Subjective Super-Resolution Model on Coarse High-Speed LED Display in Combination with Pseudo Fixation Eye Movements*

Toyotaro TOKIMOTO^{†,††}, Shintaro TOKIMOTO^{††}, Kengo FUJII[†], Shogo MORITA[†],
and Hirotugu YAMAMOTO^{†,†††a)}, Nonmembers

SUMMARY We propose a method to realize a subjective super-resolution on a high-speed LED display, which dynamically shows a set of four neighboring pixels on every LED pixel. We have experimentally confirmed the subjective super-resolution effect. This paper proposes a subjective super-resolution hypothesis in human visual system and reports simulation results with pseudo fixation eye movements.

key words: subjective super-resolution, LED, predictive-coding, fixation eye movement, visual system

1. Introduction

We proposed *CyberVision* display method [1], [2] which improves the apparent subjective resolution while fixing the position of the LED lamp to emit light on the coarse LED panel. In this method, every LED pixel dynamically shows a set of four neighboring pixel data. Subjective super-resolution effect was confirmed experimentally using a high speed LED display system at a variable frame rate [2]. In order to investigate the mechanism of the effect, this paper further discusses a hypothesis in human visual system: human recognition in text area reconstructs several frames by pseudo fixation eye movement (PFEM) function in a short time. A simulation model with the PFEM function shows perceived images that resemble some of the subjective resolution.

1.1 Previous Research 1: High-Speed Display

A complicated process is undertaken in human visual system to smoothly interpret visual data displayed at high speed. Hammett's paper [3] discusses both motion blur and motion sharpening [4], [5] as a part of the process. Although human visual system integrates information displayed approximately at 120 msec in a daylight, it does not have a capability to respond to such high amount of information instantaneously. Due to this incapability, it was expected that moving images were perceived as a camera-like motion blur

and the position of a moving image would be more uncertain than that of a still image. However, the blurring and smearing of moving objects are much smaller than the expectation based on integration time of visual system [6]. Hammett [3] introduces *Motion Sharpening*, which is a phenomenon of blurred images appearing clearer than static images in the moving state. Burr [6] concludes that spatial blur predicted from temporal characteristics of human visual system cannot be detected. A linear scroll display in *PoleVision* system [7] confirms this conclusion as its displayed image appears clear in spite of character strings varying its position continuously.

Sugihara presented a model that explains the mechanism of the illusion of arrows (Muller-Lyer illusion) [8] with dislocation of skeletons due to image blurring [9]. First step is to blur out the arrow figure. The blurred figure is binarized. Next calculate Voronoi diagram [10], [11] from boundary points of the blurred figure to obtain a skeleton diagram. The arrow inside the skeleton thus obtained explains that the length looks different and shows the mechanism of illusion. The mechanism of this illusion explains that the processing mechanism of Hammett's *Blur and Sharpening* is routinely performed in the human visual system. This can be observed not only in the processing of moving images but also in the processing of still images.

From our experience with *CyberVision* [1], [12], displaying peripheral pixel information in a dynamic manner tends to increase image resolution. However, there was insufficient understanding of a mechanism that contributes to this phenomenon. Dynamically converting RGB pixels in the similar dynamic manner is known method for increasing resolution [13]. However, this method results in a displacement of the physical pixel position, *CyberVision* method does not cause a change in the position of light emitting element, the two methods are fundamentally different and it does not explain the theory behind the increased resolution of *CyberVision* method.

In this study, experiments were carried out to confirm this subjective super-resolution. The results led to deriving a hypothesis of human visual system on *CyberVision* method.

1.2 Previous Research 2: Predictive Coding and Illusion

Watanabe *et al.* [14] succeeded in reproducing an illusion of rotation snake by making deep neural networks system (DNNs) learn from videos with camera shake. It seems

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[†]The authors are with Utsunomiya University, Utsunomiya-shi, 321-0904 Japan.

^{††}The authors are with DaoApp Technology Co., Ltd., Taipei City, 11155 Taiwan.

^{†††}The author is with JST ACCEL, Utsunomiya-shi, 321-0904 Japan.

*Earlier version of this paper was presented at IDW'18.

a) E-mail: hirotugu@yamamotolab.science

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that predictive coding theory [15] is a theoretical mechanism for generating general illusion including several motion illusions [16].

