PAPER

Stochastic Geometry Analysis of Inversely Proportional Carrier Sense Threshold and Transmission Power for WLAN Spatial Reuse*

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SUMMARY In this paper, a stochastic geometry analysis of the inversely proportional setting (IPS) of carrier sense threshold (CST) and transmission power for densely deployed wireless local area networks (WLANs) is presented. In densely deployed WLANs, CST adjustment is a crucial technology to enhance spatial reuse, but it can starve surrounding transmitters due to an asymmetric carrier sensing relationship. In order for the carrier sensing relationship to be symmetric, the IPS of the CST and transmission power is a promising approach, i.e., each transmitter jointly adjusts its CST and transmission power in order for their product to be equal to those of others. This setting is used for spatial reuse in IEEE 802.11ax. By assuming that the set of potential transmitters follows a Poisson point process, the impact of the IPS on throughput is formulated based on stochastic geometry in two scenarios: an adjustment at a single transmitter and an identical adjustment at all transmitters. The asymptotic expression of the throughput in dense WLANs is derived and an explicit solution of the optimal CST is achieved as a function of the number of neighboring potential transmitters and signal-to-interference power ratio using approximations. This solution was confirmed through numerical results, where the explicit solution achieved throughput penalties of less than 8% relative to the numerically evaluated optimal solution.

key words: IEEE 802.11ax, spatial reuse, carrier sense threshold, stochastic geometry

1. Introduction

The proliferation of wireless local area networks (WLANs) has led to the dense deployment of access points (APs). However, in such environments, APs and stations (STAs) experience substantial interference from neighboring APs and STAs, i.e., overlapping basic service sets (OBSSs). Due to the carrier sense multiple access with collision avoidance (CSMA/CA) mechanism, they time-share the channel with all neighboring APs and STAs.

To facilitate spatial channel reuse and improve system throughput, an adjustment to the carrier sense threshold (CST), the so-called clear channel assessment (CCA) threshold [1], of an AP/STA is a promising approach [2], [3]. When the CST is tuned to be greater than interference from neighbors, APs can transmit signals even when their neighbors are transmitting. However, the increased CST increases interference at the AP and the intended STA as well as at neighboring APs/STAs. In addition, the asymmetric carrier sensing relationship resulting from the CST settings can cause throughput starvation at other APs and STAs [4].

We discuss by introducing a rough example shown in Fig. 1, where two APs 1 and 2 separated by 16 m attempt to transmit to their associated STAs using the same channel.
Let transmission power and CST of AP $i$ be denoted by $p_i$ and $\theta_i$, respectively. As shown in Fig. 1(a), when $p_1 = 13$ dBm, the interference power at AP 2 is $-58$ dBm, which is greater than the CST $\theta_2 = -82$ dBm. Thus, they time-share the channel. As shown in Fig. 1(b), when AP 2 increases its CST by 26 dB to be greater than the interference power, AP 2 will transmit signals even when AP 1 transmits signals. However, AP 1 received the interference greater than its CST $\theta_1$, thus AP 1 should defer its transmission, i.e., APs 1 and 2 have the asymmetric carrier sensing relationship and throughput of AP 1 is starved.

To solve the problem of throughput starvation, the authors of [4], [5] proposed that each transmitter (TX) jointly tunes its CST and transmission power so that their product is equivalent to those of other TXs. This joint tuning ensures a symmetric carrier sensing relationship, i.e., any two TXs either detect signals from each other or they do not, and throughput starvation thus is prevented. In the example shown in Fig. 1(b), if AP 2 reduces its transmission power $p_2$ by 26 dB, the interference level at AP 1 is smaller than its CST $\theta_1$, and thus, the symmetric carrier sensing relationship holds. This is called the inversely proportional setting (IPS) of the CST and transmission power in this paper, i.e., the transmission power is set inversely proportional to the CST. In our previous works [6], [7], we had proposed connecting attenuators between an antenna connector and an antenna for APs/STAs to achieve the IPS, and had proved the effectiveness of the IPS through experiments. The IPS is used in IEEE 802.11ax standardization [8]–[10] as well as 3GPP license assisted access (LAA) as a rule to realize fair co-existence not only among OBSSs in WLANs but also between WLANs and LAA [11].

