SUMMARY

A number of battery-driven sensor nodes are deployed to operate a wireless sensor network, and many routing protocols have been proposed to reduce energy consumption for data communications in the networks. We have proposed a new routing policy which employs a nearest-neighbor forwarding based on hop progress. Our proposed routing method has a topology parameter named forwarding angle to determine which node to connect with as a next-hop, and is compared with other existing policies to clarify the best topology for energy efficiency. In this paper, we also formulate the energy budget for networks with the routing policy by means of stochastic-geometric analysis on hop-count distributions for random planar networks. The formulation enables us to tell how much energy is required for all nodes in the network to forward sensed data in a pre-deployment phase. Simulation results show that the optimal topology varies according to node density in the network. Direct communication to the sink is superior for a small-sized network, and the multihop routing is more effective as the network becomes sparser. Evaluation results also demonstrate that our energy formulation can well approximate the energy budget, especially for small networks with a small forwarding angle. Discussion on the error with a large forwarding angle is then made with a geographical metric. It is finally clarified that our analytical expressions can obtain the optimal forwarding angle which yields the best energy efficiency for the routing policy when the network is moderately dense.

key words: wireless sensor networks, network topology, routing policy, energy efficiency, energy formulation

1. Introduction

A wireless sensor network (WSN) is one of the areas of active research. In order to provide beneficial services in WSNs, a large number of sensor nodes are deployed in a physical environment, and they forward the data sensed by nodes to their sink (base station). One of the typical applications of WSNs is the environmental observation for disaster prediction such as fire, earthquake, flood and landslide [2]. On the subject of operating a WSN, power saving of nodes is a problem of vital significance. Battery replacement is required for nodes out of battery. However, when the nodes are placed underwater or in a forest, it is difficult to maintain the nodes because of the harsh geographical conditions, which may lead to an operation failure of the network. Considering that most of the energy is dissipated when nodes are transmitting and receiving data, the solution for that problem is to prolong the lifetime of the nodes by reducing the energy required during data transmission [3].

Various network topologies, such as direct data transmission, shortest path tree (SPT), Power-Efficient Gathering in Sensor Information System (PEGASIS) [4] and Low-Energy Adaptive Clustering Hierarchy (LEACH) [5], [6], have been proposed for energy-efficient data collection in WSNs. The optimal network topology for energy efficiency depends on various network parameters such as the number of nodes, network area size and data aggregation [7]. Therefore, we explore a newly considered parameter, forwarding angle, for the progress-based nearest forwarding (PNF) routing policy, which we have previously proposed for a simple yet energy-efficient data forwarding scheme [1], [8]. It determines the angular spread of neighborhood area in which nodes in a network search for their next-hop nodes. It is also noteworthy that we can metamorphose the network topology solely by changing the forwarding angle, even if the same node arrangement is given. Following this definition, we investigate the optimal forwarding angle in terms of energy efficiency for the routing scheme. Also, PNF routing is compared with other routing schemes to clarify the optimal topology which best achieves energy efficiency.

When it comes to energy budget, if we have the knowledge of how much energy consumption will be required to operate a network a priori, i.e., before deploying the nodes, the energy budget for the desired network can be clarified, leading to the determination of the optimal topology. The authors of [7] categorize network topologies according to their structures and construct the closed-form mathematical energy consumption model for each type of network. Their simulation results compare the energy consumption among the topologies to reveal the best topology for data delivery; however, the analytically derived expressions are not discussed with the simulation results. For instance, in spite of the simulation of SPT topology, a perfect M-ary tree is assumed for energy formulation. The authors also assume that for the energy formulation in cluster topology, each of the clusters includes evenly \([N/k]\) nodes, where \(N\) is the number of nodes in the network and \(k\) is the number of clusters, which is quite different from empirical cluster-based networks. These assumptions beg the question of whether the analytical models have an applicability for empirical networks.

In this paper, therefore, we construct applicable energy expressions for WSNs which employ both PNF routing and
direct communication policies. Although our analytically derived solution cannot be directly applied to networks with other routing protocols such as SPT and the greedy, it will help the insight into energy consumption modeling. The energy formulation requires the hop-count distribution for the nodes deployed in the network. In light of this, we alter and apply the closed-form expression for the hop-count distribution in random planar ad-hoc networks [9] to the progress-based topology. The theory in [9] is based on the unit disk model, where all nodes have a common transmission range, and a data forwarding between a single source node and a single destination node. We extend the theory to WSNs with transmission power control (TPC), where all nodes have different transmission power according to their communication distances. The derived expression in this paper is the function of the number of nodes, node density, network sector and forwarding angle (to be discussed in Sect. 4). The results of this analysis, if combined with the knowledge of network lifetime, can be used to investigate how much battery capacity the nodes should be equipped with.