1.3 Previous Research 3: Fixation Eye Movements

“Fixation eye movement (FEM)” known as fine movement of eyeball (Fig. 1) shows three types of motion, drift, flick (micro saccade), and tremor. These depend on their frequency, amplitude and frequency of occurrence [17]–[19].

- 1) Tremor: amplitude 20-40 arcsec, average frequency 83.68Hz (12msec cycle)
- 2) Flick: It is classified into 3 types. amplitude 3-50 arc minutes
 - 2-1) 200-300 msec reciprocating motion
 - 2-2) 40-50 msec reciprocating motion
 - 2-3) Continuously move in the same direction
- 3) Drift: 5 arcmin or less, 90-180 msec or less

Due to FEM, the retinal image is constantly shaking and requires adjustments to project a stable image. A projected image to retina has physical motion by the fixation eye movement. When there is no relative motion in the projected image, our brain makes a judgement as follows; “the physical object should be stationary only by the eye being shaken”. Human visual system has a shake correction mechanism by motion vector detection which detects a moving object when relative motion is observed. It is thought that

Fig. 1 Pattern diagram of fixation eye movement on retina.

Fig. 2 Mechanism for correcting image shake due to fixation eye movement.

a human visual system is performing a function similar to camera shake correction to a state in which an image reflected on the retina is blurred due to FEM [20].

One of the model to capture a phenomenon canceling out FEM is done by NTT Communication Science Basic Research Laboratory. They proposed that the key point of shake correction is the presence of relative motion in an image [20], [21]. Figure 2 illustrates the interpretation of the relative motion and how our brain and eyes perceive and project the data. When there is no relative motion (stationary background), the brain judges that the eyes are only vibrating but the whole image is not moving. When there is relative motion (moving objects in stationary background), the brain detects moving objects.

1.4 This Work: Subjective Super-Resolution Hypothesis

The purpose of this paper is to propose a hypothesis in human visual system to explain the subjective super-resolution effect, as mentioned in Sect. 1.1. Our model is inspired by blur & sharpening (Sect. 1.1), the predictive coding (Sect. 1.2) and FEM cancelation phenomenon (Sect. 1.3).

Section 2 describes how our high-speed LED display system is set up, and summarizes experimental results of the subjective super-resolution effect. We propose a model to explain the subjective super-resolution effect in Sect. 3 and show numerically reconstructed results obtained with the model in Sect. 4.

2. Subjective Super-Resolution Effect

CyberVision display method [1], [2], as shown in Fig. 3, displays peripheral pixel information in a dynamic manner at a high-frame rate. For LED elements with a low-density dot configuration, video data are displayed at the same LED lattice point in the order of subframes 0 to 3 at illustrated in Fig. 4, that is, in order of micro-rotation.

Figure 5 demonstrates an example of generating subframes for *CyberVision* method from a Chinese character composed of 8x8 dots font.

Fig. 3 *CyberVision* display method.

Fig. 4 Subframe generation by micro rotation subsampling in *CyberVision* display method.

Fig. 5 An example of creating subframes of the *CyberVision* method from 8x8 dots font. (a) Original sample Chinese character which is 8 × 8 font. (b) Simple average of 4 neighboring pixels for (a). (c), (d), (e), (f) are subframe 0–3 of (a) by *CyberVision* method.

Fig. 6 Previous method of increasing the resolution by dynamically changing data correspondence with RGB pixels.

2.1 Previous Work for Sub-Pixel Resolution

A conventional method to increase a resolution using the same number of pixels is by dynamically changing data correspondence with RGB pixels as shown in Fig. 6. For example, a TV signal of 30 fps is divided into four subframes and displayed clockwise as shown from Fig. 6 (a) to (d). In this display method, since the combination of RGB pixels formed as actual pixels is physically shifted by half pixel, it corresponds to twice the length and width, and resolution improvement equivalent to four times with the same number of pixels can be expected.

However, in recent years, a mainstream technology is to use 3-in-1 LED lamps incorporating RGB LED semiconductor pixel chips. Likewise, even in outdoor LED panels,

Fig. 7 (a) Experimental apparatus. (b) Original data.

a method of constructing high luminance pixels with one aggregate of RGB lamps as one pixel is mainstream. For this reason, the technique to raise the resolution by dynamic combination RGB pixels in this section is no longer used.

