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LETTER

Performance Analysis of Non-saturated IEEE 802.11 DCF Networks

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SUMMARY This letter presents a model with queueing theory to analyze the performance of non-saturated IEEE 802.11 DCF networks. We use the closed queueing network model and derive an approximate representation of throughput which can reveal the relationship between the throughput and the total offered load under finite traffic load conditions. The accuracy of the model is verified by extensive simulations.

key words: IEEE 802.11, non-saturated, queueing theory, throughput

1. Introduction

With the wide application of 802.11 devices, more analytical works for IEEE 802.11 DCF [1] have focused on real networks operating in non-saturated conditions. Most of the works [2]–[4] are based on extensions of Bianchi’s b-dimensional discrete Markov-chain model [6]. Because of the large number of state transitions involved, they are complex. In [5], by scaling the attempt rate of the saturated condition, a simple model is proposed for non-saturated networks. However, evaluating the throughput, all the proposed models use a throughput expression as in [6], which cannot explicitly show the relationship between throughput and traffic arrival rate under finite traffic load conditions. In fact, under non-saturated conditions, throughput is sensitive to traffic arrival rate. The authors of [4], by employing complex algebraic manipulations and expansions, express the throughput as a linear function of the average packet arrival rate $\lambda_a$ with the limit $\lambda_a \rightarrow 0$.

In this paper, we use queueing theory to develop a simple analytical model for non-saturated IEEE 802.11 DCF networks and give a simple but accurate throughput representation which can easily measure the relationship between throughput and traffic arrival under non-saturated conditions.

Our work begins with extending the model proposed in [7] to the non-saturated 802.11 networks. In [7], by using a closed queuing network, the authors propose a simple analytical model for saturated systems. We extend that model in two aspects: first, in order to analyze non-saturated networks, we introduce idle state queues to the closed queuing network; second, modeling the IEEE 802.11 DCF backoff mechanism more precisely, we adopt a more precise definition of "transition time". "Transition time" means the instant that a station moves from one queue to another. According to the additional definition from the extended model, the throughput of a non-saturated IEEE 802.11 DCF network is explicitly expressed in terms of packet arrival rate $\lambda_a$, which is different from other models.

This paper is organized as follows: An analytical model is described in Sect. 2. In Sect. 3, the throughput of a non-saturated IEEE 802.11 DCF network is explicitly expressed in terms of packet arrival rate $\lambda_a$. In Sect. 4, our throughput estimates agree with the simulation results closely. Finally, the conclusion remarks are stated in Sect. 5.

2. Analysis Model

Here, we adopt a more precise definition of the slotted time scale. Specifically, a new “model” slot starts whenever the backoff counter of a non-transmitting station is decremented. We assume that the network includes $N$ contention stations. There is no hidden station in the network, the transmitting channel is ideal, and the collision probability suffered by each station is identical and constant, which we denote by $p$. And we assume that the packet arrival process of each station is a Poisson process with parameter $\lambda_a$, and the buffer capacity is infinite (which will be satisfied if the buffer size is large enough not to overflow under non-saturated conditions).

The model proposed in [7] represents a saturated IEEE 802.11 WLAN as a closed queueing network, each backoff stage of IEEE 802.11 DCF as a $G/G/\infty$ queue, and each station as a customer being served at one of the $G/G/\infty$ queues corresponding to its current backoff stage. Under non-saturated conditions, however, a station will be in an idle state if, after completing a packet transmission, there is no packet waiting for transmission in its buffer. By introducing in the closed queuing network a $G/G/\infty$ corresponding to idle state, we can extend the model to analyze the performance of non-saturated IEEE 802.11 WLANs. Setting the value of the retry limit $k$, we have the extended model depicted in Fig. 1, where $Q_{i-1}$ denotes the idle queue, $Q_i$ ($0 \leq i \leq k$) denotes the backoff queue corresponding to backoff stage $i$, $\lambda_i$ ($-1 \leq i \leq k$) denotes the average arrival rate of $Q_i$ (i.e., the average number of stations arriving...
at queue $i$ per model slot) and $q$ denotes the probability that the buffer is empty when a packet is transmitted successfully or dropped due to the retry limit.

Each station in the system behaves as follows. A station at backoff stage $i$ $(0 \leq i \leq k)$ attempts to transmit at the beginning of a model slot when its backoff counter decreases to 0. After this transmitting attempt, it makes a transition from $Q_i$ at the end of the model slot. If the packet is transmitted successfully or dropped due to the retry limit, the station moves to $Q_{i-1}$ or $Q_0$ depending on whether its buffer is empty or not, otherwise it moves to $Q_{i+1}$. When packets arrive at an idle station in a random model slot, the station moves from $Q_{i-1}$ to $Q_0$ at the end of the model slot. To be consistent with 802.11 backoff rule, all transitions from one queue to another occur at the end of model slots in our model. Therefore, the transition time is different from that in [7], where transitions between queues occur at the beginning of model slots.