The rest of this paper is organized as follows. Section 2 describes data forwarding, radio propagation and energy consumption models. Section 3 presents PNF routing policy in detail, and the energy budget based on the topology is formulated in Sect. 4. Then, the evaluated results of the optimal forwarding angle, the optimal network topology and the energy formulation are given and discussed in Sect. 5, as well as the distance-based energy characteristics and network lifetime. Finally, Sect. 6 concludes this paper.

2. Preliminaries

We remark on some preliminaries, i.e., data forwarding, radio propagation and energy consumption models in this section in order to clarify both an analytical basis and a simulation environment. Environmental premises in this paper are as follows.

- A WSN under a relay architecture with a single sink is assumed, with the traffic flow from each sensor node to the sink.
- When a source generates a packet whose destination is the sink, its forwarding path from the source to the sink is reserved and every data communication between nodes is assumed to be successful. Neither transmission error nor packet loss is counted for, and data retransmissions are not performed.
- While data forwarding, all the nodes which do not become engaged in the packet forwarding are set to sleep; thus, overhearing at each node is not considered, and each node transmits data only to its parent node (fixed next-hop node).
- When a node behaves as a relay which forwards data from a source to the sink, it does not aggregate its own sensing data to the packet sent from the source; thus, when it has \( N_d \) descendent nodes, the numbers of relaying and transmissions are \( N_d \) and \( N_d + 1 \), respectively.
- Consumption energy of data sensing, reception and transmission after routing completion is considered.
- Two-ray ground reflection model is assumed as radio propagation, and the propagation loss depends only on distance attenuation.
- Each node performs TPC according to its transmission distance so that the received power is constant.
- Energy consumption at the sink is not considered, with the infinite power source for the sink in mind.

In light of the above, we consider three portions of energy consumption at each node: energy for data sensing \( E_s \), reception \( E_R \) and transmission \( E_T \). Each energy is expressed by [4]–[6]

\[
E_S = l_{sen} E_{sen}, \quad (1)
\]
\[
E_R = l_{comm} E_{elec}, \quad (2)
\]
\[
E_T = \begin{cases} l_{comm} (E_{elec} + e_{fs} r^2), & \text{for } r < r_0, \\ l_{comm} (E_{elec} + e_{mp} r^4), & \text{for } r \geq r_0, \end{cases} \quad (3)
\]

where \( l_{sen} \) is the sensed data size at a source node, \( l_{comm} \) is the transmitted/received packet size at each node; \( E_{sen} \) and \( E_{elec} \) are the energy for data sensing and transmission/reception per bit, respectively; \( e_{fs} \) and \( e_{mp} \) are the free-space propagation loss coefficient and the multipath propagation loss coefficient, respectively; \( r_0 = \sqrt{\frac{E_{fs}}{E_{mp}}} \) is the break point for the two-ray ground model, and \( r \) is the transmission distance (or Euclidean hop distance). For packet forwarding where the payload length is \( S \) and the header length is \( H \), \( l_{sen} = S \) and \( l_{comm} = S + H \). Note that when a node behaves as a relay, \( E_S = 0 \).

3. Progress-Based Nearest Forwarding [1], [8]

For efficient routing, progress should be made at each hop, i.e., the next-hop neighbor should be closer to the sink [10]. We previously proposed PNF routing policy, which is based on hop progress. Figure 1 illustrates how the PNF routing is constructed from a source to its destination. Algorithm 1 summarizes the procedure of PNF routing. It considers neighborhood area toward the sink for each node to search for its next-hop node. The area has a forwarding angle defined by \( \phi \), where \( 0 \leq \phi < \pi/2 \) to guarantee hop progress. The nearest-neighbor node from the sender inside its neighborhood area is elected as next-hop. Thus any node located outside the neighborhood area is passed over for a next-hop candidate even if it is the nearest from the previous node. The forwarding angle \( \phi \) is of topological uniqueness. That is, a multihop path from a source to the sink is likely to be linear for networks with a small \( \phi \), and zigzag for a large \( \phi \). No neighborhood area is allowed when \( \phi = 0 \), and the network has the form of direct communication topology, where all nodes in the network are directly
Algorithm 1 PNF routing

**Input:** number of nodes $N$, forwarding angle $\phi$, sink position $(x_0, y_0)$, node locations $(x_i, y_i)$

**Output:** set $\mathbf{P}$ which consists of parents $p_i$ of all $N$ nodes

1: for $i = 1$ to $N$ do
2:     $\text{nearest} \leftarrow \infty$
3:     $D \leftarrow \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$
4:     for $j = 1$ to $N$ do
5:         if $i \neq j \cap j \neq 0$ then
6:             $d \leftarrow \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
7:         if $d < D \cap$ node $j$ is inside neighborhood area defined by $\phi$ centered at node $i$ then
8:             $\text{nearest} \leftarrow d$
9:         $p_i \leftarrow j$
10:     end if
11: end if
12: end if
13: end if
14: end for
15: if $\text{nearest} = \infty$ then
16:     $p_i \leftarrow \text{sink}$
17: end if
18: end for

connected with the sink by single-hops, considered energy-inefficient when the small number of nodes are deployed in a large network. In this manner, adjusting the value of $\phi$ means the change in Euclidean hop distance (physical hop distance), the number of hops (operational hop distance) and latency in the network. Therefore, the forwarding angle is a parameter to characterize the network topology. From this viewpoint, there should be the optimal forwarding angle $\phi^*$ for energy efficiency. Therefore, we will evaluate the values of $\phi^*$ to best achieve energy efficiency in simulation to be given later.