The impact of the IPS on the system-level performance of CSMA/CA-based wireless networks should be clarified. The system-level performance of CSMA/CA networks has been analyzed based on stochastic geometry [12]–[15]. In [16]–[23], the point process of TXs at a given time was discussed based on the Matérn hard-core point process (MHCPP) type II [24]. This approach enables interference modeling of CSMA/CA networks. In [20], the optimal CST was numerically provided. In [21], [23], rate adaptation was considered and a CST adaptation scheme was provided, but an explicit CST was not given. In addition, as far as the authors know, no research to date has undertaken IPS performance analysis based on stochastic geometry.

In this paper, the IPS of the CST as well as transmission power is analyzed based on stochastic geometry. Compared to previous works, the contributions are: 1) we discuss the IPS of CST and transmission power whereas previous works discussed the adjustment of CST, 2) we apply asymptotic analysis in a dense scenario to derive the optimal CST and transmission power in explicit forms, thus, we evaluate expected throughput not outage, and 3) we discuss the CST adjustment at a single TX case as well as at all TXs case. The purpose of this paper is to show the impact of IPS on throughput and optimal CST, thus, we assume the randomness is due to location of TXs and backoff time, and not to fading and maximum transmission power. We believe these simple solutions provide insight into the optimal CST according to the number of neighbors and the received signal quality, and useful for AP installations. Note that compared to our former study [25], this paper includes new Monte Carlo simulation results and revised assumptions and analytical expressions to match the simulation results. Our recent works [26], [27] analyzed the throughput based on IPS where each TX individually adjusts its CST and transmission power based on its received power by using stochastic geometry. But in these works, explicit expressions of the optimal IPS could not be derived due to fading and nearest AP association.

The rest of this paper is organized as follows. In Section 2, we introduce a system model and IPS. In Section 3, we consider adjusting a single TX. In Section 4, we discuss the identical adjustment at all TXs. Finally, Section 5 contains the conclusions of this paper.

### 2. System Model

#### 2.1 Transmitters

The notation used here is shown in Table 1. We now consider a scenario where there is at most one TX-receiver (RX) pair in each basic service set (BSS) at a given time, and the set of potential TXs sharing a given channel follows a homogeneous Poisson point process (PPP) [15] $\Phi_p$ on $\mathbb{R}^2$ with density $\lambda_p$ as shown as all points in Fig. 2. Note that under this assumption, all simultaneous transmissions are from OBSSs. Collisions in each BSS and multiuser transmissions

<table>
<thead>
<tr>
<th>Table 1: Notation.</th>
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<td>$a : b$</td>
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<tr>
<td>$\Phi_p$</td>
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<tr>
<td>$\Phi$</td>
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<tr>
<td>$\Phi(S)$</td>
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<tr>
<td>$R_x$</td>
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<tr>
<td>$m_x$</td>
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<td>$n \in \mathbb{N}$</td>
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<td>$B \in \mathbb{R}$</td>
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<td>$W(\cdot)$</td>
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<td>$P$</td>
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<tr>
<td>$\omega$</td>
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<td>$p_x, p$</td>
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<td>$\theta_x, \theta$</td>
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<td>$G_{xy}(x \neq y)$</td>
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<td>$r(a)$</td>
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<td>$SIR_x$</td>
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where \( \Theta \) and \( P \) represent the CST and transmission power of legacy devices. For ease of expression, we introduce an auxiliary variable \( a_x \) satisfying (3) as

\[
\theta_x = \Theta a_x, \quad p_x = P/a_x, \tag{4}
\]

where \( a_x \) (\( \geq 1 \)) represents ratios \( \theta_x/\Theta \) and \( P/p_x \).

3. Adjustment at a Single Transmitter

3.1 Medium Access Probability

We first discuss a simple scenario where only one TX \( x \in \Phi_p \) adjusts its CST \( \theta_x \), transmission power \( p_x \), and \( a_x \), whereas other TXs utilize the minimum CST and maximum transmission power, i.e., \( \theta_z = \Theta \), \( p_z = P \), and \( a_z = 1 \) for all \( z \in \Phi_p \setminus \{ x \} \). For notational convenience, hereinafter we simply use \( p, \theta, \) and \( a \) for \( p_x, \theta_x, \) and \( a_x \), respectively. Here, we introduce the contention domain of TX \( x \) as

\[
S(a) := \{ z \in \mathbb{R}^2 \setminus \{ x \} \mid G_{xz}P > \theta \}. \tag{5}
\]

Note that \( S(1) \) represents the contention domain when \( a = 1 \), i.e., \( \theta = \Theta \) and \( p = P \). Fig. 2 shows an impact of IPS by adjusting \( a \), i.e., the size of contention domain \( S(a) \). Let \( \Phi_p(S) \) represent the number of points of \( \Phi_p \) in region \( S \).

