PAPER

Quantum Associative Memory with Quantum Neural Network via Adiabatic Hamiltonian Evolution

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SUMMARY There is increasing interest in quantum computing, because of its enormous computing potential. A small number of powerful quantum algorithms have been proposed to date; however, the development of new quantum algorithms for practical use remains essential. Parallel computing with a neural network has successfully realized certain unique functions such as learning and recognition; therefore, the introduction of certain neural computing techniques into quantum computing to enlarge the quantum computing application field is worthwhile. In this paper, a novel quantum associative memory (QuAM) is proposed, which is achieved with a quantum neural network by employing adiabatic Hamiltonian evolution. The memorization and retrieval procedures are inspired by the concept of associative memory realized with an artificial neural network. To study the detailed dynamics of our QuAM, we examine two types of Hamiltonians for pattern memorization. The first is a Hamiltonian having diagonal elements, which is known as an Ising Hamiltonian and which is similar to the cost function of a Hopfield network. The second is a Hamiltonian having non-diagonal elements, which is known as a neuro-inspired Hamiltonian and which is based on interactions between qubits. Numerical simulations indicate that the proposed methods for pattern memorization and retrieval work well with both types of Hamiltonians. Further, both Hamiltonians yield almost identical performance, although their retrieval properties differ. The QuAM exhibits new and unique features, such as a large memory capacity, which differs from a conventional neural associative memory.

key words: associative memory, quantum computing, adiabatic Hamiltonian evolution, neural network

1. Introduction

Quantum computing has attracted considerable research interest, because of its inherent parallel computing characteristic, which is achieved using superposition states. For realization of a quantum computer, reliable hardware and software must be developed. As a result of recent progress in nanotechnology, the implementation of a quantum computer has become a more realistic prospect. In fact, D-wave Systems Inc. has developed a quantum processor composed of 512 qubits [1], [2], and this achievement could be a breakthrough for the development of quantum computing architectures. Hence, quantum annealing (QA) [3], which is the quantum version of simulated annealing [4] and

is realized via adiabatic Hamiltonian evolution, has been demonstrated, although the range of application was restricted [5]. On the other hand, the development of new algorithms is still necessary for practical applications, despite the fact that a small number of powerful quantum algorithms have been proposed [6], [7]. Recently, neural computing techniques have successfully provided conventional computers with an effective means of acquiring new algorithms through learning [8]. It is worth considering the fusion of these two fields to enlarge the application fields of quantum computers. In fact, some pioneers have investigated the quantum analog of neural networks. Since Kak first proposed a quantum neural network (QNN) model in 1995 [9], various QNN models have been proposed (see the reviews given in Refs. [10], [11]). Quantum-inspired evolutionary algorithms have been investigated, and the concept and principles of quantum computing, such as a qubit and the superposition of states, have been incorporated into neural computing [12], [13]. Contrary to these successful proposals, which have contributed to the improvement of neural networks, Sato et al. have introduced the concept of neural computing to quantum computing [14]. These researchers have introduced the quantum dynamics of adiabatic Hamiltonian evolution [15] with a QNN to neural computing and solved combinatorial optimization problems; this approach is called neuromorphic adiabatic quantum computation (NAQC) [14], [16]. Further, Osakabe et al. have followed the analogy between a QNN and a neural network and proposed a quantum learning method for a QNN by imitating the Hebb rule [17]. Similar to these studies, several methods to introduce neural functions into quantum computing have also been explored. Ventura et al. have introduced the association function, which is a fundamental function of neural networks, to quantum computing, and proposed a quantum associative memory (QuAM) [18]. These researchers have used quantum logic gates to memorize patterns and modified Grover's database search algorithm [7] to recall memorized patterns. Peruš et al. have reported that the mathematical formalisms of an associative neural network model and quantum theory are analogous [19]. They have composed a QNN and implemented a quantum content addressable memory. Further, Allauddin et al. have incorporated the cytoskeleton of a biological neuron into a QNN and trained the QNN to implement a content addressable memory [20].

