A Study on the Market Impact of the Rule for Investment Diversification at the Time of a Market Crash Using a Multi-Agent Simulation

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SUMMARY As financial products have grown in complexity and level of risk compounding in recent years, investors have come to find it difficult to assess investment risk. Furthermore, companies managing mutual funds are increasingly expected to perform risk control and thus prevent assumption of unforeseen risk by investors. A related revision to the investment fund legal system in Japan led to establishing what is known as “the rule for investment diversification” in December 2014, without a clear discussion of its expected effects on market price formation having taken place. In this paper, we therefore used an artificial market to investigate its effects on price formation in financial markets where investors follow the rule at the time of a market crash that is caused by the collapse of an asset fundamental price. As results, we found the possibility that when the fundamental price of one asset collapses and its market price also collapses, some asset market prices also fall, whereas other asset market prices rise for a market in which investors follow the rule for investment diversification.

keywords: artificial market, multi agent based simulation, rule of investment diversification, leverage, financial market

1. Introduction

Financial products have grown in complexity and level of risk compounding in recent years. For example, mutual funds have come to choose various assets, some of which may have high risk, and there may be some funds whose performances depend on dangerously much those of a part of the assets that the funds hold. Therefore, investors have come to find it difficult to assess investment risk.

Companies managing mutual funds are increasingly expected to perform risk control and thereby prevent assumption of unforeseen risk by investors. In Japan, The Securities Investment Trust Law was revised in 2013, and the rule for investment diversification was established in December 2014 [1], [2].

The rule for investment diversification is a holding weight limitation for mutual funds; that is, mutual funds cannot hold each stock in more weight than some limitation weight∗. For example, the rule requires that mutual funds keep 1) particular companies’ securities, 2) options underlying them, or 3) their corporate bonds that are each no more than 10% of the mutual funds’ net asset value (NAV) and whose sum is no more than 20% of the mutual funds’ NAV. A similar rule was previously established in Europe [3].

There are many empirical studies on diversified investment.

Cremers et al. [4] showed that the funds with the highest Active Share∗∗ significantly outperform their benchmark indexes, while the non-index funds with the lowest Active Share underperform. However, there has been no clear discussion of the effects on market price formation when restrictions are imposed on diversified investments. An empirical study cannot isolate the pure contribution of these regulations to price formation because many kinds of traders can affect price formation in actual markets.

One way of analyzing how particular transactions influence the market is to use an artificial market. An artificial market is a multi-agent based model of financial markets [5]–[7]. Each of the agents is assigned a specific trading (i.e., buying and selling) rule and then set to trade financial assets as an investor. The market can then be observed to see how the agents behave. At the same time, by modeling and incorporating certain restrictions on the market side (e.g., limitations to ensure market stability and efficiency such as a short selling regulation), it is possible to examine how investors behave and also what kinds of effects their behaviors induce in the market.

Artificial market models that are too complex and have too many parameters are often criticized as being too difficult to evaluate [6]. The validity of the model is evaluated in terms of its ability to reproduce stylized facts such as a fat tail and volatility clustering obtained by empirical analysis [8]. However, in most cases, neither the precision of the reproduction of a stylized fact nor the number of kinds of stylized facts that can be reproduced increases when the model is made more complicated. Thus, Chen et al. [6] suggested that as simple a model which can reproduce the styl-
ized facts should be made as possible.

Chiarella et al. [5] succeeded in structuring a simple agent model which reproduced the statistical properties of the kinds of long-term price fluctuations observed in empirical analyses. This model was not intended to completely reproduce a real market, but was built to be as simple as possible while still being able to achieve the goal of the analysis, and did not cover all cases, such as investors becoming bankrupt.

On the other hand, previous studies using an artificial market implemented various kinds of learning processes. For example, agents switched strategies and/or tuned their strategy parameters based on their performance, market price, etc. [9]–[11]. Empirical studies [12], [13] also showed switching strategies existing in real financial markets.