We consider the setting of \( a \) based only on the number of neighboring potential TXs when \( a = 1 \), \( n = \Phi_p(S(1)) \). Note that \( n \) is a natural number. We also assume that \( n \) TXs are uniformly and independently distributed in \( S(1) \), i.e., the set of potential TXs follows a binomial point process (BPP) [12].

**Lemma 1**: The distribution of the number of neighboring potential TXs when applying IPS, \( \Phi_p(S(a)) \) for \( a \geq 1 \), is given by

\[
\mathbb{P}(\Phi_p(S(a)) = k \mid \Phi_p(S(1)) = n) = \binom{n}{k}(a^{-2/\alpha})^k(1 - a^{-2/\alpha})^{n-k}, \tag{6}
\]

for \( k = 0, 1, \ldots, n \).

**Proof 1**: Since we assume that the set of potential TXs follows a BPP, the distribution is given by

\[
\mathbb{P}(\Phi_p(S(a)) = k \mid \Phi_p(S(1)) = n) = \binom{n}{k}(1 - \rho)^{n-k}, \tag{7}
\]

where

\[
\rho := \frac{|S(a)|}{|S(1)|} \tag{8}
\]

and \(|\cdot|\) represents the area of the region [15, Theorem 2.9]. To evaluate (7), we need to discuss \( \rho \). Substituting (2) and (4) into (5), we get

\[
S(a) = \{ z \in \mathbb{R}^2 \setminus \{ x \} \mid \|z - x\| < (AP/\Theta a)^{1/\alpha} \}. \tag{9}
\]
Thus, the radius of $S(a)$ is $\delta(a) = (AP/\Theta a)^{1/\alpha}$, and
\[
\rho = \frac{|S(a)|}{|S(1)|} = \frac{a^{2/\alpha}}{\pi d(1)^2} = \frac{a^{2/\alpha}}{\pi d(1)^2}, \quad (10)
\]
Substituting (10) into (7), we get (6).
\[\square\]

Let backoff time of each potential TX $x \in \Phi_p$ be denoted by $m_x$ and it is assumed that $m_x$ is an independent random variable uniformly distributed on $[0, 1]$ as in [14, Section 18]. TX $x$ would transmit if $m_x < m_z$ for all $z \in \Phi_p \cap S(a)$.

**Lemma 2:** When $\Phi_p(S(1)) = n$, the medium access probability for $a \geq 1$ is given by
\[
\text{MAP}(a, n) = 1 - \left(1 - \frac{(1 - a^{-2/\alpha} + 1)}{(n + 1)a^{-2/\alpha}} \right)^{n+1}, \quad (11)
\]
where $\rho$ is defined in Lemma 1.

**Proof 2:** Since in contention domain $S(a)$, the probability that backoff time is less than $t$ is given by $t a^{-2/\alpha}$, substituting $\rho = t a^{-2/\alpha}$ into (7), and evaluate the probability that there are no points in $S(a)$, we get
\[
\text{MAP}(a, n) = \int_0^1 \Pr[\Phi_p(S(1)) = n | \Phi_p(S(1)) = n] dt
\]
\[
= \int_0^1 (1 - t a^{-2/\alpha})^n dt
\]
\[
= \frac{1 - (1 - a^{-2/\alpha})^{n+1}}{(n + 1)a^{-2/\alpha}}. \quad \square
\]

We then discuss an asymptotic expression in a dense scenario.