PNF routing is a cherry-picking scheme; its features applied are progress-based routing, nearest-neighbor forwarding and variable-range TPC. Progress-based routing leads to an efficient framework in terms of energy efficiency and latency. Nearest-neighbor forwarding is often a design choice for packet routing [9], and applied to the literature [11]–[13]. The literature based on the nearest forwarding [11], [12] claims that the short hop routing achieves a good performance under energy consideration for most sce-

narios. TPC sets the radio transmission power of each node to an appropriate level according to the communication distance, which enables preventing the networks from having an isolated node which does not have a path to reach the sink. In [14], TPC mechanism is implemented in the ad-hoc on-demand distance vector (AODV) routing protocol for mobile WSNs, and it is shown that implementation of TPC has some positive impact on networks in respect to energy consumption. In [15], the minimum spanning tree rooted at one source node is constructed on the basis of TPC, and it is clarified that compared to the disk model, the variable-range strategy can save transmission power, as well as improving the traffic capacity. To verify the validity of PNF routing, we will compare the performance of PNF with those of other routing protocols.

When location-based packet forwarding topologies such as SPT and PNF are constructed in actual applications, location information on all nodes including the sink is required. It is achieved by measuring the received signal strength from other nodes and the sink, and/or using geographical location service [16]. In addition, considering a location-based TPC mechanism for routing, overhead energy for channel state estimation has an effect on the total energy consumption. Giving the same number of nodes and the same node arrangement, however, such routings have the same number of links in the network, leading to the same additional energy budget for location information retrieving and channel state estimation [8]. Therefore, we can have a consequence that the fair premise in any routing policy results in an impartial energy addition, and we omit the consideration of those kinds of extra energy budgets.

4. Energy Formulation

This section formulates the energy budget required for all $N$ nodes in a WSN which employs the PNF routing policy. We start with obtaining the distance between the sink and its $n$-th nearest neighbor in the network. Consider the node appearance process according to the two-dimensional Poisson point process with intensity $\rho$, and then the Euclidean distance $D_n$ to the $n$-th nearest neighbor of the sink within a sector with angle $2\theta$ follows the distribution such that [10]

$$D_n \sim p_{D_n}(r) = \frac{2(\rho \theta r^2)^n}{r \Gamma(n)} e^{-\rho \theta r^2},$$

where $\Gamma(\cdot)$ is the Gamma function. The expected distance $E[D_n]$ and the higher moments are

$$E[D_n] = \sqrt{\frac{1}{\rho \theta}} (n)_{1/2},$$

$$E[D_n^2] = \left(\frac{1}{\rho \theta}\right)^{1/2} (n)_{3/2},$$

respectively, where $(n)_m$ is the Pochhammer symbol

$$(n)_m = \frac{\Gamma(n + m)}{\Gamma(n)}.$$

and \( \alpha \) denotes the path loss exponent \([10]\). \( \alpha \) takes the value of two when \( r < r_0 \), and four when \( r \geq r_0 \).

Here, we must refer to the fact that when \( n \) is large, \( n > 171 \) in our computation environment, \( \Gamma(n) \) diverges, making the computation of \((n)_{1/2} \) intractable. This invokes the necessity of approximation. By applying the approximate expression of the Gamma function \([17]\)

\[
\Gamma(z) \approx \sqrt{2\pi} \left( \frac{1}{z} \left( z + \frac{1}{12z - \frac{1}{10z}} \right) \right)^z, \quad \text{for } \Re[z] > 0,
\]

and performing the Taylor expansion for \( n \to \infty \), we obtain

\[
(n)_{1/2} = \frac{1}{3n} \left( \frac{1}{\sqrt{\pi}} - \frac{1}{4n \sqrt{\pi}} + O\left(\left(\frac{1}{n}\right)^2\right)\right) \\
\times \exp\left\{ n \log 3 + \frac{1}{2} \left( 1 - \log \frac{1}{n} \right) \right\} \\
+ \frac{1}{8n} - \frac{1}{16n^2} + O\left(\left(\frac{1}{n}\right)^3\right).
\]

Terminating the calculation of negligible orders in Eq. (9) yields

\[
(n)_{1/2} \approx \sqrt{n} \left( 1 - \frac{1}{4n} \right) \exp\left( \frac{2n - 1}{16n^2} \right).
\]

In this approximate expression, the error from the true value gets smaller as \( n \) grows. Better yet, the error is negligibly small even for a small \( n \), since Eq. (10) converges quickly.