Taking into account the hardware compatibility of adi-

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adiabatic Hamiltonian evolution, which is demonstrated by D-wave Systems Inc., this paper proposes a novel QuAM model realized on a QNN, the dynamics of which is based on QA and NAQC. In the following, we explain some of the fundamental theory of our proposed QuAM model and discuss its memorization and retrieval procedures. To investigate the detailed dynamics of our QuAM, we examine the performance of two Hamiltonians in the context of multiple pattern recognition. Based on presented numerical simulation results, we evaluate the QuAM performance.

2. Basic Theory

In this section, we explain adiabatic Hamiltonian evolution, which is utilized in the dynamics of our QuAM model.

2.1 Quantum Annealing

The first formulation shown in Eq. (1) was proposed by Kadowaki et al. in 1998 [3], to solve combinatorial optimization problems such as determining the ground state of a spin glass. This method is known as QA and the adiabatic time evolution of its Hamiltonian is defined as

$$H(t) = \Gamma(t)H_{\text{tun}} + H_{\text{fin}}, \quad (1)$$

where H_{tun} and H_{fin} denote the tunnel and final Hamiltonians, respectively. H_{tun} allows state changes via tunneling and yields faster state transition to global minima. H_{fin} is chosen according to the desired operation or target problem. Because $\Gamma(t)$ is a decreasing function in time, the effect of H_{tun} is reduced with increasing t . Finally, the quantum state converges to a ground state of H_{fin} .

2.2 Adiabatic Quantum Computation

Farhi et al. proposed another formulation of adiabatic Hamiltonian evolution, which is shown in Eq. (2) and which has been used to solve the 3-satisfiability (3-SAT) problem [15]. This algorithm is known as adiabatic quantum computing (AQC) and its computing performance is considered to be closely related to that of QA. The time evolution of the Hamiltonian is given as

$$H(t) = \left(1 - \frac{t}{T}\right)H_{\text{init}} + \frac{t}{T}H_{\text{fin}}, \quad (2)$$

where T denotes the period of the Hamiltonian evolution. The initial Hamiltonian H_{init} is given as a superposition of all states, and the final Hamiltonian H_{fin} includes solution states. The initial state $|\Psi(0)\rangle$ is given as the ground state of H_{init} , which includes all possible states, such that

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2^N}} \sum_{z=0}^{2^N-1} |z\rangle, \quad (3)$$

where N is the number of qubits. $|z\rangle$ is the decimal notation of the quantum system and is expressed as

$$|z\rangle = |x_N\rangle \otimes \cdots \otimes |x_2\rangle \otimes |x_1\rangle = |x_N \cdots x_2 x_1\rangle, \quad (4)$$

where $|x_i\rangle$ denotes a single qubit state and takes $|0\rangle$ or $|1\rangle$ exclusively. If a sufficiently large T is chosen, the quantum system evolves adiabatically, and the adiabatic theorem [21] indicates that the quantum system state remains near the ground state during the evolution. Along with increasing t , non-desired states vanish gradually. Finally, desired states as optimal solutions given by H_{fin} can be found by measuring $|\Psi(T)\rangle$. However, if any energy level crossing occurs during the evolution, successful operation is not guaranteed.

2.3 Neuromorphic Adiabatic Quantum Computation

Sato et al. introduced a neuro-inspired approach to AQC, namely, NAQC [14], [16], and proposed a new method of generating H_{fin} .

In order to clarify the relation between a qubit and a neuron, let us consider a 1/2-spin system as sample quantum system, where $|0\rangle$ and $|1\rangle$ correspond to a down- and up-spin, respectively. The Hamiltonian of this system is written as

$$H = -\frac{1}{2} \sum_i \sum_j J_{ij} \sigma_i \cdot \sigma_j, \quad (5)$$

where J_{ij} is the magnitude of the spin-spin interaction between the i -th and j -th 1/2-spins, and σ_i and σ_j denote the Pauli spin matrices. The network behavior is determined by this Hamiltonian, which is called a Heisenberg Hamiltonian. This is similar to the fact that the behavior of the Hopfield network [22] is given by the energy function, such that

$$E = -\frac{1}{2} \sum_i \sum_j w_{ij} o_i o_j, \quad (6)$$

where w_{ij} is the synaptic weight between the i -th and j -th neurons and o_i and o_j are neuron outputs. Inspired by this analogy, one can introduce quantum mechanical dynamics to the Hopfield network. By comparing Eqs. (5) and (6), the synaptic weight matrix is converted into a Hamiltonian, which corresponds to H_{fin} in Eq. (2). Thus, it becomes possible to relate a neural network with a QNN. The conversion method is described in detail in Sect. 3.1.2.

