Efficient Algorithm for Sentence Information Content Computing in Semantic Hierarchical Network

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SUMMARY We previously proposed an unsupervised model using the inclusion-exclusion principle to compute sentence information content. Though it can achieve desirable experimental results in sentence semantic similarity, the computational complexity is more than \(O(2^n)\). In this paper, we propose an efficient method to calculate sentence information content, which employs the thinking of the difference set in hierarchical network. Impressively, experimental results show that the computational complexity decreases to \(O(n)\). We prove the algorithm in the form of theorems. Performance analysis and experiments are also provided.

**key words:** information content, sentence IC, inclusion-exclusion principle, difference set, hierarchical network

1. Introduction

Nowadays semantic textual sentence similarity becomes a research hotspot \([1], [2]\) in short text related area of natural language processing (NLP). From the view point of information theory, the essence of natural language is the carrier of information. The amount of information can be calculated by information content (IC)\([3]\). IC has been successfully applied in word similarity computation\([3]–[5]\). In sentence similarity computation research, Wu and Huang \([6]\) proposed a sentence IC computational model utilizes the inclusion-exclusion principle from combinatorics. To the best of our knowledge, it’s the first model that can compute non-overlapping sum IC for a sentence\([1], [2], [6]\). It is a fully unsupervised computational model and obtains desirable experimental results. But the computational complexity is over \(O(2^n)\)\([6]\) which becomes the bottleneck for its further applications.

To address the above-mentioned efficient issue, we propose a new model to compute sentence IC which employs the thinking of the difference set and makes use of the features of hierarchical network. Actually, many combinations of nodes share the same subsumer (the node subsumes the other nodes) which respects common IC of nodes. In the inclusion-exclusion principle model, the same common IC has been continuously added and subtracted, which causes the unreasonable waste of computation, but it is difficult to decide which combinations should be abolished. In order to avoid double counting, we add the words into the information space one by one and add information gain of the newly input one each time, which is the idea of the difference set rather than the inclusion-exclusion principle. The proof and experimental results demonstrate the consistency of IC values computing between the two models. The computational complexity decreases dramatically by employing the new method.

The contributions of this work are summarized as follows: 1) it presents a high-efficiency computational model by exploiting the thinking of the difference set for computing sentences IC, 2) it establishes a theoretical system with lemmas and theorems for sentence IC computing, and 3) the elaborated algorithms, comparative analysis and experiments about computational complexity are given.

2. Preliminaries

Following the standard argumentation of information theory, Resnik \([3]\) defines information content (IC):

\[
IC(c) = -\log P(c),
\]

where \(P(c)\) refers to statistical frequency of concept \(c\). The implementation of \(P(c)\) is

\[
P(c) = \frac{\sum_{w\in\text{words}(c)}\text{count}(w)}{N},
\]

where \(\text{words}(c)\) is the set of the words contained in concept \(c\) and sub-concepts of \(c\) in the hierarchy of semantic net, \(N\) is the sum of frequencies all the words contained in semantic hierarchial net.

Let \(c_1, \cdots, c_n\) be the collection of concepts, we defined the quantity of common information of \(n\)-concepts*:

\[
\text{commonIC}(c_1, \cdots, c_n) = \text{IC}\left(\bigcap_{i=1}^n c_i\right) = \text{IC}\left(\bigcup_{j=1}^m c_j\right),
\]

where \(c_j \in \text{subsum}(c_1, \cdots, c_n)\), \(m\) is the total number of \(c_j\). For physical meaning of \(\cap\) and \(\cup\), see Sect. 3.1 for details.

*In previous work\([6]\), we define common IC of \(n\)-concepts is

\[
\text{commonIC}(c_1, \cdots, c_n) = \text{IC}\left(\bigcap_{i=1}^n c_i\right) \equiv \max_{c_j \in \text{subsum}(c_1, \cdots, c_n)} [-\log P(c_j)],
\]

where, \(\text{subsum}(c_1, \cdots, c_n)\) is the set of concepts that subsume all the concepts of \(c_1, \cdots, c_n\). In consideration of accurate calculation for multiple subsumers of \(c_1, \cdots, c_n\), here, we change it to Eq. (4).
Specially, when \( n = 1 \), Eq. (4) becomes IC of one single concept: \( \text{commonIC}(c_1) = \text{IC}(c_1) \).