Mizuta et al. [7] added a learning process to the model of Chiarella et al. and reproduced large-scale market confusion such as a bubble or a financial crisis in their artificial market model. This model also includes investors following different investment strategies, such as fundamental investments or technical investments. Furthermore, no investor sticks to a single strategy when making investment decisions, but rather each switches between strategies according to market price information. Studies on artificial markets to investigate market regulations have had some success in market analysis in recent years [7], [14], [15]; however, the only study on the effects of the rule for investment diversification on the market using an artificial market has been Yagi et al. [16].

Previously, Yagi et al. [16] investigated the market impact of the rule for investment diversification in an artificial two-asset market when there is a sudden drop in asset price. The results showed that in markets in which the rule for investment diversification is imposed, if the fundamental price of one of the two assets falls sharply, then the asset price of the other asset will similarly fall. When an asset price falls sharply in Yagi et al.’s artificial market model, the value of all assets held is very small compared to NAV. On the other hand, an actual mutual fund manager adjusts the total value of assets held relative to NAV by buying more of assets held in small quantities, in order to prevent the value of an asset falling below some lower limit that is set in advance according to the level of asset price fluctuation. Thus, in this study, we set out to examine market impact when a third asset (asset 3) which has not been held by any mutual fund is added to an artificial two-asset market and the price of a particular asset drops sharply, both when there is a lower limit on the total amount of assets relative to NAV and also when no such limit is imposed.

First, we explain our proposed artificial market used in this study in Sect. 2. Specifically, the order process and the learning process of our artificial market are explained in Sects. 2.1 and 2.2, respectively. In Sect. 2.3, we model the rule for investment diversification and the leverage limitation (i.e., the upper and lower limits on the total amount of assets relative to NAV) in our artificial market. Next, we perform some simulations using our artificial market and analyze the results in Sect. 3, as follows. In Sect. 3.1, we check the validity of our artificial market. In Sect. 3.2, we observe market price transitions of three assets whose fundamental prices are constant in a market where agents follow the rule for investment diversification and in a market where agents do not follow the rule for investment diversification. In Sect. 3.3, we also observe the market price transitions of an asset whose fundamental price falls and those of the other assets, whose fundamental prices are stable, in both a market where agents follow the rule for investment diversification and in a market where agents do not follow the rule for investment diversification. In Sect. 3.4, we examine the market price transitions under the same conditions as in Sect. 3.3 but with a lower limit on the total amount of assets relative to NAV. Finally, we discuss our conclusions regarding this study and our future work in Sect. 4.

2. Artificial Market Model

In view of the discussion of artificial market models in Sect. 1, for the present study, we built a new artificial market model that is based on the artificial market model of Mizuta et al. [17].

In the proposed model, only three risk assets are available for trading. Hereinafter, we call risk assets simply assets, while we refer to the non-risk asset as cash because a non-risk asset is equivalent to cash in our artificial markets.

The mechanism for determining the price in this model being a continuous double auction (continuous trading session) means that if there are some sell (buy) order prices in the order book that are lower (higher) than the agent’s buy (sell) order price, then the agent’s order is immediately matched to the lowest sell order (highest buy order) in the order book. We call this a “market order.” If there are no such orders in the order book, the order does not match any other order and remains in the order book. We call this a “limit order.” The remaining orders in the order book are canceled $t_i$ after the order was placed.

2.1 Order Process

There are $n$ agents, agents $j = 1, \ldots, n$, each of which places an order in sequence, that is, from agent 1 to $n$. When the final agent, agent $n$, has placed an order, the first agent, agent 1, places the next order. The time $t$ is incremented by 1 each time an agent places an order. Thus, the process moves one step forward even when a trade does not occur and this new order is placed on the order book.