2.2 Experiments on Our Proposed Micro-Rotation Subsampling

A device used for the experiment a high-speed LED display system (Fig. 7) that capable of displaying full color images at an arbitrary frame rate of 960/k fps where k is a positive integer representing consecutive repeated frame number [23]. In order to input display data numerically and output it, experiment was carried out under the condition that $\gamma = 1.0$ and gamma inverse correction is not performed.

For the six subjects, the following four patterns were evaluated by the two-pair comparison method and Scheffe's analysis of variance was performed. The main purpose of the experiment is to confirm the difference in superiority between Case 1 and Case 3. Additionally, Case 2 and Case 4 were tested by inserting interlaced black frames. These cases are in conjunction with a protocol used in the previous experiment [23] for the two-pair comparison method.

[Case 1] 40fps MR (micro-rotation) luminance 40%
 [Case 2] 80fps MR/black (= insert black display frame next to each MR subframe) luminance 80%
 [Case 3] 40fps average frame (= repeat the average subframe of 4 pixels 4 times) luminance 40%
 [Case 4] 80fps average/black (= insert black display next to each average subframe) luminance 80%
 where the average frame is calculated from a simple average of four adjacent pixels;

$$D_{i,j} = 1/4(x_{2i,2j} + x_{2i,2j+1} + x_{2i+1,2j+1} + x_{2i+1,2j}).$$

A preliminary experiment using this experimental apparatus, confirmed improvement in subjective resolution at 120 fps or less and significant degradation in image quality which may cause visual impairment to the subject due to strong flickering at 15fps or less. Hence the frequency at 40fps was selected from range between 15fps to 120fps. Since the average luminance is reduced by half by inserting the black frame, the adjustment is conducted so that the average luminance is made substantially equal by displaying the Case 1 and Case 3 with half the luminance when black insertion is performed in advance.

Fig. 8 Scale of perceived resolution.

Fig. 9 (a) The original image data (16x16 dots font), (b) case1: average display photograph, (c) case3: CyberVision 40fps display photograph (1/8 shutter speed)

2.3 Results

Comparing the scale values of each method obtained as a result of the experiment, Case 1 results in the difference of about 0.875 for Case 3 and about 0.854 for Case 4, where yardstick $Y_{0.01} = 0.657$ or more. For this reason, it is recognized that Case 1 has a clear significant difference in subjective resolution up with no significant difference was obtained for Case 2.

Since the same light emitting point is flashing continuously from subframe 0 to subframe 3, Case 1 and Case 3 displayed the same image data when photographed with a fixed camera (Fig. 9 (b), (c)). In the previous study [24], [25], human visual system is supposed to integrate 120 msec information. As the display speed of 40 fps displays 4.8 frames at 120 msec, this means that Case 1 and Case 3 are recognized as the same resolution in the human visual system as in the fixed camera. However, the experiment shows that Case 1 and Case 3 have significant differences in recognition by subjects. This suggests that in the Cyber Vision display method, the human visual system cannot be explained with a fixed camera model. It is confirmed that when the peripheral pixel information is temporally shifted to the physically fixed same point and displayed on a dynamic basis at a certain frequency, subjects perceived an image with a higher resolution than the averaged display.

3. Super-Resolution Cognitive Mechanism Hypothesis

What kind of cognitive mechanism in the human visual system supports the subjective super-resolution by the CyberVision display method? We propose the following two hypotheses from previous studies (as mentioned in Sects. 1.1-1.3) and this validation experiment (Sect. 2.3).

Fig. 10 (a) Original data (16x16 dots font), (b) micro-rotation reconstruction ($r = 4$, $\delta = 2$), (c) simple PFEM function reconstruction ($r = 4$, $\delta = 1$) cf. (b) and (c) are simulation images in Sect. 4. Please refer to Sect. 4 for simple PFEM function. See Appendix B for a description of analytic resolution (r) and movement width (δ).

3.1 Cognitive Mechanism Hypothesis 1

[Hypothesis 1]

Step 1: Perform a blurring process within a certain range.

Step 2: Perform and simplify Sharpening process.

Motion vector detection is performed on a simplified target.

Step 3: Compare the simplified image in Step2 with memorized images.