Next we consider the hop-count of a route between a source \( s \) (i.e., the \( n \)-th nearest neighbor of the sink, \( 1 \leq n \leq N \)) to the sink \( t \) separated by distance \( D_n \). Let node \( h_t \) be the nearest neighbor of \( h_{t-1} \) inside the neighborhood area toward \( t \), with \( h_0 = s \). Then, the projected node \( h'_t \) is an imaginary node located on the baseline between \( h_{t-1} \) and \( t \), resulting from the projection of \( h_t \) onto the baseline along the arc centered at \( h'_{t-1} \). Therefore, the projected node \( h'_t \) is the nearest-neighbor from \( h'_{t-1} \). Figure 2 illustrates a brief concept of node projection. As a result of iterating the projection, we obtain a set of projected nodes \{\( s, h'_1, \ldots, h'_{t}, \ldots, h'_{N_p(n)} \), \( t \)\}, where \( N_p(n) \) is the number of projected nodes initiated from \( s \). This operation to obtain the set of projected nodes is defined in \([9]\) as linear equivalent. Notice that due to the projections, the node separation of linear equivalent is equated to the random variant \( R \) of Euclidean hop distance to the nearest neighbor \( h_t \), such that

\[
R \sim p_D(r) = 2\rho \phi \exp(-\rho \phi z^2),
\]

which can be reduced from Eq. (4) substituting \( n = 1 \) and \( \theta = \phi \). Since Eq. (11) is the Rayleigh distribution, its \( k \)-th moment \( \mu_k \) can be expressed from definition by

\[
\mu_k = 2^{k/2} \sigma^k \Gamma\left( 1 + \frac{k}{2} \right),
\]

where \( \sigma = \sqrt{1/2\rho \phi} \). It is clear that \( \mu_k \) is a function of node density \( \rho \) and forwarding angle \( \phi \).

We here consider a random linear (one-dimensional) network with a source-destination pair \( (s,t) \) and \( N_l \) relay nodes. \( (s,t) \) is \( D \) away from each other. Additionally, the node separation \( r_i \) between node \( h_i \) and \( h_{i-1} \) is assumed to be a random variate with an arbitrary continuous PDF \( p_r(r) \), and node \( h_j \) is located at the distance \( z_j \) away from \( s \), where \( z_j = \sum_{i=1}^{j} r_i \), i.e., \( j \)-hop distance. Plus, let \( P_z(r,j) \) denote the cumulative distribution function (CDF) of \( z_j \). Then, the number of nodes \( N_l \) follows a difference of the two CDFs \( P_z(r,j) \) and \( P_z(r,j+1) \), such that \([9]\)

\[
N_l \sim (P_z(D_j) - P_z(D_j + 1)).
\]

In this manner, Eq. (13) gives the hop-count distribution required for \( s \) to reach \( t \) at distance \( D \) for any node separation distribution. For the planar network under linear equivalent performed, it is shown in \([9]\) that the Nakagami-\( m \) CDF can approximate \( P_z(D_j) \) and \( P_z(D_j+1) \) in Eq. (13). Therefore, the number of projected nodes \( N_p(n) \) follows a difference of the Nakagami-\( m \) CDFs, such that

\[
N_p(n) \sim \left( P_{Nak}(D_n; m(j; \rho, \phi), \Omega(j; \rho, \phi)) - P_{Nak}(D_n; m(j+1; \rho, \phi), \Omega(j+1; \rho, \phi)) \right) \\
= P_{Nak}(j; D_n; \rho, \phi),
\]

\[
P_{Nak}(x; m, \Omega) = \frac{\gamma(m, mx^2/\Omega)}{\Gamma(m)},
\]

\[
\Omega(j; \rho, \phi) = E[X^2] = \eta \mu_2 + (j^2 - j)\mu_1^2,
\]

\[
m(j; \rho, \phi) = \frac{(E[X^2])^2}{E[(X^2)^2]} = \frac{(\Omega(j; \rho, \phi))^2}{Z(j; \rho, \phi) - (\Omega(j; \rho, \phi))^2},
\]

\[
Z(j; \rho, \phi) = \eta \mu_4 + 4(j^2 - j)\mu_3 \mu_1 + 3(j^2 - j)\mu_2^2 \\
+ 6(j^3 - j^2)\mu_1 \mu_2 \\
+ (j^4 - 6j^3 + 11j^2 - 6j)\mu_1^4,
\]