The order prices of agent $j$ by transaction are determined as shown below. The rate of change of the price expected by agent $j$ at time $t$ (the expected return) $r_{j,k}^t$ is given by

$$ r_{j,k}^t = \frac{1}{w_{1,j,k} + w_{2,j,k} + u_{j,k}} \left( w_{1,j,k} \log \frac{P_{j,k}}{P_{j,k-1}} ight. $$

$$ + \left. w_{2,j,k} r_{j,k}^{t-1} + u_{j,k} \epsilon_{j,k} \right) $$

(1)
where $k$ is the asset, and $w_{i,k}$ is the weight of the $i$-th term for asset $k$ and agent $j$, and is set according to the uniform distribution between 0 and $w_{i,max}$ at the start of the simulation and then varied by using the learning process described later. Furthermore, $u_{j,k}$ is the weight of the third term and is set according to the uniform distribution between 0 and $u_{max}$ at the start of the simulation and kept constant thereafter; $P^t_{j,k}$ is the fundamental price; $P^*_{j,k}$ is the market price for asset $k$ at time $t$, and is set to the most recent price at the time if no trading is occurring and to $P^0_{j,k}$ at $t = 0$; $e_{j,k}$ is a normally distributed random error with mean zero and standard deviation $\sigma_{e_{j,k}}$; and $r_{j,k}$ is the past return measured by agent $j$, given by $r_{j,k}^t = \log(P^t_{j,k}/P^{t-\tau}_{j,k})$, in which $\tau_j$ is set according to the uniform distribution between 1 and $\tau_{max}$ at the start of the simulation.

The first term on the right-hand side of Eq. (1) represents the fundamental strategy, which reflects that an agent expects a positive (negative) return when the market price is lower (higher) than the fundamental price. The second term represents the technical strategy, which reflects that an agent expects a positive (negative) return when the historical return is positive (negative). The third term represents noise.

Based on the expected return $r_{j,k}^t$, the expected price $P^t_{j,k}$ is found using the equation

$$P^t_{j,k} = P^{t-1}_{j,k} \exp(r_{j,k}^{t-1})$$

The order price $P^o_{j,k}$ is set according to the uniform distribution between $P^{t-\tau}_{j,k} - P_d$ and $P^{t-\tau}_{j,k} + P_d$, where $P_d$ is a constant. The minimum unit for the price is $\delta P_d$ and fractional values smaller than this are rounded down. The choice between buying and selling is determined by the relative sizes of the expected price $P^{t-\tau}_{j,k}$ and the order price $P^o_{j,k}$.

- An agent places a buy order for one share if $P^{t-\tau}_{j,k} > P^o_{j,k}$.
- An agent places a sell order for one share if $P^{t-\tau}_{j,k} < P^o_{j,k}$.

Agents confirm whether their positions violate the rule for investment diversification and the upper and lower limits on the amount of each asset that are defined in detail in Sect. 2.3 after placing their orders. The orders are sent only if they satisfy the rule and the upper limit. The orders in Sect. 3.4 additionally have to satisfy a lower limit on the total amount of assets relative to NAV.

2.2 Learning Process

The learning process in the present study is implemented to switch the strategy between fundamental and technical strategies. We modeled the learning process as follows. Learning is performed by each agent immediately before the agent places an order. The expected return $r_{i,j,k}^t = \log(P^t_{j,k}/P^t_{k})$ is set for only the fundamental strategy, and the expected return $r_{e,j,k}^t = r_{j,k}^t$ is set only for the technical strategy. When $r_{i,j,k}^t$ and $r_{e,j,k}^t$ are of the same sign, $w_{i,j,k}$ is updated as follows:

$$w_{i,j,k} = w_{i,j,k} + k_i r_{j,k}^t (w_{i,max} - w_{i,j,k})$$

(3)

When $r_{i,j,k}^t$ and $r_{e,j,k}^t$ have opposite signs, $w_{i,j,k}$ is updated as follows:

$$w_{i,j,k} = w_{i,j,k} - k_i r_{j,k}^t q_j^t w_{i,j,k}$$

(4)

where $k_i$ is a constant and $q_j^t$ is set according to the uniform distribution between 0 and 1.

Separately from the process for learning based on past performance, $w_{i,j,k}$ is reset with a small probability $m$, according to the uniform distribution between 0 and $w_{i,max}$. This means that learning is random, and this, in combination with learning based on performance, allows objective modeling of the situation in which agents find the weights of strategies by trial and error.