However, subjective super-resolution of this time cannot be explained only with this hypothesis 1. If motion vector detection is properly performed and micro-rotation is detected, it should be reconstructed to a resolution almost similar to that of the original image as shown in Fig. 10 (b) as a reconstruction process. However, the display on the experimental device looks like Fig. 10 (c).

The motion time of micro-rotation in the CyberVision display system is estimated approximately 25 msec (40 fps) from results of the experiment. It seems that human visual system is incapable of tracking the motion vector of micro-rotation perfectly. For instance, if the motion vectors are correctly detected and reconstructed 4 subframe, the subjective resolution will theoretically be four times as high as the original image.

The subjective super-resolution (Sect. 2) confirmed in the experiment was subjectively recognized as super-resolution, as the subframes displayed dispersively on the time axis were reconstructed on the space axis. However, the function that performs reconstruction process does not appear to be as refined to detect a motion vector for the reconstruction of subframes. Human visual system utilizes FEM (previous research 1.3) which is an example of an image processing that performs eye blur correction without detecting motion vectors. There might be a recognition process that samples frames on the time axis then on the space axis using a reconstruction function that resembles FEM. FEM are categorized in three types of movement; Tremor, Drift, and Flick (Micro-saccade). Each movement vibrates for 300msec or less. The time of 300msec or less for reconstruction is referred to as 'micro-time'.

Fig. 11 Image diagram of reconstruction by pseudo fixation eye movement function in human visual system.

3.2 Cognitive Mechanism Hypothesis 2

There is a previous research [26]–[28] to simulate the movement of FEM (see Appendix A) as well as the research on FEM in Sect. 1.3. This is called pseudo fixation eye movement (PFEM).

In the character area, we propose the following hypothesis as a reconstruction process within micro time.

[Hypothesis 2]

When micro-vibration between frames is detected in a character area, several frames within a micro time are reconstructed by a pseudo fixation eye movement function.

The reason why it was clearly separated from “character area” in Hypothesis 2 is as follows. In the preliminary experiment, subjective super-resolution is easily generated in the case of a monochromatic background image than the case of superposing a character string on a natural image.

4. Simulation by Pseudo Fixation Eye Movement

Although PFEM in Appendix A tries to simulate the whole of FEM, local movement within a short time of FEM could be expressed easily and pseudo by a simple function. Focusing on Flick and Drift among fixation micromotions, we propose the simple function shown in Fig. 12 (simple PFEM function).

A simple PFEM can be expressed in a cyclic function as illustrated in Fig. 12. First frame in four subframes is set at the origin and the rest are set on a circumference of the circle. The latter three subframes are then shifted by $3/4\pi$ on the circumference in a counterclockwise direction, resulting in eight patterns of movements as shown in Fig. 13.

Based on the number of dots in the display image, in order to increase the display analytical resolution, the dots are divided into r and $r - 1$ dots are added. Let δ be the displacement of subframe at this virtual resolution. In virtual resolution space, the units of r and δ are dots (Appendix B). In the following, the discussion of simulation results proceeds with $r = 12$ or $r = 6$, but as shown in Appendix B,

Fig. 12 Simple PFEM function.

Fig. 13 Eight kinds of movement patterns by our simple pseudo fixation eye fine movement function.

Fig. 14 simple PFEM eight pattern, $r = 3$, $\delta = 1$

the diagram is used with we use $r = 3$ or $r = 4$. The diagram is the simulation result of the δ value corresponding to the new r value. When the displacement $\delta = 4$ on analytic resolution $r = 12$, shown in Fig. 14 (*), the perceived resolution appears higher than the average value for any of the 8 patterns of the simple PFEM function. When the displacement is doubled ($\delta = 8$), some characters appear to

Fig. 15 simple PFEM eight patter, $r = 3$, $\delta = 2$

Fig. 16 Magnified images of (a) P4 in Fig. 14 and (b) P8 in Fig. 15.

improve the apparent resolution, but variations in the light emitting points in any pattern become noises, the subjective resolution is decreasing as shown in Fig. 15. Thus, the recommended displacement is limited from 3 to 6 in this simulation.