Through the inclusion-exclusion principle [6], the quantity of total information of \( n \)-concepts is

\[
\text{totalIC}(c_1, \ldots, c_n) = \text{IC} \left( \bigcup_{i=1}^{n} c_i \right) = \sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \leq i_1 < \cdots < i_k \leq n} \text{commonIC}(c_{i_1}, \ldots, c_{i_k}).
\] (5)

For sentence \( S = \{c_i | i = 1, 2, \ldots, n; n = |S|\} \), where \( c_i \) is the concept of the \( i \)-th concept in \( S \), \( |S| \) is concept count of \( S \), the quantity of the information in \( S \) is

\[
\text{IC}(S) = \text{totalIC}(c_1, \ldots, c_n).
\] (6)

We can see Eqs. (4) and (5) are indirect recursion. Formally, let \( \text{Set}(p) \) be the set of nodes in path \( p \), \( \text{Root}(n) = \{p_i | \forall \text{Root}(c_i), p_i \in \text{Root}(c_i)\} \). Formally, \( \text{Root}(n) = \{p_i | \forall \text{Root}(c_i), p_i \in \text{Root}(c_i), \text{Set}(p_i) \neq \emptyset, i = 1, 2, \ldots, n\} \), \( \text{|Root}(c_i) \) means the number of paths in \( \text{Root}(c_i) \).

3. Sentence IC Computing

By employing the idea of the difference set, let \( \text{ICG}(c_n) \) be IC gained by introducing concept \( c_n \) to the set of \( n - 1 \) concepts and \( \text{intersectIC}(n|n-1) \) be the common IC shared between concept \( c_n \) and previous \( n - 1 \) concepts. Formally,

\[
\text{ICG}(c_n) = \text{IC}(c_n) - \text{intersectIC}(n|n-1)
\] (7)

Specially, define \( \text{intersectIC}(1|0) = 0 \), and \( \text{ICG}(c_1) = \text{IC}(c_1) \). Thus, sentence IC can be defined as

\[
\text{IC}(S) = \sum_{i=1}^{n} \text{ICG}(c_i)
\] (8)

The following sections will show how to compute \( \text{intersectIC}(i|i-1) \).

3.1 Basic Concepts and Functions

For convenience in the discussion, we name some concepts and define some functions:

1) HSN: Hierarchical semantic network is a semantic knowledge base with hierarchical structure such as WordNet [7]. In WordNet, content words are grouped into sets of cognitive synonyms (synsets), each expressing a distinct concept (a node in HSN). The most frequently encoded relation among synsets is the super-subordinate relation (is-a relation). All noun synsets ultimately go up to the root synset (the concept of entity).

2) SIS: Semantic information space is the space mapping of HSN through Eqs. (1) and (4). Concepts (Nodes) with the super-subordinate relation in HSN are the space with inclusion relation in SIS. The space of super concept is subsumed by that of subordinate one. SIS isn’t a traditional space which uses orthogonality multidimensional to construct, while it utilizes the inclusion relationship of the information to represent.

Physical meaning of IC is the space size of concepts in SIS: the space size of concept \( c \) is \( \text{IC}(c) \), the common space size of \( n \)-concepts is \( \text{commonIC}(c_1, \ldots, c_n) \), the total space size of \( n \)-concepts is \( \text{totalIC}(c_1, \ldots, c_n) \) and the intersection space size between concept \( c_{n+1} \) and \( n \)-concepts is \( \text{intersectIC}(n+1|n) \).

3) \( \text{Root}(c_i) \) indicates the set of paths, each path consists of sequence of nodes from \( c_i \) to the root in HSN. \( \text{Root}(n) \) is the short form of \( \text{Root}(c_1, \ldots, c_n) \).

From the definition of \( \text{ICG}(c_n) \), the quantity of the information in \( S \), the physical meaning of IC is the space size of concepts in SIS: the space size of concept \( c \) is \( \text{IC}(c) \), the common space size of \( n \)-concepts is \( \text{commonIC}(c_1, \ldots, c_n) \), the total space size of \( n \)-concepts is \( \text{totalIC}(c_1, \ldots, c_n) \) and the intersection space size between concept \( c_{n+1} \) and \( n \)-concepts is \( \text{intersectIC}(n+1|n) \).