where \( \gamma(\cdot, \cdot) \) is the lower incomplete Gamma function, \( m(j) \) and \( \Omega(j) \) are the shape and scale parameters of the Nakagami-\( m \)-distribution, respectively, and \( Z(j) \) denotes the fourth moment of sum of \( j \) i.i.d. random variants \( R \)-s. It
is noteworthy that the Nakagami-\(m\) parameters are direct functions of its second and fourth moments, which can be derived from the \(k\)-th moment in Eq. (12). According to the above results, the closed-form solution of the hop-count distribution for a route from a node to the sink can be derived. The expected number of relay nodes \(E[N_p(n)]\) and the estimated average number of hops \(\bar{N}_H(n)\) required to reach from \(s\) to \(t\) can be expressed by

\[
E[N_p(n)] = \sum_{j=1}^{\infty} j P_{\bar{N}}(j; D_n, \rho, \phi),
\]

\[
\bar{N}_H(n) = E[N_p(n)] + 1,
\]

respectively. The infinite summation in Eq. (19) can be truncated by an upper bound \(N\) since the upper bound of hop-count is \(N\), where all nodes belong to one single path (chain-like topology) toward the sink.

After deriving these analytical values, we reach the final step of formulation, the energy budget for all \(N\) nodes in the network. We separately derive the budget for \(\phi = 0\) and \(\phi > 0\). First for the former case, the hop-count equals one for all nodes in common. According to the value of \(E[D_n]\), energy for data transmission \(E_T\) for the source (the \(n\)-th nearest-neighbor node of the sink) is calculated by Eq. (3) substituting \(r = E[D_n]\). Then, the energy budget \(E(n)\) of one source is the sum of the energy for its data sensing \(E_S\) and transmitting \(E_T\). No energy is required for data receiving, \(i.e., E_R = 0\). Therefore, the total energy budget per node \(E_{\text{total}}\) required for all \(N\) nodes in a network to transmit their sensed data directly to the sink is calculated by

\[
E_{\text{total}} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} E(n).
\]

Note that taking the limit of \(N\) to infinity is required to evaluate the expected energy budget at a steady state as it is solely dependent on network size, independent of the number of nodes, when \(\phi = 0\).

And for the latter case of \(\phi > 0\), we calculate the probability that the distance \(R\) of an arbitrary communication link between two terminals (node-to-node or node-to-sink) is the break point \(r_0\) or over. The probability is given by

\[
\Pr[R \geq r_0] = 1 - \int_0^{r_0} [p_{D_n}(r)]_{\alpha=1, \phi=\phi} dr = e^{-\frac{\phi}{\rho}}.
\]

Here, a conflict between Eq. (22) and an actual network is that \(\Pr[R \geq r_0] = 0\) for \(D_n < r_0\); i.e., any node located at a distance of less than \(r_0\) away from the sink always dissipates energy proportional to \(r^2\) for data transmission (as in Eq. (3)). Equation (22) holds true if we consider an arbitrary area in an infinite two-dimensional plane with intensity \(\rho\). Then, Eq. (22) must be modified such that

\[
\Pr[R \geq r_0] = \begin{cases} 
0, & \text{for } L < r_0, \\
1 - \frac{r_0^2}{L^2} e^{-\frac{\phi}{\rho}}, & \text{for } L \geq r_0,
\end{cases}
\]

where \(L\) is the radius of the network sector centered at the sink; \(i.e., no nodes are located beyond the network bounds. Note that \(\frac{r_0^2}{L^2}\) equals the ratio of the area within distance \(r_0\) from the sink to the network area, \(i.e., the probability that a node in the network is not as far from the sink as \(r_0\). Therefore, the average energy \(E_T(n)\) required for the nodes belonging to a multihop path from the source to the sink to transmit data to their parent nodes (excluding the energy for data sensing and receiving) is

\[
E_T(n; \rho, \phi) = E_p(n; \rho, \phi) + E_Q(n; \rho, \phi),
\]

\[
E_p(n; \rho, \phi) = (1 - \Pr[R \geq r_0]) \bar{N}_H(n) E_{\text{com}}
\]

\[
\times \left( E_{\text{elec}} + \epsilon_{\text{fs}} E[R^2] \right),
\]

\[
E_Q(n; \rho, \phi) = \Pr[R \geq r_0] \bar{N}_H(n) E_{\text{com}}
\]

\[
\times \left( E_{\text{elec}} + \epsilon_{\text{mp}} E[R^4] \right),
\]

where \(E[R^2] = 1/\rho\phi\) and \(E[R^4] = 2/(\rho\phi)^2\) are the second and fourth moments of \(R\), respectively, which can be obtained by substituting \(n = 1, \theta = \phi\) and \(\alpha = 2, 4\) in Eq. (6). Then, the energy budget \(E(n)\) for one source to forward its data to the sink is equivalent to the energy required for the nodes belonging to the multihop path to forward data which the source generates, such that

\[
E(n; \rho, \phi) = E_S + (\bar{N}_H(n) - 1) E_R + E_T(n).
\]