**Proposition 1:** As $\lambda_p \to \infty$,
\[
\text{MAP}(a, n) \simeq \frac{1}{1 + na^{-2/\alpha}}. \quad (12)
\]

**Proof 3:** As $\lambda_p \to \infty$, $n \to \infty$,
\[
\lim_{n \to \infty} \left[ 1 - (1 - a^{-2/\alpha})^{n+1} - \frac{1}{1 + na^{-2/\alpha}} \right] = 0. \quad \square
\]

### 3.2 Interference and SIR

Let the location of RX associated with TX $x \in \Phi_p$ be denoted by $R_x \in \mathbb{R}^2$. The interference-plus-noise power level at $R_x$ is assumed to be approximate to that at TX $x$, and it is also assumed to be approximate to the CST $\Theta$ as in [4] (hereinafter referred to as the “SIR approximation”). This assumption is reasonable because the interference level is guaranteed to be less than the CST due to the carrier sense mechanism. In this case, the signal-to-interference power ratio (SIR) is given by
\[
\text{SIR}(a) := \frac{G_{R_x}P}{\theta} = \frac{G_{R_x}P}{\Theta a^2} = \frac{SIR_1}{a^2}, \quad (13)
\]
where $SIR_1 := G_{R_x}P/\theta$ represents SIR when $a = 1$.

The SIR approximation is equivalent to the approximation that the nearest TX is on the border of $S(a)$, thus, it underestimates the distance and results in underestimation of the SIR (13). To compensate them, we discuss the nearest neighbor distance after applying IPS. In Section 3.1, we consider the situation where $n$ TXs are in $S(1)$ before applying IPS. For the ease of evaluation, we assume a PPP $\Phi'$ with the same density,
\[
\lambda' := \frac{n}{|S(1)|}. \quad (14)
\]
outside the contention domain $S(a)$. According to [15, 2.9.1], the distribution of the distance to the nearest TX in $\Phi' \setminus S(a)$ is given by
\[
G(r) = 1 - \Pr[\Phi'(b(x, r) \setminus S(a)) = 0]
\]
\[
= 1 - \exp(-\lambda'(r^2 - \delta(a)^2)), \quad r > \delta(a) \quad (15)
\]
\[
g(r) := \frac{d}{dr}G(r) = 2\lambda' r \exp(-\lambda'(r^2 - \delta(a)^2)), \quad (16)
\]
where $b(x, r)$ represents a disk of radius $r$ centered at $x$. Thus, the expected distance is given by
\[
E[r] = \int_{\delta(a)}^{\infty} r g(r) dr
\]
\[
= \delta(a) + \frac{\sqrt{\pi} \delta(1)}{2 \sqrt{n}} \exp(na^{-2/\alpha}) \erfc\sqrt{na^{-2/\alpha}}. \quad (17)
\]
Finally, to compensate the underestimation, the nearest neighbor distance should be reduced from $\delta(a)$ to $E[r]$. In this case, the SIR is estimated by
\[
\text{SIR}'(a) := \text{SIR}(a) \frac{A\delta(a)^{-\alpha}}{A \left(E[r]\right)^{-\alpha}}
\]
\[
= \frac{SIR_1}{a^2} \left[ 1 + \frac{\sqrt{n}a^{1/\alpha}}{2 \sqrt{n}} \exp(na^{-2/\alpha}) \erfc\sqrt{na^{-2/\alpha}} \right]^\alpha. \quad (18)
\]

### 3.3 Throughput Modeling

We evaluate the long-term throughput of WLANs under saturated traffic conditions and interference-limited situations by the product of the medium access probability (11) and Shannon capacity with SIR (18) assuming that interference is equivalent to Gaussian noise,
\[
r(a) := \text{MAP}(a, n) \cdot \log_2(1 + \text{SIR}'(a)). \quad (19)
\]
The IPS reduces the number of neighboring potential TXs while also reducing SIR. That is, there is a trade-off between the number of neighboring potential TXs and SIR, and the careful adjustment of parameter $a$ is required to achieve high throughput.