(*) Note that analyses of the simulation were conducted with resolution of 12 or 6 dot. However, the figures below are presented in resolution of 3 or 4 dots for printing purpose. The relational expressions used in this simulations are;

- (1). resolution $n = 12/k$,
- (2). movement amount δ/k , where k is positive integer.

Because Fig. 14 and Fig. 15 are smaller, Fig. 16 shows a magnified view. In the former, the degree of opening of the upper part of the character γ has been improved and the subjective resolution has been raised since the central Chinese character can be read without being crushed. Although the noise component increases considerably in the latter case, it can also be considered that the subjective resolution increased since γ and Chinese character are not collapsed.

5. Discussion

Through this confirmation experiment, preliminary experiment, and computer simulation, it demonstrated a great potential to the subjective super resolution that samples frames on a time axis and reconstructs them on a space axis using PFEM function. The reconstruction process does not

Fig. 17 (a) Neighborhood 4-pixels average (photograph), (b) Simulation result with simple PFEM ($r = 4$, $\delta = 1$), (c) micro-rotation reconstruction ($r = 4$, $\delta = 2$)

appear as accurate micro-rotating motion vectors for the reconstruction of subframes. Displacement, δ , tends to be inversely proportional to the display speed. Although preliminary experiments have suggested these possibilities, it is necessary to establish a form of scaling and its experimental method. These can be used as an evaluation tool similar to the confirmation experiment discussed in Sect. 2.2. In fact, it was reported that the image quality was not observed than optimal micro-rotating reconstruction values ($r = 4$, $\delta = 2$) in Sect. 3.1. However, this observation may be corrected by possible optimal micro-rotating reconstruction value of $r = 12$, $\delta < r/2$ if a more detailed evaluation in image quality is established. The scaling of the subjective super-resolution is still under examination and further consideration are required including fundamental concepts such as fractal dimensions. Also, in Sect. 2.3, it was observed that the subjective resolution tends to decrease when interlaced black frames are inserted. Farther examination of this experiment may lead to comprehend the sampling rate of human visual system. The examinations include detailing the conditions of the experiment and defining a relationship of display frequencies to the number of additional black frames.

6. Conclusion

Experiments have shown that the *CyberVision* display method improves subjective resolution. If motion vectors of micro-rotating subframes can be detected from the previous research, the reconstructed image by the predictive coding system of human visual system should be close to the original image as the subjective resolution. However, the simulation results suggested that other reconstruction mechanisms may be adopted to enhance the subjective resolution. In this research, we focused on examination of basic hypotheses of human visual function and its simulation based on the results of preliminary experiments. As shown in Sect. 5. Discussion, some trends was able to be grasped. In the near future, we would like to clarify the cognitive mechanism of human visual system by examining the definition of the subjective resolution scale and its experimental method, and checking the subjective super resolution in more detail. Future work will include the correlation between display frame rate (Nfps) and the displacement (δ) of the reconstruction in subjective super resolution.

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Appendix A:

The following models have been proposed as simulation models for FEM.

- 1) 1/f fluctuation
- 2) random walk model [28]
- 3) fractional Brown movement [26]
- 4) Fractal dimension analysis [27]

An measurement example of FEM is in the paper of Tokudome et al. [26]. Engbert et al. [28] proposed a self-avoidance random walk model to explain the nature of FEM. 2) The random walk model is defined by Mean Square Displacement (MSD) Eq. (A·1), and is mainly used for evaluation of particles that randomly walk as an index for evaluating the degree of divergence of trajectories that move stochastically.

$$D^2(l) = \frac{1}{N-1} \sum_{i=1}^{N-1} \|\vec{x}(i+l) - \vec{x}(i)\| \quad (\text{A} \cdot 1)$$

3) Tokudome et al. [26] proposed fBm with $H = 0.7$ as a pseudo fixation micromotion function among fBm with arbitrary Hurst function H .

4) Fractal dimension analysis

Yoshimatsu et al. [27] proposed fractal dimension analysis for the drift and tremor components of FEM.

Appendix B:

In order to increase the display analytical resolution (described as r below), dots in the display frame (subsampling frame) are divided by r . The consequent matrix of the dot space becomes $rn \times rn$ dots when the original matrix is $n \times n$ dots. The unit of r in this new space is a dot. δ is the displacement of the reconstructed frame in this new resolution space. The unit of δ is a dot.