4) \( \text{HSN}(c_i) \) expresses the set of nodes in any of path in \( \text{Root}(c_i) \). \( \text{HSN}(n) \) is the short form of \( \text{HSN}(c_1, \ldots, c_n) \).

5) \( \text{SIS}(c_i)/\text{SIS}(n) \) denotes the space occupied by the nodes of \( \text{HSN}(c_i)/\text{HSN}(n) \). \( \text{SIS}(n) \) is also the shortened form of \( \text{SIS}(c_1, \ldots, c_n) \). \( \text{SIS}(c_i) \) and \( \text{SIS}(n) \) is the size of the space \( \text{SIS}(c_i) \) and \( \text{SIS}(n) \) respectively. From the physical meaning of totalIC, we have

\[
\text{totalIC}(\text{HSN}(c_i)) = \text{IC}(c_i);
\] (9)

\[
\text{totalIC}(\text{HSN}(n)) = \text{totalIC}(c_1, \ldots, c_n).
\] (10)

3.2 Method Proving

Suppose \( \Omega \) is the universal set of all the nodes in HSN, define \( \text{Outer}(c_i) = \{c_k | c_k \in \Omega, c_j \notin \text{HSN}(c_i)\} \), \( \text{Outer}(n) = \{c_k | c_k \in \Omega, c_i \notin \text{HSN}(n)\} \). For \( \text{Outer}(n) \), we have

\[
|\text{SIS}(c_i)| = \text{totalIC}(\text{HSN}(c_i)) > 0;
\] (11)

\[
|\text{SIS}(n)| = \text{totalIC}(\text{HSN}(n)) > 0.
\] (12)

Lemma 1 (Only Outer Node Expands Space). If \( n \in \mathbb{N}^+ \), then \( c_{n+1} \in \text{Outer}(n) \Leftrightarrow |\text{SIS}(n+1)| > |\text{SIS}(n)| \).

Proof. From the relationship between HSN and SIS, we know each node in HSN holds a space in SIS. Equations (9) and (10) show the space owned by subordinate nodes embody the space possessed by super nodes. From the definition of \( \text{Outer}(c_i)/\text{Outer}(n) \), only nodes in \( \text{Outer}(c_i)/\text{Outer}(n) \) are not the super nodes of any node in \( \text{HSN}(n) \), so only \( \text{Outer}(c_i)/\text{Outer}(n) \) provide additional space for \( \text{SIS}(c_i)/\text{SIS}(n) \) and vice versa. According Eqs. (11) and (12) we can have Lemma 1. □

For the space of \( \text{SIS}(c_i)/\text{SIS}(n) \) is already held by nodes of \( \text{HSN}(c_i)/\text{HSN}(n) \), we can easily infer the following corollary from Lemma 1:

Corollary. If \( n \in \mathbb{N}^+ \), then \( c_{n+1} \in \text{HSN}(n) \Leftrightarrow |\text{SIS}(n+1)| = |\text{SIS}(n)| \).

Let \( \text{Deepest}(p) \) be the deepest node in HSN from path set \( p \). Define \( \text{Intersect}(n+1|n) = \text{Deepest}(\text{Set}(p) \land \text{Root}(c_i)) \).
Proof. From Theorem 1 and Eq. (8), sentence IC is

\[ IC(S) = \sum_{i=1}^{\infty} [IC(c_i) - totalIC(Intersect(i|i-1))] \tag{13} \]

Specially, when network degenerates to tree structure, \( \forall c_{n+1} \in Outer(n), \|Root(c_{n+1})\| \equiv 1 \). Thus, \( |Intersect(n + 1|n)| = totalIC(Intersect(n + 1|n)) \).

Let Subordinate(c_i) denotes the set of subordinate nodes of c_i in HSN. Define leaf nodes of HSN(n):\( \text{Leaf}(n) = \{ c_i \mid \forall c_i \in HSN(n), \text{Subordinate}(c_i) \wedge HSN(n) = \emptyset \} \).

Lemma 2 (Leaf Nodes Represent Space). If \( n \in \mathbb{N}^+ \), then \( |SIS(n)| = |SIS(\text{Leaf}(n))| \).