If we set the value of parameters $\alpha$, $n$, and $SIR_1$, we can numerically find the optimal value $a^* := \arg\max_ar(a)$, i.e., the optimal CST $\Theta a^*$ and transmission power $P/a^*$, but an explicit expression for $a^*$ cannot be derived.
3.4 Explicit Expression of Optimal CST and Transmission Power

To achieve an explicit solution, we first use the SIR approximation, i.e.,
\[ r(a) \approx MAP(a, n) \cdot \log_2(1 + SIR(a)), \quad (20) \]

and then discuss the asymptotic expression in a dense scenario. As \( \lambda_p \to \infty \),
\[ (20) \approx MAP'(a, n) \cdot \log_2(1 + SIR(a)). \quad (21) \]

In addition, assuming a high SIR regime, we can ignore the fixed value, 1, inside the logarithm in (21) and get
\[ (21) \approx MAP'(a, n) \cdot \log_2(SIR(a)) = \frac{\log_2(SIR_1/a^2)}{1 + na^{-2/\alpha}}. \quad (22) \]

(22) is useful for attaining the optimal solution, as shown in Proposition 2.

**Proposition 2:** The unique maximum of (22) is attained at
\[ a^* := \arg \max_a \frac{\log_2(SIR_1/a^2)}{1 + na^{-2/\alpha}} = \max \left\{ \left[ nW(SIR_1^{1/\alpha}/en) \right]^{\alpha/2}, 1 \right\}, \quad (23) \]
i.e., at CST \( \Theta a^* \) and transmission power \( P/a^* \), where \( W(\cdot) \) represents the principal branch of the Lambert \( W \) function [28].

**Proof 4:** The result follows from (22) by differentiation with respect to \( a \). Following a few manipulations, we get
\[ (a^{*2/\alpha}/n) \exp(a^{*2/\alpha}/n) = SIR_1^{1/\alpha}/en. \]

Using the Lambert \( W \) function, defined as the inverse of function \( u \mapsto w \exp(w) \) [28], we get
\[ a^{*2/\alpha}/n = W(SIR_1^{1/\alpha}/en). \]

Since \( a \geq 1 \), we get (23). \( \square \)

3.5 Numerical Evaluation

Fig. 3 shows numerical examples of throughput (19). It also presents results of Monte Carlo simulations with 10,000 trials, where we determine the location of RX associated with \( x \) so that the SIR is equal to \( SIR_1 \). Although there is a gap between the numerical results (19) and simulation results, we can confirm a similar trend and there is a unique optimal value in terms of throughput. The gap is due to the SIR approximation and some of the assumptions used to derive \( SIR'(a) \) in (18).

Fig. 3 also shows the approximated throughput (20)–(22). As has been discussed, the approximation is valid for high SIR regions, i.e., when \( a \) is sufficiently low to maintain \( SIR_1/a^2 \gg 1 \).

Fig. 4(a) shows the contour plot of the numerically calculated optimal value \( a^* \) in terms of throughput (19). We can see that \( a^* \) depends both on the number of neighbors \( n \) and \( SIR_1 \), i.e., SIR at \( \alpha = 0 \) dB. Fig. 4(b) shows the explicit expression \( a^* \) (23).

To confirm the effectiveness of (23) in terms of throughput estimation, Fig. 5 shows the contour plot of the following normalized loss due to approximation:
\[ \frac{r(a^*) - r(a^*)}{r(a^*)}. \quad (24) \]

The maximum loss is 15% for \( 10 \leq n \leq 100 \) and \( 10 \) dB \( \leq SIR_1 \leq 30 \) dB. Thus, we can conclude that the explicit expression (23) is useful for the IPS of the CST and transmission power for these situations.

4. Adjustment at All Transmitters

4.1 Medium Access Probability

We now consider a scenario where all TXs are assumed to
Proposition 3: As \( \lambda_p \to \infty \),
\[
MAP(a) \approx \frac{1}{1 + Ba^{-4/\alpha}} \equiv MAP'(a).
\] (28)

Proof 5: Similar to that of Proposition 1. \( \square \)

4.2 Interference and SIR

The average interference at RX \( R_x \) is approximated by the average interference at TX \( x \), and we assume \( x \) is at the origin \( o \) without loss of generality:

\[
I(a) := E \left[ \sum_{z \in \Phi_p\setminus S(a)} G_{R_x z} p \right] \approx E \left[ \sum_{z \in \Phi_p\setminus S(a)} G_{o z} p \right] = \frac{A P}{a} E \left[ \sum_{z \in \Phi_p\setminus S(a)} \|z\|^{-\alpha} \right] = \frac{A P \lambda_p}{a} \int_{\delta(a)}^{\infty} r^{1-\alpha} dr = \frac{A P \lambda_p}{a(a-2)} \delta(a)^{2-\alpha},
\] (29)

where transformation (a) follows the Campbell’s theorem [15]. Therefore, the average interference is reduced by the following factor

\[
\frac{I(a)}{I(1)} = \left( \frac{\delta(1)}{\delta(a)} \right)^{2-\alpha} = a^{-1}(a^{-2/\alpha})^{2-\alpha} = a^{1-4/\alpha}.
\] (30)