2.3 Model with the Rule for Investment Diversification

In this model, we implement a leverage limitation (i.e., upper and lower limits on the total amount of assets relative to NAV) and the rule for investment diversification. The upper limit on the total amount of assets relative to NAV is defined as follows:

$$\sum_{k=1}^{3} |P_k^t S_{j,k}^t| \leq v_{upper} NAV^t_j$$

(5)

where $S_{j,k}^t$ is the quantity possessed of asset $k$ at time $t$ and agent $j$. Note that the position of agent $j$ is long when $S_{j,k}^t > 0$ and that the position of agent $j$ is short position when $S_{j,k}^t < 0$. Furthermore, $v_{upper}$ is the upper limit on the ratio of the total amount of assets relative to NAV and is set as $v_{upper} = 1$ in this model. If $C_j^t$ is the quantity of cash possessed by agent $j$ at time $t$, then $NAV^t_j$, the net assets of agent $j$ at time $t$, is given as follows:

$$NAV^t_j = \sum_{k=1}^{3} (P_k^t S_{j,k}^t + C_j^t)$$

(6)

Next, the rule for investment diversification is defined as follows:

$$\frac{|P_k^t S_{j,k}^t|}{NAV^t_j} \leq w_{dir}$$

(7)

where $w_{dir}$ is an upper limit on the ratio of the amount of asset $k$ that agent $j$ can own at $t$ to $NAV^t_j$ and is set as $w_{dir} = 0.5$.
in the model. When agent $j$’s order for asset $k$ at time $t$ matches an order in the order book but Formula (7) is not satisfied, then this order by agent $j$ is canceled. When Formula (7) is not satisfied for some other reasons than those referred to above (e.g., when $NAV^{t}_{j}$ is much less than $NAV^{t-1}_{j}$), then agent $j$ sends an order for asset $k$ to satisfy Formula (7). Hereinafter, we refer to such an order as a rule contravention resolution order, which is a kind of market order. We further differentiate this by referring to the order as a “rule for investment diversification contravention resolution buy order (RCRBO)” if an agent sends a buy order and a “rule for investment diversification contravention resolution sell order (RCRSO)” if an agent sends a sell order.

Finally, the lower limit on the total amount of assets is defined as follows:

$$v_{i}^{\text{NAV}}_{j} \leq \sum_{k=1}^{3} |P_{t}^k S_{j,k}^t|$$

where $v_{i}^{\text{NAV}}_{j}$ is the lower limit on the ratio of the total amount of assets relative to NAV. When Formula (8) is not satisfied, agent $j$ sends an order for asset $k$ to satisfy Formula (8). Hereinafter, we call this a “lower limit contravention resolution buy order (LLCRBO)”, which is a kind of market order. Note that the expected return of asset $k$ at time $t$ is the highest among all assets that satisfy Formula (7)\(^{11}\).

### 3. Simulation Results and Discussions

In this study, we set model parameters as listed in Table 1. Furthermore, agent $j$ has initial quantity of cash $C_{j}^{0}$ = 100,000, initial quantities of assets 1 and 2 $S_{j,1}^{0}$ = $S_{j,2}^{0}$ = 48, respectively, and initial quantity of asset 3 $S_{j,3}^{0}$ = 0. We ran simulations from $t = 0$ to $t = 1,000,000$.

#### 3.1 Validation of Our Artificial Market

As many empirical studies have mentioned\(^{18},^{19}\), a fat tail and volatility clustering appear in actual markets; therefore, we set our artificial market parameters to replicate these features.

Table 2 shows statistics for stylized facts that are averages of 100 simulation runs, for which we calculated the price returns at intervals of 100 time units. All the following figures also use averages of 100 simulation runs. Table 2 shows that both kurtosis and autocorrelation coefficients for square returns with several lags are positive, which means that all runs replicate a fat tail and volatility clustering. This shows that the model replicates long-term statistical characteristics observed in real financial markets.

#### 3.2 Case When the Fundamental Prices of All Assets Are Constant

In this section, we describe the simulation results for the case when the fundamental prices of all assets are constant ($P_{f,1}^{t} = P_{f,2}^{t} = P_{f,3}^{t} = 10,000$). Table 3 shows the statistical values of the market price transitions for assets 1, 2, and 3, both with and without the rule for investment diversification imposed.

Table 3 confirms that when the fundamental prices of all assets are constant, the market price transitions of all assets are stable, regardless of whether the rule for investment diversification is imposed.