Therefore, the total energy budget per node \(E_{\text{total}}\) for all \(N\) nodes to forward the data generated at each node to the sink in a multihop way is then derived by

\[
E_{\text{total}} = \frac{1}{N} \sum_{n=1}^{N} E(n; \rho, \phi).
\]

5. Performance Evaluations

This section clarifies the optimal routing topology to best achieve energy efficiency, and assesses the validity of energy formulation for PNF routing. Table 1 shows the simulation specification. For circle networks, the area is defined as a circle with radius \(L\). For fan-shaped networks, it is defined as a fan-shaped sector with radius \(L\) and central angle of \(\pi/2\), \(i.e.,\) the first quadrant of a circle with radius \(L\). It is to be noticed that \(L\) does not indicate the transmission range of nodes, but the largest possible distance at which

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Simulation specification.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network radius, (L)</td>
<td>20–500 m</td>
</tr>
<tr>
<td>Number of nodes, (N)</td>
<td>20–300</td>
</tr>
<tr>
<td>Forwarding angle, (\phi)</td>
<td>0–(\pi/4) (0°–45°)</td>
</tr>
<tr>
<td>Data size, (S)</td>
<td>500 bytes</td>
</tr>
<tr>
<td>Header size, (H)</td>
<td>25 bytes</td>
</tr>
<tr>
<td>(E_{\text{elec}})</td>
<td>50 nJ/bit</td>
</tr>
<tr>
<td>(E_{\text{vas}})</td>
<td>5 nJ/bit</td>
</tr>
<tr>
<td>(\epsilon_{\text{fs}})</td>
<td>10 pJ/bit/m²</td>
</tr>
<tr>
<td>(\epsilon_{\text{mp}})</td>
<td>0.0013 pJ/bit/m⁴</td>
</tr>
<tr>
<td>(r_0)</td>
<td>87.7 m</td>
</tr>
<tr>
<td>Number of trials</td>
<td>10,000</td>
</tr>
</tbody>
</table>
nodes are distributed away from the sink. \( N \) nodes are randomly distributed in the network, and the sink is at the origin \((0,0)\). We use the energy model in [6] to determine the energy for data sensing, reception and transmission. Evaluations for PNF routing are scanned with five-degree intervals of \( \phi \), and all evaluations are made through MATLAB_2013a and C language coding.

For investigating the optimal network topology, we compare PNF routing (including direct communication as \( \phi = 0 \)) with SPT topology. Direct communication to the sink is one simple approach as each node in the network directly sends its data to the sink; however, when the sink is distant from the node, the node consumes relatively much energy for data delivery [2]. On the other hand, one of the most primitive yet effective methods for energy-efficient routing is SPT, which constructs a tree structure rooted at the sink. SPT-based topologies [18] generally optimize distance-based costs for constructing multihop paths. The simplest approach to construct such an SPT is Dijkstra’s algorithm [19]. The compared SPT method we prepare minimizes the summed cost \( \sum_{i,j \in L} r^2_i \) by Dijkstra’s algorithm, where \( r_i \) is the \( i \)-th Euclidean hop distance, \( N_H \) is the hop-count and \( \alpha \) is the path-loss exponent, to establish an SPT among all nodes in a network.

### 5.1 Optimal Network Topology and Forwarding Angle

Figure 3 illustrates the optimal network topology which leads to the best energy efficiency for \( 15 \times 25 \) combinations of \( N \) and \( L \). Figure 3 (a) is the result for circle networks, and Fig. 3 (b) is the counterpart for fan-shaped networks. PNF routing in Fig. 3 also gives the optimal forwarding angles \( \phi^* \) in degrees. We can see that the optimal topology is dependent on node density in the network. For circle networks (Fig. 3 (a)), direct communication topology is superior to other multihop topologies when the network is relatively small (\( L \sim 140 \) m). Given the same \( N \), the expected hop distance is short for small networks and direct communications to the sink are more energy-efficient than routing multihop paths to shorten hop distances. As the network becomes wider, however, direct communication loses the superiority, and in contrast, the multihop topology by PNF routing outperforms the others. It is noteworthy that the value of \( \phi^* \) gets larger as \( L \) grows, i.e., as the network gets sparse. A low density puts direct communication at a disadvantage for energy-efficient data forwarding, and thus a larger \( \phi \) is of efficacy. When the network becomes further sparse, the superiority of hop progress by PNF routing is lost, leading to the superiority of SPT. It is observed that the border density between PNF with \( \phi = \pi/4 \) and SPT in Fig. 3 (a) is around \( 10^{12} \) nodes/m\(^2\). When the node density is quite small, signals often encounter a harsh attenuation, i.e., the energy for data transmission is likely to be more proportional to \( r^4 \) than \( r^2 \). As mentioned above, SPT minimizes the distance-based costs while increasing the number of hops required for reaching the sink. Therefore, SPT can suppress energy dissipating thanks to its shorter hops. This is the reason why SPT outperforms PNF for very sparse networks. In this way, the optimal network topology is deeply related to the node density in the network.