3. Model of Quantum Associative Memory

3.1 Memorization Procedure

Let us assume that the QuAM model composed of N qubits memorizes a set $\mathcal{S}$ of M patterns. The patterns are N -dimensional random vectors, the elements of which take ± 1 randomly. One of the vectors is denoted by $\xi_\mu = (y_1, y_2, \dots, y_N)^T$, where μ is the pattern index ($1 \leq \mu \leq M$). y_i satisfies

$$y_i = 2x_i - 1, \quad (7)$$

where i is the index of qubits, for calculation convenience.

The QuAM memorizes a pair comprised of an original pattern and its inverted pattern (ξ_μ and $\bar{\xi}_\mu$, respectively) with equal probability, which is also true for the Hopfield network. Thus, the maximum of M is given as $M_{\max} = 2^{N-1}$. In a conventional neural network, a pattern is memorized with an autocorrelation matrix regarded as a synaptic weight matrix. Therefore, we relate an autocorrelation matrix of memorized patterns with a Hamiltonian. To study the detailed dynamics of the QuAM, we examine two types of Hamiltonians for pattern memorization H_{mem} ; one is based on QA and the other is based on NAQC.

3.1.1 H_{mem} Based on Ising Hamiltonian (QA)

The first method of composing H_{mem} is based on QA. Thus, we formulate this expression by following the Hamiltonian for the database search problem [23] and obtain

$$H_{\text{mem}}^{\text{IS}} = I^{(2^N)} - \sum_{\mu} |z_{\mu}\rangle \langle z_{\mu}|, \quad (8)$$

where $|z_{\mu}\rangle$ is the decimal notation of ξ_{μ} , and $I^{(2^N)}$ is the $2^N \times 2^N$ identity matrix. In this formulation, $H_{\text{mem}}^{\text{IS}}$ has diagonal elements only. Because $H_{\text{mem}}^{\text{IS}}$ is similar to the cost function of a Hopfield network, it is confirmed that the $H_{\text{mem}}^{\text{IS}}$ ground states are given as superposition states of all patterns in $\mathcal{S}$. Thus, the total probability $P_{\text{total}}^{\text{IS}}$ of all memorized patterns (i.e. the ξ_{μ} s) always takes its maximum (50%), as explained in the next section.

3.1.2 H_{mem} Based on Neuro-Inspired Hamiltonian (NAQC)

The other method of composing H_{mem} follows NAQC. Here, a neuro-inspired Hamiltonian $H_{\text{mem}}^{\text{NI}}$ for pattern memorization is converted from the autocorrelation matrix of the patterns in $\mathcal{S}$. The autocorrelation matrix W for the QuAM is defined as

$$W = \frac{1}{N} \left(\sum_{\mu=1}^M \xi_{\mu} \xi_{\mu}^T - M I^{(N)} \right). \quad (9)$$

We follow the method proposed in [14], which enables us to convert neuron-neuron interactions into qubit-qubit interactions, and define $H_{\text{mem}}^{\text{NI}}$ based on these interactions, such that

$$H_{\text{mem}}^{\text{NI}} = \lambda I^{(2^N)} + \sum_i \sum_{j < i} |w_{ij}| \{ H_{ij}(w_{ij}) + H_{ij}^{\dagger}(w_{ij}) \}, \quad (10)$$

where λ is a constant. $H_{ij}(w_{ij})$, which denotes the interaction between the i -th and j -th qubits, is given as

$$H_{ij}(w_{ij}) = \begin{cases} (I^{(2)} \otimes \cdots \otimes \overbrace{a \otimes a}^{i\text{-th}} \cdots \otimes \overbrace{a \otimes a}^{j\text{-th}} \cdots \otimes I^{(2)}) & (w_{ij} > 0), \\ O^{(2^N)} & (w_{ij} = 0), \\ (I^{(2)} \otimes \cdots \otimes \overbrace{a \otimes a}^{i\text{-th}} \cdots \otimes \overbrace{a^{\dagger} \otimes a^{\dagger}}^{j\text{-th}} \cdots \otimes I^{(2)}) & (w_{ij} < 0), \end{cases} \quad (11)$$

where a and $a^{\dagger}$ denote the fermion annihilation and creation operators, respectively, and $O^{(2^N)}$ is the $2^N \times 2^N$ zero matrix. As explained in previous reports [14], [16], both excitatory ($w_{ij} > 0$) and inhibitory ($w_{ij} < 0$) interactions can be realized effectively.