#### 3.3 Case with a Crash of the Asset 1 Fundamental Price

In this section, we describe the simulation results when the fundamental price of asset 1 drops sharply ($P_{f,1}^{t} = P_{f,2}^{t} = P_{f,3}^{t} = 10,000$; $P_{f,1}^{t} = 10,000$ for $t = 0$ to $t = 1,000,000$ and $P_{f,1}^{t} = 7,000$ for $t \geq 100,001$). Figures 1 and 2 show the market price transitions without and with the rule for investment diversification imposed, respectively.

These figures reveal that the market prices of assets 1 and 3 do not diverge significantly from the fundamental prices of assets 1 and 3, regardless of whether the rule for investment diversification is imposed. On the other hand, a fall in the market price of asset 2 lags a little behind that of asset 1 and converges to a price that is lower than the fundamental price (9,000 in this case) with the rule for investment diversification imposed, although the market price of asset 2 is also stable without the rule for investment diversification imposed.

The reason that the market price of asset 2 falls in line with the market price of asset 1 can be explained as follows. First, after the market price of asset 1 begins to drop sharply, the NAV of agents decreases. For this reason, the total value of asset 2 held by many agents rises to a proportion that contravenes the rule. Consequently, to resolve the situation, so that the rule is satisfied, the agents place RCRSOs, which has the effect of lowering the market price of asset 2.

As a concrete example, consider Fig. 3. NAV before the sharp drop in the market price of asset 1 is 1,000,000, with the total values of assets 1 and 2 held equal to 500,000 each. Thus, $w_{\text{dir}}$ is 0.5 (Fig. 3 left). Suppose then that the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1,000</td>
</tr>
<tr>
<td>$w_{1,\text{max}}$</td>
<td>10</td>
</tr>
<tr>
<td>$w_{2,\text{max}}$</td>
<td>10</td>
</tr>
<tr>
<td>$\tau_{\text{max}}$</td>
<td>10,000</td>
</tr>
<tr>
<td>$\sigma_{\text{e}}$</td>
<td>0.03</td>
</tr>
<tr>
<td>$P_{d}$</td>
<td>1,000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>13,000</td>
</tr>
<tr>
<td>$k_{i}$</td>
<td>4</td>
</tr>
<tr>
<td>$m$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(^{11}\)In the real world, it is considered that $w_{\text{dir}}$ is almost 0.9, but we set it lower than the real world value in order to emphasize the influence of the rule for investment diversification on the market.

\(^{12}\)It is considered that the agents buying an asset with the highest price are following an LLCRBO strategy. However, if the asset which they bought is coming down, Formula (7) is not satisfied and they will send an RCRSO in the near future. Therefore, in our model, an agent buys an asset with the highest expected return.
Table 2  Stylized facts of an artificial market where the fundamental prices of three assets are stable.

<table>
<thead>
<tr>
<th></th>
<th>Asset 1 Without the rule</th>
<th>Asset 1 With the rule</th>
<th>Asset 2 Without the rule</th>
<th>Asset 2 With the rule</th>
<th>Asset 3 Without the rule</th>
<th>Asset 3 With the rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kurtosis</td>
<td>4.18</td>
<td>3.89</td>
<td>4.09</td>
<td>4.04</td>
<td>4.16</td>
</tr>
<tr>
<td></td>
<td>Lag</td>
<td>0.12</td>
<td>0.15</td>
<td>0.12</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>Autocorrelation coefficients</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>for square returns</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3  Statistical values of the market price transitions for assets 1, 2, and 3, both with and without the rule for investment diversification.