Proof. From the definition of Leaf(n): Any leaf concept can’t be subsumed by any other concepts in HSN(n), including any other leaf concepts. On the contrary, any non-leaf concept can be subsumed by at least one other concept in HSN(n). In other words, only nodes in Leaf(n) don’t have any subordinate node in HSN(n). The space of subordinate nodes subsumes the space of their super nodes. From Eqs. (10) and (12), the space size of SIS(n) can be represented by all leaf nodes of HSN(n).

Let \( \text{Leaf}(n + 1|n) \) be leaf nodes of \( HSN(n + 1|n) \). Formally, \( \text{Leaf}(n + 1|n) = \{ c_i \mid \forall c_i \in HSN(n + 1|n), HSN(n + 1|n) \wedge \text{Subordinate}(c_i) = \emptyset \} \). Then we have

Theorem 2. If \( n \in \mathbb{N}^+ \), then \( \text{intersectIC}(n + 1|n) = totalIC(\text{Leaf}(n + 1|n)) \).

Proof. From Lemma 2, \( |SIS(n + 1|n)| = |SIS(\text{Leaf}(n + 1|n))| \). According to the physical meaning of \( totalIC \), \( totalIC(\text{Leaf}(n + 1|n)) = |SIS(\text{Leaf}(n + 1|n))| \). Thus, \( \text{intersectIC}(n + 1|n) = totalIC(\text{Leaf}(n + 1|n)) \).
efficient form of Eq. (13). However, when concepts have specific features and cannot be removed at will for the further modification of algorithms, we should use full set of $\text{Intersect}(n + 1|n)$ instead of $\text{Leaf}(n + 1|n)$. In this case, Step 3 in Algorithm 1 should be deleted and the algorithm becomes the computation of sentence IC using Eq. (13).

### 3.4 Complexity Analysis and Experiments

Searching subsumers between concepts, which consists of deepest intersected nodes between paths of two nodes in HSN, is the most time-consuming computing. Let one time of comparing between two nodes be the minimum computational unit ($O(1)$).

The previous method uses Eqs. (3) and (5). According to the Binomial Theorem, the amount of combinations among concepts to find subsumers of them can be deduced [6]:

$$C(n, 1) + C(n, 2) + \cdots + C(n, n) = 2^n - 1.$$  \hfill (15)

where $n$ is the amount of the concepts in the sentence pair, $C(n, 1)$ is the number of 1-combinations from n-concepts. Actually, this method is approximate and the real computational times of precise method are more than $(0 \times C(n, 1) + 1 \times C(n, 2) + \cdots + (n - 1) \times C(n, n))$. Therefore, the computational complexity of previous method is between $O(2^n)$ and $O(n^2)$.

Efficient method employs Eq. (13) or (14). There are two layers of loops to find intersected nodes between nodes from Algorithm 1 and 2. The function Leaf($S$) at Step 3 in Algorithm 1 can be realized by no more than two layers of loops based on its definition. Generally, $|\text{Intersect}(n + 1|n)| \leq 2$ and Step 2 in Algorithm 1 could be deemed as an ordinary step rather than recursive statement. Hence, the computational complexity is about $O(n^2)$.

We use all dataset of English from [2] to setup our experiments. Because total IC of two sentences are required in sentence similarity computing, we use each joint of two sentences as one computing unit which has the max computational complexity. For convenience in IC computing using WordNet, only real nouns are employed. The experimental results show the consistency of IC values from two models.

Table 1 show the efficiency contrasted between the models. To our surprise, the computational complexity of efficient algorithm is only $O(n)$ according to the polynomial index from curve fitting of experimental results utilizing Matlab toolkit. This complexity decrease may be caused by employing LeafRoot($n$) to efficiently represent $HSN(n)$.

### 4. Conclusion

This work proposes an efficient model to compute sentence IC by utilizing the thinking of the difference set in hierarchical network. It solves the waste of computation by employing the inclusion-exclusion principle. Theoretical system with lemmas and theorems has been established for supporting the correctness of sentence IC computing. Algorithms based on the theorems are elaborated. The computational complexity decreases to $O(n)$ from more than $O(2^n)$. Efficiency improvement indicates that sentence IC model could be applied to long texts such as paragraphs or even documents.

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### References