In this formulation, $H_{\text{mem}}^{\text{NI}}$ has non-diagonal elements and differs from $H_{\text{mem}}^{\text{IS}}$. One of the aims of examining two types of Hamiltonians is evaluating the influence of the diagonal and non-diagonal elements in H_{mem} on the QuAM dynamics. In contrast to $H_{\text{mem}}^{\text{IS}}$, it is not obvious that the ground state of $H_{\text{mem}}^{\text{NI}}$ can be given as a superposition state of the memorized patterns. In order to confirm the memorization capability of $H_{\text{mem}}^{\text{NI}}$, we conducted numerical simulations, for which a sufficiently large amount of $\mathcal{S}$ s were prepared. For each $\mathcal{S}$, we calculated the eigenstates of $H_{\text{mem}}^{\text{NI}}$ and obtained its ground state $|\Psi_{\text{gs}}^{\text{NI}}\rangle$. The total probability $P_{\text{total}}^{\text{NI}}$ was calculated as $\sum_{\mu} |\langle z_{\mu} | \Psi_{\text{gs}}^{\text{NI}} \rangle|^2$. In other words, $P_{\text{total}}^{\text{NI}}$ means the sum of the probabilities of the quantum states corresponding to the memorized patterns ξ_{μ} s. Because of the symmetric property of W , the QuAM cannot avoid memorizing inverted patterns $\bar{\xi}_{\mu}$. Hence, the maximum of $P_{\text{total}}^{\text{NI}}$ must be 50% at most, following the same reasoning presented in Sect. 3.1.1. As shown in Fig. 1, $\langle P_{\text{total}}^{\text{NI}} \rangle$, which is the average of $P_{\text{total}}^{\text{NI}}$, increases with increasing M and reaches 50% in the case of $2 \leq M \leq 5$. In comparison with the Hopfield network, this M -dependency seems strange at a glance. Theoretically, the sum of all synaptic weight matrices for $M_{\max}$ patterns is identical to the zero matrix. As a special case, the zero matrix gives the superposition of all possible states; hence, the QuAM memorizes all possible states simultaneously. This suggests that $\langle P_{\text{total}}^{\text{NI}} \rangle$ for $N = 6$ reaches 50% at $M = M_{\max}$. However, we cannot confirm such cases numerically, because our computer has insufficient memory (128 GB). Here, we found that the nominal memory capacity M_{cap} of the QuAM is $M_{\max} = 2^{N-1}$, because $\langle P_{\text{total}}^{\text{NI}} \rangle$ takes its maximum, and confirmed that $H_{\text{mem}}^{\text{NI}}$ can store multiple patterns in its ground states. We should note that this estimation was calculated without executing an adiabatic Hamiltonian change. The actual retrieval per-

Fig. 1 Sum of nominal estimated probabilities of all memorized patterns with $H_{\text{mem}}^{\text{NI}}$, $\langle P_{\text{total}}^{\text{NI}} \rangle$.

formance is discussed in Sect. 4.

3.2 Retrieval Procedure

3.2.1 Associative Key Input

A conventional neural associative memory converges to a certain state corresponding to one of the memorized patterns according to the initial state given as the key input, which is usually a degraded target pattern. On the other hand, the QuAM state is given as a superposition state and does not converge to a specific state unless some specific treatment is applied. Therefore, we apply an external Hamiltonian H_{fix} to partially fix the state of some qubits, which imitates the role of the key input for a neural associative memory. First, we choose ξ'_μ as a target pattern from the ξ_μ s and generate the key input ξ_{key} by replacing one qubit in ξ'_μ with an unfixed qubit. Thus, the retrieval process is regarded as recalling the original target pattern ξ'_μ from ξ_{key} . H_{fix} is given as

$$H_{\text{fix}} = (\kappa_1 \otimes I^{(2)} \otimes \cdots \otimes I^{(2)} + I^{(2)} \otimes \kappa_2 \otimes \cdots \otimes I^{(2)} + \cdots + I^{(2)} \otimes I^{(2)} \otimes \cdots \otimes \kappa_N). \quad (12)$$