Figure A·2 shows a P4 pattern display of (a) $r = 4$ and $\delta = 1$, and (b) $r = 12$ and $\delta = 3$. (a) and (b) have the same display pattern but the analytical resolution is different. As the analytical resolution increases, the light emitting point becomes relatively smaller, making it difficult to see in

Fig. A-1 analytical resolution example. $r = 4$

(a) $r=3, \delta=1$ (b) $r=6, \delta=2$

Fig. A-2 Analytical resolution comparison for pattern 4

prints. Normally our simulation is performed with analytic resolution $r = 6$ or 12 , but in this paper, we use $r = 3, 4$ as a figure for a printing purpose.

The above is a theoretical explanation and will be applied to the display of the actual experimental equipment in the Sect. 2.2. Since this display consists of 6 mm pitch display dots, it can not be displayed between these dot pitches. If analytical resolution $r = 12$, the unit of analytical resolution $= 6 \text{ mm}/12 = 0.5 \text{ mm}$. Displacement $\delta = 4$ at that time is $4 \times 0.5 \text{ mm} = 2 \text{ mm}$. When the viewer sees the display of the *CyberVision* method, it looks like the reconstruction with the simple PFEM function with $r = 12, \delta = 4$ (same as $r = 3, \delta = 1$). In that case, the human visual system may be reconstructed with 3 times pseudo resolution of the dot pitch for a 6mm pitch display.

Toyotaro Tokimoto was born in Tokyo, Japan, in 1955. He received his B.S. and M.S. in Mathematics from Rikkyo Univ. and Gakushuin Univ. in 1981 respectively. He received Ph.D. degree from Utsunomiya Univ. in 2019. He engaged in research and development for audio and visual products at Casio Computer Co., Ltd. during 1981-1988. He founded venture company AVIX inc. at 1989 and he developed LED display systems and services by after image technology, and he was CEO during 1989-

2009, EVP & CTO during 2009-2014. He is with DaoApp Technology Co., Ltd., Co-founder and CTO since 2014. His recent work activities have included digital signage, high-speed LED display system, subjective super-resolution and human visual system. He was recipient of Excellent Poster Award 2nd Prize at IEEE GCCE 2013, and Outstanding Poster Paper Award at IDW'18.

Shintaro Tokimoto received a B.E. with Honors and M.Sc.Eng. in Civil Engineering from the University of Adelaide and the University of Western Australia respectively. His areas of expertise include Civil Engineering whereby he has over 7 years of experience working in mining industry throughout Australia as a consultant and an academic researcher. He is a researcher with DaoApp Technology since Oct. 2018.

Kengo Fujii received his B.E. degrees from Utsunomiya University in 2017. Currently, he is a graduate student in Dept. of Optical Engineering at Utsunomiya University. His research interests include aerial imaging on display, heat and sound respectively. He is a member of The Japan Society of Applied Physics (JSAP) and The Laser Society of Japan (LSJ).

Shogo Morita received his B. E. degrees from Utsunomiya University in 2017. Currently, he is a graduate student in Dept. of Optical Engineering at Utsunomiya University. His research interests include implementation of high-speed LED driving circuit by use of FPGA and its application for single pixel imaging.

Hirotugu Yamamoto was born in Wakayama, Japan, in 1971. He received his B.E., M.E., and Ph.D. degrees from the University of Tokyo. From 1996 to 2008, he was an assistant professor at Tokushima University. From 2009 to 2014, he was an associate professor at Tokushima University. Since 2014, he has been an associate professor at Utsunomiya University. His recent work activities have included aerial display, digital signage by use of high-speed LED panels, 3D display, secure display, and information photonics for emerging information communications technologies.

Dr. Yamamoto was a recipient of Young Scientist Award for the Presentation of an Excellent Paper, The Japan Society of Applied Physics, Outstanding Poster Paper Award at IDW'03, IDW'04, IDW'07, IDW'08, IDW'09, IDW'10, IDW'11, IDW'12, IDW'13, IMID2014, IDW'14, IMID2015, IDW'15, IMID2016, and IDW'16, Best Paper Award at DHIP2011, IDW'11, and IDW'15, Best 3D Demonstration Award at Stereoscopic Displays and Applications 2012, and the Gen-Nai Grand Prize from the Ozaki Foundation of Japan.