From Fig. 1, the average number of stations arriving at $Q_i$ per model slot can be expressed as:

$$\lambda_i = p_i \lambda_{i-1} = p_i \lambda_0 \quad 1 \leq i \leq k$$

$$\lambda_{i-1} = \sum_{i=0}^{k-1} (1-p)q_i + q_i \lambda_k = q \lambda_0$$

Except for the transition from $Q_{i-1}$ to $Q_0$ (equal to $\lambda_{i-1}$), the other transitions are produced by stations' transmitting attempts. Therefore, the total average number of stations that make transitions from a backoff queue $Q_i$ $(0 \leq i \leq k)$ at the end of a random model slot is equal to the total average number of stations that attempt to transmit and remaining of a random model slot, that is, the total transmission rate of the system, which is $\lambda_0$. Thus from (1) we have:

$$\tau_{total} = \sum_{i=0}^{k} \lambda_i = 1 - p^{k+1} \frac{1}{1-p} \lambda_0$$

Let $\tau$ be the probability that a station transmits in a model slot. Then we express $\tau$ as:

$$\tau = \frac{\tau_{total}}{N}$$

Let $N_i$ be the average number of stations in $Q_i$. By Little’s theorem [8], $N_i$ can be expressed as

$$N_i = \lambda_i T_i \quad 1 \leq i \leq k$$

where $T_i$ denotes the mean service time of $Q_i$. Due to the discrete time scale adopted in the synchronized system, $T_i$ is measured in model slots.

For $0 \leq i \leq k$, $T_i$ is the average interval from the instant that a station enters backoff queue $Q_i$ until it finishes a transmission attempt and moves to another queue. Therefore, it equals the total number of backoff slots at stage $i$ plus a transmission slot. $T_i$ can be expressed as the average number of backoff slots at stage $i$, $T_i$, as well as

$$T_i = b_i + 1$$

According to IEEE 802.11 DCF binary exponential backoff mechanism, $b_i = \min(2^{\tau W_0} - 1)/2$, where $W_0$ is the size of original backoff window that determines the maximum backoff window.

Defining $P_a$ as the probability that at least one packet arrives at an idle station in a random model slot, we can express the average number of model slots spent by a station at idle state as

$$T_i = \sum_{n=1}^{\infty} n P_a(1 - P_a)^{n-1} = \frac{1}{P_a}$$

Substituting (1), (5), and (6) into (4), we have:

$$\lambda_0 = \sum_{i=1}^{k} q \lambda_0 + \sum_{i=0}^{k-1} (b_i + 1) p \lambda_0$$

Substituting (2), (8) into (3), we have

$$\tau = \frac{1 - p^{k+1}}{(1-p) \left( \sum_{i=0}^{k-1} (b_i + 1) p \right)}$$

where $p$ is defined as in [6]:

$$p = 1 - (1-\tau)^{-1}$$

Next we express $P_a$ and $q$ as functions of $\tau$ and $p$. Let $t_i$ be the average duration of one model slot experienced by idle stations. For a Poisson arrival process with parameter $\lambda_0$, the probability that at least one packet arrives during the period of $t_i$ is

$$P_a = 1 - e^{-\lambda_0 t_i}$$

Under the synchronization assumption, during a station stays in idle state, the average duration of one model slot in the system is determined by the other $N - 1$ stations. For a random model slot, if none of the $N - 1$ stations transmits, the slot will be an empty slot $\sigma$ with the probability $P_{idle} = (1-\tau)^{N-1}$, and if only one of them transmits, the slot will be a successful transmission slot ($T_s$ = PHY header + MAC header + $T_{sIFS}$ + ACK + $DIFS$ + $s$) with the probability $P_s = N\tau(1-\tau)^{N-2}$, otherwise, the slot will be a collision slot ($T_c$ = PHY header + MAC header + $T_{cIFS}$ + $EIFS$) with the probability $P_c = 1 - P_{idle} - P_s$, where $T_L$ denotes the time to transmit...
the payload in a packet, SIFS, DIFS, and EIFS denote the lengths of Inter-Frame Spaces following 802.11 DCF. So we have

\[ t_i = P_{idle} \sigma + P_s T_s + P_c T_c \]  \hspace{1cm} (12) \]