κ_i is defined as

$$\kappa_i = \begin{cases} +\sigma^z & (\text{if } y_i \text{ is } +1), \\ I^{(2)} & (\text{if } y_i \text{ is unfixed}), \\ -\sigma^z & (\text{if } y_i \text{ is } -1), \end{cases} \quad (13)$$

where σ^z is the z -component of the Pauli spin matrices. When we add H_{fix} during the Hamiltonian evolution, we can retrieve a target pattern according to ξ_{key} .

3.2.2 H_{fix} Application Method

We re-formulate Eqs. (1) and (2) in consideration of the H_{fix} application. Equation (1) can be replaced with

$$H(t) = \Gamma(t)H_{\text{tun}} + (H_{\text{mem}}^{\text{IS}} + A_{\text{fix}}H_{\text{fix}}), \quad (14)$$

where A_{fix} is a scaling factor of H_{fix} . H_{tun} is the tensor product of the σ^x s. Equation (2) can be replaced with

$$H(t) = \left(1 - \frac{t}{T}\right)H_{\text{init}} + \frac{t}{T}(H_{\text{mem}}^{\text{NI}} + A_{\text{fix}}H_{\text{fix}}). \quad (15)$$

Note that the quantum state of a QNN also evolves with t as

$$|\Psi(t + \Delta t)\rangle = U(\Delta t)|\Psi(t)\rangle = e^{-\frac{i(H(t) + \Delta H)\Delta t}{\hbar}}|\Psi(t)\rangle, \quad (16)$$

where the operator $U(\Delta t)$ is given by the Padé approximation [24]. ΔH denotes a slight energy fluctuation and facilitates convergence of a QNN to its ground state; this approach has been proposed and verified by Kinjo et al. [16].

3.3 Summary of QuAM Operations

In this subsection, we summarize the memorization and retrieval operations of our QuAM model, as follows.

<Memorization Procedure>

1. A set $\mathcal{S}$ of memory patterns, i.e., ξ_μ s, is prepared.
2. The autocorrelation matrix W is generated according to Eq. (9).
3. The memorization Hamiltonian H_{mem} is generated according to Eq. (8) or (10).

<Retrieval Procedure>

1. A target pattern ξ'_μ is selected from $\mathcal{S}$.
2. The unfixed qubits in the key input ξ_{key} are defined for retrieval.
3. H_{fix} is generated to fix some qubits according to ξ_{key} with Eq. (12).
4. The quantum state is changed following Eq. (16), with the adiabatic Hamiltonian evolution given by Eq. (14) or (15).

4. Retrieval Performance of 4-Qubit QuAM

We performed pattern recall simulations in order to evaluate the QuAM retrieval performance. The setup was almost identical to that for the numerical simulations for nominal estimation of the memory capacity discussed in Sect. 3.1.2. We composed a 4-qubit QuAM and executed all QuAM operations. The QuAM state evolved with $T = 3000$ according to Eq. (16). The magnitude of ΔH was $O(10^{-2})$, while those of H_{mem} and H_{fix} were $O(10^0)$. By measuring $|\Psi(T)\rangle$, we evaluated $\langle P_{\text{ret}} \rangle$, which is the average probability of retrieving a correct ξ'_μ associated with ξ_{key} . In the following, we statistically analyze all cases of $\mathcal{S}$, i.e., we examine all combinations of M patterns chosen from 2^4 patterns.