<table>
<thead>
<tr>
<th></th>
<th>Asset 1 Without the rule</th>
<th>Asset 1 With the rule</th>
<th>Asset 2 Without the rule</th>
<th>Asset 2 With the rule</th>
<th>Asset 3 Without the rule</th>
<th>Asset 3 With the rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>9,993.65</td>
<td>9,977.83</td>
<td>9,989.97</td>
<td>9,974.04</td>
<td>9,997.29</td>
</tr>
<tr>
<td></td>
<td>S.D.</td>
<td>15.35</td>
<td>21.99</td>
<td>17.36</td>
<td>24.32</td>
<td>14.93</td>
</tr>
<tr>
<td></td>
<td>Max.</td>
<td>10,035</td>
<td>10,028</td>
<td>10,039</td>
<td>10,047</td>
<td>10,039</td>
</tr>
<tr>
<td></td>
<td>Min.</td>
<td>9,946</td>
<td>9,917</td>
<td>9,934</td>
<td>9,920</td>
<td>9,956</td>
</tr>
</tbody>
</table>

Fig. 1  Price transitions of markets where no agent follows the rule for investment diversification when the fundamental price of asset 1 declines.

Fig. 2  Price transitions of markets where agents follow the rule for investment diversification when the fundamental price of asset 1 declines.

The market price of asset 1 falls by 30%, thereby causing NAV to drop to 850,000 and the total value of asset 1 held to decrease to 350,000 (Fig. 3 right). The total value of asset 2 held does not change, however, so the amount of asset 2 relative to NAV now exceeds 50%, thereby contravening the rule for investment diversification. Accordingly, to resolve the situation, RCRSOs amounting to 75,000 for asset 2 are placed. As many agents take this action, the market price of asset 2 naturally falls.

The reason that the market price of asset 2 increases after falling is likely to be as follows. As the decline in the market price of asset 1 levels off, the conditions causing the contravention of the rule with regard to asset 2 are gradually resolved. As a result, the placing of RCRSOs stops, causing the decline in the price of asset 2 to stop. At this point, the market price of asset 2 is much lower than the fundamental price, so many of the agents decide that asset 2 is a good buy, promising a positive return on investment. Thus, the number of market buy orders for asset 2 increases, leading to an increase in the market price of asset 2.

The reason that the market price of asset 2 levels off at a price lower than the fundamental price can be explained as follows. Since the market price of asset 2 rises somewhat after dropping, the number of agents contravening the rule for investment diversification increases again, and this leads to a situation in which the number of RCRSOs and the number of market buy orders are approximately equal, causing the market price to level off.

Of the above hypotheses regarding the price transition of asset 2, we first examine whether the reason suggested for why the market price of asset 2 falls in line with that of asset 1 is correct. Here, we examine three types of or-
ders from immediately after the fundamental price of asset 1 drops sharply to when the price of asset 2 levels off: RCRSO, market sell order, and market buy order (Fig. 4).

As shown in Fig. 4, a short time after the fundamental price drops sharply, a large quantity of RCRSOS of asset 2 are placed. Note that although the decline in the market price of asset 2 continues until time 200,000, the number of RCRSOS peaks around time 140,000 before falling. The likely reason for this is that, for many agents, the rule contravention regarding the total value of asset 2 gets resolved naturally in the course of trading.

Here we discuss the reason why the market price of asset 2 overshoots. When the number of RCRSOS of asset 2 increases, the market price of asset 2 starts to fall. When a market price continues to fall for a certain amount of time, agents whose technical strategy weights increase and whose expected returns become negative in their order processes increase. They send market sell orders. Therefore, the market price of asset 2 continues to decline even after the number of RCRSOS starts to drop, as the market sell orders of asset 2 increase, which can be seen in Fig. 4. Mizuta et al. [17] discovered the fact that the market price overshoots and then converges to the equilibrium price such as the fundamental price when a market price continues to decline for a certain amount of time and agents’ technical strategy weights rise on their order processes. Figure 5 shows time series of the fundamental strategy weight (\( \sum_{j=1}^{n} w_{1,j,2} \)) and technical strategy weight (\( \sum_{j=1}^{n} w_{2,j,2} \)) of asset 2. As can be seen from this figure, the technical strategy weight rises when the market price of asset 2 overshoots around time 180,000.

Next, we examine whether the suggested reason for why the market price of asset 2 rises after falling and then levels off at a price lower than the fundamental price is correct. Thus, we examine three types of orders from when the market price of asset 2 begins to fall to its minimum value to the time that the price levels off to a stable value: RCRSO, market sell order, and market buy order (Fig. 6).