Also, desired signal level is reduced by factor \( 1/a \) due to IPS. Therefore, the SIR after IPS is given by

\[
SIR(a) := SIR_1 \frac{1/a}{a^{4-\alpha}} = SIR_1 a^{4/\alpha - 2},
\] (31)

where \( SIR_1 \) represents the SIR when \( a = 1 \) as has been defined in Section 3.

4.3 Throughput Modeling

As with (19) in the single TX adjustment scenario, individual throughput is given by

\[
r(a) := MAP(a) \log_2(1 + SIR(a)) = \frac{1 - \exp(-Ba^{-4/\alpha})}{Ba^{-4/\alpha}} \log_2(1 + SIR_1 a^{4/\alpha - 2}).
\] (32)

Note that the differences between the adjustment at a single TX discussed in Section 3 and that of all TXs are: 1) BPP is used in Section 3, whereas PPP is used in Section 4, and 2) the contention domain is reduced by a factor of \( a_x^{-2/\alpha} \) in Section 3, but by a factor of \( a^{-4/\alpha} \) in Section 4 because the surrounding TXs also reduce their transmission power.

4.4 Optimal CST and Transmission Power

We would like to find the optimal value of \( a \) that maximizes
expected throughput, i.e., \( a^* = \arg \max_a r(a) \). We thus consider the approximated throughput of (32) assuming a high SIR and dense regime

\[
   r(a) \approx \frac{\log_2(SIR_1 a^{4/\alpha - 2})}{1 + Ba^{-4/\alpha}}. \tag{33}
\]

**Proposition 4:** The unique maximum of (33) is attained at

\[
   a^* = \max \left\{ B W \left( \frac{SIR_1^{2/(\alpha - 2)}}{eB} \right) \right\}, \tag{34}
\]

This condition is equivalent to \( p^* = P/\alpha^* \) and \( \theta^* = \Theta a^* \).

**Proof 6:** The approach is the same as the one adopted in the proof of Proposition 2. Differentiating (33) with respect to \( a \) and following some manipulations, we get

\[
   a^{4/\alpha} \frac{B}{eB} \exp \left( a^{4/\alpha} \frac{B}{eB} \right) = \frac{SIR_1^{2/(\alpha - 2)}}{eB}.
\]

Using the Lambert \( W \) function,

\[
   a^{4/\alpha} \frac{B}{eB} = W \left( \frac{SIR_1^{2/(\alpha - 2)}}{eB} \right).
\]

Taking \( a \geq 1 \) into account, we get (34). \( \square \)

### 4.5 Numerical Evaluation

Fig. 6 shows the throughput (32) and its approximation (33) for deriving the optimal setting as shown in Proposition 4. It also presents Monte Carlo simulation results with 1,000 trials. We can see that all of these have a unique maximum. Although the difference between the throughput and its asymptotic throughput is not small, they have maximum values around \( a^* \) given by (34). Thus, we can conclude that despite approximations, (34) can be used for the CST and transmission power setting to achieve high throughput.

Fig. 7 shows \( a^* \) and \( a^* \), and we can see the similarity in trends. Similarly to (24), we calculated the normalized loss \( [r(a^*) - r(a^*)]/r(a^*) \) as shown in Fig. 8. We confirmed that the loss was less than 10% for high SIR condition (\( SIR_1 > 21 \text{ dB} \)). Thus, the explicit solution \( a^* \) provides a satisfactory guideline for designing the IPS.

### 5. Conclusion

In this paper, we discussed the joint adjustment of CST and transmission power. Throughput was formulated by assuming that the set of potential TXs forms a PPP. The explicit expressions of the optimal IPS of the CST and transmission power in terms of throughput were obtained as a function of SIR and the number of neighbors. Numerical results confirmed that the explicit solutions are applicable to the inversely proportional setting despite approximations.

### References


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