$\langle P_{\text{ret}}^{\text{IS}} \rangle$, which is $\langle P_{\text{ret}} \rangle$ obtained with $H_{\text{mem}}^{\text{IS}}$, is shown in Fig. 2. $\langle P_{\text{ret}}^{\text{IS}} \rangle$ s with 1 and 2 unfixed qubits are shown in Figs. 2 (a) and (b), respectively. These results indicate that the QuAM performance degrades if the number of unfixed qubits increases. It is also apparent that $\langle P_{\text{ret}}^{\text{IS}} \rangle$ declines along with M in both cases. When A_{fix} takes a non-zero value, the QuAM performance is improved. Note that $A_{\text{fix}} = 0$ indicates that H_{fix} is not used, and corresponds to the random selection of one pattern among the $2M$ memorized patterns. In fact, the $\langle P_{\text{ret}}^{\text{IS}} \rangle$ s with $A_{\text{fix}} = 0$ in Figs. 2 (a) and (b) meet $1/2M$. Therefore, the results indicate the retrieval efficacy of H_{fix} . However, we note here that $\langle P_{\text{ret}}^{\text{IS}} \rangle$ becomes saturated if A_{fix} takes 0.1 or higher.

The upper limit of $\langle P_{\text{ret}}^{\text{IS}} \rangle$ is caused by the conflict between the memorized patterns and their inverted patterns. For example, we consider the memorization of a set $\mathcal{S}^{\text{eg}} = \{\xi_1, \xi_2\}$, where $\xi_1 = (-1, -1, -1, +1)^T$ and $\xi_2 = (-1, +1, +1, -1)^T$. Thus, the QuAM also memorizes the inverted patterns $\bar{\xi}_1 = (+1, +1, +1, -1)^T$ and $\bar{\xi}_2 = (+1, -1, -1, +1)^T$. These 4 patterns are realized as a superposition state in the ground state of H_{mem} , and their probabilities are the same. If we choose ξ_1 as a target pattern and select the last qubit y_4 as an unfixed qubit, the key input is $\xi_{\text{key}}^{\text{eg1}} = (-1, -1, -1, y_{\text{un}})^T$, where y_{un} denotes an unfixed qubit. In this case, the QuAM successfully retrieves

Fig. 2 A_{fix} -dependency of retrieval probabilities of correct target pattern $\langle P_{\text{ret}} \rangle$ obtained with $H_{\text{mem}}^{\text{IS}}$. (a) $\langle P_{\text{ret}}^{\text{IS}} \rangle$ associated with a 1-unfixed-qubit key input and (b) $\langle P_{\text{ret}}^{\text{IS}} \rangle$ associated with a 2-unfixed-qubit key input.

ξ_1 , because there is no conflict. On the other hand, if we select the first qubit y_1 as an unfixed qubit, the key input is $\xi_{\text{key}}^{\text{eg}2} = (y_{\text{un}}, -1, -1, +1)^T$. In this case, the QuAM cannot distinguish ξ_1 from $\bar{\xi}_2$ according to $\xi_{\text{key}}^{\text{eg}2}$, and the probabilities of ξ_1 and $\bar{\xi}_2$ take the same value (50% each). In this way, the QuAM often fails to retrieve a target pattern, even if H_{fix} succeeds in fixing the qubit states correctly. This inherent conflict of patterns occurs inevitably and its frequency increases with increasing M . The conflict occurs necessarily, especially in the case of $M = 8$, because the QuAM memorizes all 2^4 patterns. Thus, $\langle P_{\text{ret}}^{\text{IS}} \rangle$ at $M = 8$ should be 50%. In the same manner, $\langle P_{\text{ret}}^{\text{IS}} \rangle$ for the case of two unfixed qubits should be 25%. In Fig. 2, the actual probabilities are obtained via simulation, as expected. Thus, we claim that our results are reasonable.

$\langle P_{\text{ret}}^{\text{NI}} \rangle$, which is P_{ret} obtained with $H_{\text{mem}}^{\text{NI}}$, is shown in Fig. 3. The difference between Figs. 3 (a) and (b) is the number of unfixed qubits. The M - and A_{fix} -dependency of $\langle P_{\text{ret}}^{\text{NI}} \rangle$ are also confirmed. Their tendencies are the same as for the case of QuAM with $H_{\text{mem}}^{\text{IS}}$, although its changes along with M or A_{fix} are partially non-monotonic. Unlike $H_{\text{mem}}^{\text{IS}}$, $H_{\text{mem}}^{\text{NI}}$ memorizes patterns based on the interactions between two qubits. Because $H_{\text{mem}}^{\text{NI}}$ is composed of non-diagonal elements, its ground states are given as mixed states composed of desired and undesired states. Therefore, the QuAM with $H_{\text{mem}}^{\text{NI}}$ often retrieves fake patterns, which are irrelevant to the memorized and inverted patterns. However, the efficacy of using H_{fix} for the QuAM with $H_{\text{mem}}^{\text{NI}}$ is again confirmed.