Figure 6 confirms that the period from when the market price of asset 2 reaches its minimum value to when the market price stabilizes (approximately the period from 200,000 to 300,000) and the period during which market buy orders are higher than sell orders match quite closely. It also confirms that the period during which market buy orders are approximately equal in number to sell orders corresponds very closely to the period when the market price of asset 2 begins to level off (also see Table 4). These results indicate that our hypothesis about the price transition of asset 2 is correct. However, the result that the market price of asset 2 levels off at a value lower than the fundamental price is very unlikely to be observed in practice. This is because, in our experiment, the rule for investment diversification is imposed on all agents, whereas in a real-world market, there are many investors on which the rule for investment diversification is not imposed.

To access whether the above hypothesis is correct, we performed a simulation in a market where a subset of the agents (20 percent) does not follow the rule for investment diversification. Figure 7 shows price transitions of markets where a subset of the agents does not follow the rule for in-

![Fig. 4 Numbers of asset 2 market orders from time 100,000 to time 200,000.](image1)

![Fig. 5 Time series of fundamental strategy weight and technical strategy weight of asset 2.](image2)

![Fig. 6 Numbers of asset 2 market orders from time 200,000 to time 500,000.](image3)

![Table 4 Numbers of asset 2 market orders from time 400,000 to time 500,000.](table4)
Investment diversification when the fundamental price of asset 1 declines. As can be seen from this figure, the market price of asset 2 in the market where agents who do not follow the rule for investment diversification participate declines less than that of asset 2 in the market where only agents who follow the rule for investment diversification participate after a great crash of the fundamental price of asset 1. Moreover, the former converges to its fundamental price more closely than the latter.

We can see that the number of RCRSOS of asset 2 is less than that of asset 2 in the market where only agents who follow the rule for investment diversification participate, when the market price of asset 2 begins to fall in Fig. 8. In Fig. 9, we can also see that the technical strategy weight of asset 2 is not larger than that of asset 2 in the market where only agents who follow the rule for investment diversification participate.

Therefore, it is considered that the market price of asset 2 in the market where agents who do not follow the rule for investment diversification participate declines less than that of asset 2 in the market where only agents who follow the rule for investment diversification participate. The reason why the former converges to its fundamental price more closely than the latter is that the number of sell orders almost equals that of the market buy orders (around from time 400,000 to time 500,000 in Fig. 9) when the market price of asset 2 becomes higher than that of asset 2 in the market where only agents who follow the rule for investment diversification participate.

These results confirm the above hypothesis.

3.4 Case with a Crash of the Asset 1 Fundamental Price and a Lower Limit of Each Asset

In our artificial market model, each agent trades with the aim of maximizing the value of the assets that they hold. In real-world markets, however, institutional investors that operate mutual funds do not necessarily trade in the same way. The reason is that, since many mutual funds are obligated to use most of the funds entrusted to them for the purchase of assets (full investment) and thus hold only small amounts of cash, they engage in trading so that the total amount of held assets relative to NAV exceeds a specified value.

In this section, we describe an experiment in which a lower limit on the total amount of assets relative to NAV was added to the experiment conditions described in Sect. 3.3.

Figure 10 shows the price transition when a lower limit is set on the total amount of assets relative to NAV ($v_{lower} =$...
Comparing Figs. 2 and 10, we see that the market price of asset 1 in Fig. 10 substantially overshoots after reaching the fundamental price relative to in Fig. 2. In addition, we see that the market price of asset 2 in Fig. 10 declines by a large degree, again relative to in Fig. 2, before increasing and then leveling off at a price below that of Fig. 2. Finally, asset 3 in Fig. 10, unlike in Fig. 2, increases over nearly the same period that the market price of asset 2 is decreasing.

The reason why the amount of overshoot in the market price of asset 1 becomes large is likely to be as follows. Since the initial quantity of shares of asset 3 held by agents is 0, there are also agents that short-sell asset 3, and so an increase in the price of asset 3 causes a reduction in the NAV of these agents. As a result, rule contravention occurs for asset 1, which leads to the placing of RCRSOs. This then leads to a further decline in the market price of asset 1. The large drop in the market price of asset 2 is likely for the same reason.