Under the assumption of Poisson packet arrival process with parameter \( \lambda_a \) and an infinite MAC buffer capacity, each station in the system can be treated approximately as a single M/G/1 queue. The probability that the buffer is empty at the instant when a packet leaves the station can be expressed as in [8]

\[ q = \max((1 - \lambda_a t_{serv}), 0) \]  \hspace{1cm} (13) \]

where \( t_{serv} \) is the mean service time of the M/G/1 queue, corresponding to the average interval from the instant that a packet gets the head of the MAC buffer ready for transmission until it is transmitted successfully or dropped ultimately due to the retry limit. Let \( P_i \) denote the probability that a packet is transmitted successfully at stage \( i \), \( P_{drop} \) denote the probability that a packet is dropped due to the retry limit, \( t_i \) denote the mean time spent by a packet successful transmitted at stage \( i \) and \( t_{drop} \) denote the mean time spent by a dropped packet. According to the IEEE 802.11 DCF backoff rule, they can be expressed respectively as follows:

\[
\begin{align*}
P_i &= (1 - p)^{pi} \quad 0 \leq i \leq k \\
P_{drop} &= p^{k+1}
\end{align*}
\]  \hspace{1cm} (14) \]

\[
\begin{align*}
t_i &= \sum_{e=0}^{i} b_e t_e + i T_e + T_s \quad 0 \leq i \leq k \\
t_{drop} &= \sum_{e=0}^{k} b_e t_e + (k + 1) T_e
\end{align*}
\]  \hspace{1cm} \hspace{1cm} (15) \]

Then \( t_{serv} \) can be written as:

\[ t_{serv} = \sum_{i=0}^{k} P_i t_i + P_{drop} t_{drop} \]  \hspace{1cm} (16) \]

Substituting the above equations of \( P_i \) and \( P_{drop} \) (9), we get a nonlinear system for \( t_{serv} \) (9) and \( t_{drop} \). \( \tau \) and \( p \) can be uniquely determined. As \( \lambda_a > 0 \), it can be used for analyzing saturated network.

3. Throughput Analysis

Under non-saturated conditions, for each station in the system (that is, each M/G/1 queue), the packet arrival rate is less than the packet service rate. According to queue theory, the departure rate is equal to the arrival rate for the queue in statistical equilibrium [8]. We can use \( \lambda_a \) and \( \mu \) to denote the probability that a packet is dropped due to the retry limit. Then the number of packets transmitted successfully by each station per second can be expressed approximately as

\[ n = (1 - P_{drop}) \lambda_a = (1 - p^{k+1}) \lambda_a \]  \hspace{1cm} (17) \]

Denoting the payload size by \( L \), the throughput of each station can be obtained as:

\[ S^{\text{true}} = nL = (1 - p^{k+1}) \lambda_a L \]  \hspace{1cm} (18) \]

And then we get the throughput of the system:

\[ S = (1 - p^{k+1}) \lambda_a N L \]  \hspace{1cm} (19) \]

Clearly, (18) and (19) can explicitly describe the approximate relationship between the throughput and the total offered traffic load in a non-saturated system.

We can easily derive the applying range of (18) and (19). First, setting \( q = 0 \), the probability that packet is dropped due to the retry limit. Let \( \tau \), denote the mean time spent by a packet successful transmitted at stage \( i \) and \( t_{drop} \) denote the mean time spent by a dropped packet. According to the IEEE 802.11 DCF backoff rule, they can be expressed respectively as follows:

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4. Model Verification

The proposed model is verified using the 802.11 simulator in ns-2 [9]. The physical and MAC parameter values are set according to 802.11b, where data rate = 11 Mbps, \( W_0 = k = m = p = 1 \), and \( L = 1000 \) bytes. The size of buffer is 1000 packets. We compare the numerical result obtained with the approximate model in [5], and also with the basic model in [7].

Figure 2 shows the numerical and simulated results of the collision probability with variation of the normalized total offered load for \( N = 6, 12, 18 \). It indicates that the collision probability evaluation of our model is very close to the simulation result and has almost the same result as the approximate model in [5]. Figure 3 depicts the results of the throughput. It indicates that the throughput representations proposed in Sect. 3 characterize the throughput accurately under both non-saturated and saturated conditions.

![Fig. 2](image-url) The collision probability versus normalized total offered load.
and have the same accuracy as the expression used in [5] and most other models. Figure 3 exhibits the approximate linear relationship between throughput and unsaturated traffic load. Equations (17) and (18) can give an intuitive and reasonable explanation for the fact that the collision probability is very small under non-saturated condition. Small \( p \) leads to very small \( P_{\text{drop}} \) (which equals \( p^{k+1} \)), that is, most packets can be transmitted successfully before reaching retry limit. We can obtain \( S = N\lambda_0 L \) from Eq. (19). Therefore, we can evaluate the throughput of non-saturated systems approximately.

Figure 4 shows the saturation throughput of the whole 802.11 system versus the number of stations. From Fig. 4, we can observe that our model estimates the performance of the IEEE 802.11 DCF protocol more accurately than the basic model in [7]. This improvement can be explained as follows: all transitions between queues occur at the end of model slots in our model, which is consistent with 802.11 backoff rule. In [7], however, the transition is at the beginning of model slots. Therefore, the number of model slots used for one packet transmission in the basic model is less than that in 802.11 DCF, which may overestimating the saturation throughput of 802.11 DCF.

5. Conclusion

In this paper, a closed queueing network model has been considered for non-saturated 802.11 networks, and an approximate representation of throughput has been derived which can reveal the relationship between the throughput and total offered load under finite traffic load conditions. Extensive simulations validate the accuracy of our model.

References