At the end of this section, we discuss the difference between the A_{fix} values for the experiments using $H_{\text{mem}}^{\text{IS}}$ and $H_{\text{mem}}^{\text{NI}}$. The magnitude of A_{fix} in the case of $H_{\text{mem}}^{\text{IS}}$ is smaller

Fig. 3 A_{fix} -dependency of retrieval probabilities of correct target pattern $\langle P_{\text{ret}} \rangle$ obtained with $H_{\text{mem}}^{\text{NI}}$. (a) $\langle P_{\text{ret}}^{\text{NI}} \rangle$ associated with a 1-unfixed-qubit key input and (b) $\langle P_{\text{ret}}^{\text{NI}} \rangle$ associated with a 2-unfixed-qubit key input.

Fig. 4 Comparison between $\langle P_{\text{ret}}^{\text{IS}} \rangle$ with $A_{\text{fix}} = 0.1$ (blue lines) and $\langle P_{\text{ret}}^{\text{NI}} \rangle$ with $A_{\text{fix}} = 10$ (purple lines). The solid and dotted lines denote the cases of 1 and 2 unfixed qubits, respectively.

than that for $H_{\text{mem}}^{\text{NI}}$. During the evolution, the QuAM amplifies the probability of one specific state according to H_{fix} and retrieves a correct target pattern. A_{fix} determines the strength of the qubit state fixing and controls the degree of amplification. Because the ground states of $H_{\text{mem}}^{\text{NI}}$ are given as mixed states, the energy difference between the target pattern and the other patterns is small, and thus, a relatively large A_{fix} is required to definitely retrieve the correct pattern. Sufficient $\langle P_{\text{ret}}^{\text{NI}} \rangle$ is obtained with $A_{\text{fix}} = 10$. In Fig. 4, we compare two $\langle P_{\text{ret}} \rangle$ s; $\langle P_{\text{ret}}^{\text{IS}} \rangle$ with $A_{\text{fix}} = 0.1$ (Fig. 2) and $\langle P_{\text{ret}}^{\text{NI}} \rangle$ with $A_{\text{fix}} = 10$ (Fig. 3). When M is small, $\langle P_{\text{ret}}^{\text{IS}} \rangle$ (blue line

with squares) is higher than $\langle P_{\text{ret}}^{\text{NI}} \rangle$ (purple line with circles), and this difference is clearly confirmed when the number of unfixed qubits is 2 (dotted lines). However, although this difference exists, these two types of QuAM exhibit almost the same performance overall. Thus far, we have evaluated the retrieval performance and confirmed that our QuAM can memorize multiple patterns and can retrieve one specific pattern associated with a given ξ_{key} . Because the M_{cap} of the Hopfield network is limited to $0.14n$ [25], where n is the number of neurons, our QuAM model exhibits superior performance, if we allow some retrieval errors.

5. Conclusion

We have proposed a new quantum associative memory (QuAM) model with a quantum neural network by relating neural and quantum computing. The association function is realized via adiabatic Hamiltonian evolution, and the Hamiltonians are composed of H_{mem} for pattern memorization and H_{fix} to provide a key input. We studied two types of H_{mem} , i.e., an Ising Hamiltonian $H_{\text{mem}}^{\text{IS}}$ used for quantum annealing and a neuro-inspired Hamiltonian $H_{\text{mem}}^{\text{NI}}$ used for neuromorphic adiabatic quantum computation. Numerical simulation results indicate that the proposed memorization and retrieval operations work properly with both types of Hamiltonian. Because their performances are very similar, one should select a Hamiltonian for pattern memorization based on the hardware feasibility. Surprisingly, the QuAM can memorize 2^{N-1} patterns at once because of its quantum mechanical property; however, it cannot always successfully retrieve one specific target pattern because of the inherent conflicts between the memorized and inverted patterns. Although some errors in the retrieval process are inevitable, our QuAM exhibits new and unique features that differ from a conventional neural associative memory.

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