To assess whether the above argument is correct, we examine four types of orders from immediately following the sharp drop in the fundamental price of asset 1 until the declines in the market prices of assets 1 and 2 level off: RCRSO, market sell order, market buy order, and LLCRBO (Figs. 11 and 12).

From Figs. 10 and 11, we can confirm that the period from when the market price of asset 1 reaches the fundamental price until the price decline has leveled off matches closely the period during which the number of RCRSO increases. In Fig. 12, we see that the number of RCRSOs is high compared to in Fig. 4, and that consequently the number of market sell orders increases, due to the strong influence of technical factors. The above observations suggest that our hypothesis about the large drop in the market prices of both assets 1 and 2 is valid.

Next, the reason that the market price of asset 3 increases can be explained as follows. As the fundamental price of asset 1 falls sharply, the market prices of assets 1 and 2 decline, thereby reducing the amount of each asset relative to NAV. If the total amount of assets relative to NAV falls below the lower limit, LLCRBOs are then placed. However, since the rule for investment diversification is contravened for assets 1 and 2, it is not possible to buy any more of these assets. Accordingly, since LLCRBOs are placed for asset 3, the market price of asset 3 is likely to rise.

As a concrete example, consider Fig. 13. Suppose is set at 0.95. Consider the case when RCRSOs are placed from the state shown on the right of Fig. 3, after which the rule contravention for asset 2 is resolved (left of Fig. 13). The NAV of agents at this time is 850,000, the amount of cash held is 75,000, the value of asset 1 held is 350,000, and the value of asset 2 held is 425,000. At this point, the total amount of assets relative to NAV is 0.91, which is now below the lower limit. For this reason, 32,500 LLCRBOs for asset 3 are placed, thereby driving up the market price of asset 3. The increase in the market price of asset 3 lagging behind the drop in the market price of asset 1 is probably due to the fact that since the size of each order in this model is always 1, the selling of assets following the sharp drop in fundamental price takes time.

To assess whether the above hypothesis is correct, we examine four types of orders from immediately following the sharp drop in the fundamental price of asset 1 until the increase in the market prices of asset 3 levels off: RCRSO, market sell order, market buy order, and LLCRBO (Fig. 14).

From Fig. 14, we can confirm that LLCRBOs are placed from when the market price of asset 3 starts to increase (approximately time 110,000). Accordingly, our hypothesis seems to be valid. The number of LLCRBOs peaks
like to consider the LLCRBO strategy when the lower limit on the total amount of assets is not satisfied, for example, agents buy not just one asset but assets which satisfy the rule for investment diversification and whose expected returns are positive in terms of asset risk diversification.

4. Conclusion

In this study, we added an asset which has not been held until the fundamental price of a particular asset drops sharply to our artificial market model in order to examine the impact of the rule for investment diversification on markets for the case when fundamental prices of all assets are constant and the case when the fundamental price of a particular asset drops sharply. We furthermore explored scenarios under the rule for investment diversification in which the asset price of a particular asset drops sharply and a lower limit on the total amount of assets relative to NAV either is or is not imposed.

The results showed that when the fundamental price of all assets is constant, the market prices of all assets transition stably, regardless of whether the rule for investment diversification is imposed. Next, we verified that if there is a sharp drop in the fundamental price of asset 1 in a market with the rule for investment diversification, then the market price of asset 2 falls in line with the decline in the market price of asset 1. In contrast, in a market without the rule, only the market price of asset 1 declines. Furthermore, in the case when a lower limit is set on the total amount of assets relative to NAV, the market prices of assets 1 and 2 both decline, whereas the market price of asset 3 increases.

Looking ahead, we plan to extend this line of research as follows. This artificial market model has three assets, but since numerous assets can be traded in real-world financial markets, we wish to conduct experiments with artificial markets having a greater number of assets. In addition, since the number of orders per agent per timestep is fixed at 1, the movement of the market can be slower than that of a real-world market. Accordingly, we will try increasing the number of orders per agent per timestep. Finally, we would

Acknowledgments

This research was supported by JSPS KAKENHI Grant Number 15K01211.

References


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