 17$: Any factor can be excluded. For example, $[17]$ is deleted.
- 3) $m = 16$: The column that forms 0.6-triplets with the above two columns is deleted. In the above example, $[14]$ is deleted since it forms 0.6-triplets with $[17]$ and $[19]$.
- 4) $m = 15$: Any column from the remaining column is deleted. For example, $[1]$ is deleted.
- 5) $m = 14$: A column is deleted to form many 0.6-triplets with previously deleted columns. In this example, $[3]$ is deleted to form 0.6-triplet with $[1]$ and $[17]$.

In the case of $m = 13$ to $m = 10$, similar procedure is continued in principle. Specific procedure is shown in Figure 4. The red boxes with circle in below is success examples. If other columns are deleted, the number of 0.6-triplets is larger than the others. This delete nine columns from 19 columns. Therefore, removing the factor from $m = 15$ is a slightly complicated procedure. From $m = 10$, the factors are eliminated so that the number of 0.6-triplets is reduced as well. Table 6 summarizes the design in which the selected columns for assigning m factors including the obvious case of $m = 1, \dots, 5$. There are several designs that have the same number of 0.6-triplets.

Figure 4. Outline of deleting columns from $m = 13$ to $m = 10$

4.3. Verification of the assignment method

In order to verify the effectiveness of assignment method shown in Table 6, a numerical evaluation is executed whether the number of containing 0.6-triplets is enough small. This verification is executed by a statistical software R as follows: For m factors, we randomly select m columns from PB design. Among the selected m columns, the number of 0.6-triplets is counted. This procedure is repeated 10,000 times for each m . The results of this verification are summarized in Table 7. It shows the distribution of the number of 0.6-triplets. As a reference, assigning factors based on the order of column number PB design. In this table, + and – indicates the difference of number of 0.6-triplets comparing to the reference method, where – implies less number of triplets. This table indicates that there is room for improvement on the reference method. In particular, results in $m = 9, 10$ and 11 because of large variation of the difference. Table 8 shows the number of included 0.6-triplets to compare among the reference design, the best case of randomly generated design and the design based on our assignment method in terms of the number of 0.6-triplets. This table indicates that the design based on our assignment method is always the best in terms of number of 0.6-triplets. In particular, a better design than the randomly generated design is obtained for $m = 10$. Therefore, the effectiveness of our proposed design is verified.

Table 6. Proposal for assigning factors to columns

m	Column number																		
1	1																		
2	1	2																	
3	1	2	3																
4	1	2	3	4															
5	1	2	3	4	5														
6		2		4		6	7	8					13						
7		2		4		6	7	8					13		15				
8		2		4		6	7	8	9				13		15				
9		2		4		6	7	8	9				12	13		15			
10		2		4		6	7	8	9				12	13		15	16		
11		2		4		6	7	8	9				12	13		15	16		18
12		2		4		6	7	8	9		11	12	13		15	16		18	
13		2		4		6	7	8	9	10	11	12	13		15	16		18	
14		2		4	5	6	7	8	9	10	11	12	13		15	16		18	
15		2	3	4	5	6	7	8	9	10	11	12	13		15	16		18	
16	1	2	3	4	5	6	7	8	9	10	11	12	13		15	16		18	
17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		18	
18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Table 7. Distribution of the number of 0.6-triplets in randomly generated designs

m	-4	-3	-2	-1	0	+1	+2	+3	+4
1					10000				
2					10000				
3					9370	630			
4					8266	1734			
5					4663	4740	597		
6				1782	5082	2845	291		
7			272	2310	4466	2504	448		
8			276	1616	4078	3022	968	40	
9		53	610	2577	3931	2240	576	12	1
10	0	7	596	2253	3951	2523	637	33	
11		41	1026	2992	4135	1536	270		
12			424	2408	4655	2260	253		
13			297	2727	5170	1806			
14			578	4649	4773				
15				2344	7656				
16				620	9380				
17					10000				
18					10000				
19					10000				

*Difference from the assignment from the first column to m th columns

Table 8. Comparison of number of 0.6-triplets in each design

m	Assigning from the first	Minimum in Random designs	Proposed design
6	1	0	0
7	2	0	0
8	3	1	1
9	5	2	2
10	7	4	3
11	10	7	7
12	13	11	11
13	17	15	15
14	22	20	20
15	27	26	26
16	33	32	32
17	40	40	40
18	48	48	48
19	57	57	57

5. Conclusion

As shown in a case study of radio-controlled car simulation, the alias relation between main effect and two-factor interaction may bring incorrect result in screening. We propose an assigning method of factors to the columns in $N = 20$ PB design that avoids including highly correlated columns as called 0.6-triplets. Specifically, the appearance pattern of 0.6-triplets can be described by a similar manner of BIBD. The effectiveness has been verified by the numerical comparisons. Extension to other N and application to data analysis would be a problem for future researches.

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[DOI : 10.17929/ tq.5.81]

Received, April 12, 2018

Revised: June 27, 2019

Accepted: July 24, 2019