For fan-shaped networks (Fig. 3 (b)), the data map has more superiority of PNF with a small \( \phi \) including direct communication. We explain the reason for the observations. The node density for circle networks equals \( N/\pi L^2 \), on the other hand, the counterpart for fan-shaped networks equals \( 4N/\pi L^2 \). Therefore, fan-shaped networks have four times as much node density as circle networks, given the same combination of \( N \) and \( L \). A higher density means shorter hops in the network, and thus there is not the urgent necessity of spreading the forwarding angle \( \phi \) compared to circle networks, resulting in more small \( \phi \) cases emerging in Fig. 3 (b).

### 5.2 Validity of Energy Formulation

We validate our analytical formulation results from the aspects of energy budget, geographical metric and optimal forwarding angle. Figure 4 shows the analytically derived and empirically simulated results of energy budget for circle networks with \( N = 300 \), focusing on the cases of \( \phi = 0 \) (direct communication), \( \pi/36 \) (5°) and \( \pi/4 \) (45°). It can be seen that the result for \( \phi = 0 \) well approximates the empirical one. The formulation accuracy for direct communication depends on the computational cost \( N \) in Eq. (21), with the

![Fig. 3](image-url)  
Data map illustrating the best network topology for energy efficiency. The optimal forwarding angles in degrees are also given for PNF routing. (a) For circle networks, (b) for fan-shaped networks.
average percentage error of 24%, 2.3% and 0.22% for the cost $N = 10^1$, $10^2$ and $10^3$, respectively. The result of $\phi = 0$ in Fig. 4 is for a relatively low cost ($N = 10^2$). Thus, we can calculate the energy budget for $\phi = 0$ with a small computational cost. On the other hand, the accuracy of the result for $\phi = \pi/36$ depends on node density, with a larger error for a larger network. This is because the actual distance distributions are not precisely consistent with the analytical ones. In our analysis (in Eq. (4)), the node distribution is assumed to abide by an infinite two-dimensional Poisson point process, where the sink location is not considered. In an actual case, however, hops for nodes near the sink are likely to be last-hops, which are shorter than the average-hops [20]. Especially, the distance deviation is more noticeable when $\phi$ becomes small, i.e., when hop distance becomes longer in average. Plus, a larger network means longer hop distance, resulting in a harsh radio attenuation and a relatively large error in energy formulation.

Figure 4 also shows that the formulation result for $\phi = \pi/4$ always underestimates the empirical one, with the errors of approximately 10%. The characteristics of this lower-bounding are described in [9], claiming that this is due to the underestimation of the hop-count distribution resulting from linear equivalent. Next we discuss the error of our formulated expression caused by linear equivalent in a geographical way. Consider a single hop from a sender $h_i$ and its next-hop node $h_{i+1}$, with node $h'_{i+1}$ projected onto the baseline between $h_i$ and the sink by linear equivalent. Let $\delta$ denote the angle between the baseline and the hop link from $h_i$ to $h_{i+1}$, i.e., the deviation of node position from the baseline. We now define a new metric in terms of hop progress, namely, progress maintenance ratio (PMR). PMR is defined as a ratio of virtual hop progress toward the sink to actual hop progress. This ratio can be calculated from trigonometry by $PMR = \cos \delta$. PMR indicates how much progress in an actual routing can be maintained compared to hop progress made by linear equivalent. It is known that the paths in random networks with high densities typically approach straight lines [21]. Moreover, we know that a smaller $\phi$ in PNF routing also results in nearly straight paths as previously described in Sect. 3. If there is no deviation in a node position, i.e., the next-hop node is located just on the baseline, or the network topology is direct communication, $PMR = 1$. Also, PMR for the last-hop is always one since the last-hop always connects to the sink. Meanwhile, when the next-hop node is located on the boundary of neighborhood area, PMR takes the worst value for the forwarding angle. We can easily obtain the mean value of PMR:

$$PMR = \frac{1}{\phi} \int_0^\phi \cos \delta \, d\delta = \text{sinc} \phi,$$

Here, $PMR \approx 1$ for PNF routing with $\phi = \pi/36$, and $PMR \approx 0.90$ for $\phi = \pi/4$. This means that the hop progress in an actual routing with $\phi = \pi/4$ degrades by approximately 10% in average compared to the progress by linear equivalent. It is also to be noted that $PMR$ reduces to a constant value, independent of node density. In fact, we can confirm in Fig. 5 that the empirical values of $PMR$ obtained by simulation are also insensitive to node density, constant in the range of interest. In this manner, we can attribute the energy derivation of roughly 10% to the degraded hop progress. Therefore, in order to improve the accuracy of energy formulation for a large $\phi$, it is of efficacy that we utilize PMR as a scaling parameter for analyzing energy budget in the whole network.

As a final remark on the validity of energy formulation, we show the analytically derived optimal forwarding angles for PNF routing. The energy budget for each $\phi$ can be obtained by substituting an arbitrary forwarding angle $\phi$ in Eq. (28). We compare each of the budget, choose the optimal forwarding angle $\phi^*$ for each combination of $N$ and $L$, and list them in Fig. 6. Note that Fig. 6 does not include the result for SPT, since our energy formulation covers only PNF routing. Instead, it can show the result for direct communication topology by Eq. (21). Comparing Figs. 3 and 6, it can be seen that our analytical expressions can tell the optimal forwarding angle $\phi^*$ when the network density is fairly high. In Fig. 6, the result for circle networks is less able in accuracy than that for fan-shaped network, again because of the difference in node density. The validity ratio is 71% (268/375) for circle networks, and 90% (337/375) for fan-shaped networks. Therefore, we conclude that our formulation results can be well applied to determining the optimal forwarding angle for PNF routing policy in WSNs.
with a moderately large node density.

5.3 Distance-Based Energy and Network Lifetime

We have thus far discussed the results of the optimal network topology and forwarding angle, as well as the validity of energy formulation, on the basis of total energy budget per node $E_{\text{total}}$. Here, however, energy consumed by each node in a sensing field is considered and discussed, in conjunction with the lifetime of the network.

Figure 7 shows distance-energy characteristics in a circular sensing field with $N = 160$ and $L = 500$ m. Recall that in such an environment, PNF routing with $\phi = \pi/4$ achieves the best performance as seen in Fig. 3 (a), as direct communications to the sink are inefficient due to a harsh attenuation and SPT is less energy-efficient due to the rich node density ($\rho > 10^{-4}$ nodes/m$^2$). Node-to-sink distance $D_{ns}$ is the Euclidean distance between each node and the sink. Table 2 shows the energy characteristics obtained from the same simulation environment as that of Fig. 7. Lifetime of the network is defined as the number of data retrieving rounds, which indicates how many times the node which consumes the most energy in the network can transmit its data, given that the initial energy residue is 1,000 J. As we can see these results, energy consumption in direct communication is solely dependent on transmission distance, which is equivalent to $D_{ns}$ in that routing policy. The further node consumes more energy to deliver its data to the sink. Not surprisingly, therefore, direct communication realizes the worst performance in terms of both load balancing and network lifetime.

Unlike direct communication, it can be seen from Fig. 7 and Table 2 that SPT and PNF routings perform load balancing in the networks thanks to their multihop communications. Nodes near the sink may exhaust energy faster than the others in these routing strategies, as they become hub nodes which frequently receive data from their child nodes and also frequently relay data to the sink. Comparing SPT and PNF in Table 2 reveals that the proposed PNF outperforms SPT in terms of energy efficiency, load balancing and lifetime for $N = 160$ and $L = 500$ m. It is assumed that the superiority of PNF in unbiased energy variance is due to the flexibility of transmission (hop) distance $r$ by changing forwarding angle $\phi$. As mentioned in Sect. 3, adjusting $\phi$ means changing $r$, which should lead to load balancing. After all, the observed energy efficiency of PNF complies with the results in Figs. 3 (a) and 6 (a), and thus the aforementioned discussions based on total energy budget can be deemed to be reasonable and significant.

6. Conclusions

In this paper, we focused on our proposed PNF routing policy to clarify the best energy-efficient topology in WSNs. The unique topology parameter, forwarding angle, was employed to determine the network routing structure, and the optimal angle for the best energy efficiency of the routing policy was investigated. Moreover, we modeled and formulated the energy budget in the network which employs PNF routing. In this formulation, the stochastic-geometric theories of hop distance and hop-count distributions in random planar ad-hoc networks were applied to WSNs. Simula-
tion results show that the optimal network routing method depends on the node density in the networks. The optimal forwarding angle becomes larger as the node density gets lower since direct communication has an inaptitude for long-distance transmissions with a harsh radio attenuation. It is also clarified that our energy expressions well approximate the empirical energy consumption, especially for small networks with narrow neighborhood area. However, when the forwarding angle is large, with the neighborhood area with central angle of $\pi/2$, the budget error is approximately 10%. It is indicated that this error is attributed to deterioration of hop progress by the deviation of next-hops. And it is remarked that utilizing the derived expressions enables us to determine the optimal value of forwarding angle for a desired network. The distance-based energy characteristics and network lifetime were also evaluated to find that the multihop routing policy achieves an effective load balancing and a durable lifetime.

Improvement of the PNF mechanism for more scalability and energy efficiency, and improvement of the energy formulation especially for large networks are open issues to be discussed. Accuracy improvement in energy formulation with a large forwarding angle by utilizing PMR is also desired. We let these problems be included in our future work.

References


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