A Weighted Overlapped Block-Based Compressive Sensing in SAR Imaging

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SUMMARY The compressive sensing (CS) theory has been widely used in synthetic aperture radar (SAR) imaging for its ability to reconstruct image from an extremely small set of measurements than what is generally considered necessary. Because block-based CS approaches in SAR imaging always cause block boundaries between two adjacent blocks, resulting in namely the block artefacts. In this paper, we propose a weighted overlapped block-based compressive sensing (WOBCS) method to reduce the block artefacts and accomplish SAR imaging. It has two main characteristics: 1) the strategy of sensing small and recovering big and 2) adaptive weighting technique among overlapped blocks. This proposed method is implemented by the well-known CS recovery schemes like orthogonal matching pursuit (OMP) and BCS-SPL. Promising results are demonstrated through several experiments.

key words: SAR imaging, block-based compressive sensing, weighted overlapped reconstruction, the block artefacts reduction

1. Introduction

High-resolution synthetic aperture radar (SAR) which is widely applied in military and civil utilization, produces huge amount of data and brings on-board storage burden. SAR imaging is assumed to start with a demodulated baseband signal and attempts to estimate ground reflectivity. The CS theory proposed by Donoho et al[1, 2], has been widely used in SAR imaging for its ability to reconstruct images from an extremely small set of measurements than what is generally considered necessary. When backscatter coefficients of the target reflectivity are sparse and compressible, a CS reconstruction algorithm can be adopted to estimate the ground reflectivity. Several works on applying CS to SAR/ISAR imaging have been reported [3]–[5]. Though a block-based CS approach is more practical for SAR imaging, it also causes block boundaries between two adjacent blocks, resulting in namely the block artefacts.

In this paper, we propose a weighted overlapped block-based compressive sensing (WOBCS) algorithm to reduce the block artefacts. A weighted recovery method was proposed by using its neighbouring recovered blocks in [6] and it brought poor reconstruction quality of blocks. The overlapped sensing method with a dual sensing matrix was used by Coluccia[7], but its limited in non-overlapping block-based compressed sensing. WOBCS takes the advantage of both overlapped CS recovering method and adaptive weighting technique to improve the performance, so that WOBCS is able to achieve block artefacts reduction through weighted averaging.

This paper is outlined as follows. CS theory is briefly introduced in Sect. 2, and the CS-based SAR imaging is described in Sect. 3. In Sect. 4, WOBCS is proposed to achieve block artefacts reduction. The performance of the proposed method in SAR imaging is tested in several experiments in Sect. 5. We draw our conclusions in Sect. 6.

2. Compressive Sensing Theory

Consider a one-dimensional, discrete-time signal $x$ and an orthonormal basis, represented in terms of the vectors $\Psi \in \mathbb{C}^{N \times N}$, thus $x = \Psi s$ can be obtained, where $s$ is an $N \times 1$ vector of coefficients. Supposing $K (K \ll N)$ elements of those coefficients are nonzero or largest, and the $N - K$ remaining elements are zero or negligible, then $x$ is called $K$-sparse or compressible. The measurements $y$ can be written as follows:

$$y = \Phi x = \Phi \Psi s = \Theta x \quad (1)$$

Where $\Phi \in \mathbb{C}^{M \times N}$ is the measurement matrix (MM). Because the dimension of $y$ is far lower than $s(M \ll N)$, so that obtaining $x$ from measurement $y$ by solving the linear equation in (1) is NP-hard. Many algorithms including OMP, CoSaMP, BP, etc, were proposed to solve this problem.

3. CS-Based SAR Imaging

SAR imaging is assumed to start with a demodulated baseband signal and attempt to estimate ground reflectivity. The sending chirp signal is

$$p(\tau) = \text{rect}(\frac{\tau}{T_r}) \cos(2\pi f_0 \tau + \pi K_r \tau^2)$$

(2)

where $T_r$ is the pulse width, $f_0$ is the carrier frequency and $K_r$ is the frequency modulation rate.

There are $b^2$ stationary point targets, each with a target reflectivity $r_{ij}$ ($i = 1, 2, \ldots, b$ and $j = 1, 2, \ldots, b$) which is located at the position $(x_i, y_j)$. To obtain the expression of the 2D SAR echo data in the discrete domain, the imaging area is divided into $b \times b$ grids as shown Fig. 1. We denote the target reflectivity function of the scene of interest as

$$f(x, y) = \sum_{i=1,j=1}^{i=b,j=b} r_{ij} \delta(x - x_i, y - y_j)$$

(3)
According to the theory of SAR, the echo signal of a point target in position \((x_i, y_j)\) \((x_i)\) denotes the range coordinate and \(y_j\) denotes the azimuth coordinate) can be written as

\[
s(\tau, \eta) = \sum_{i,j} f(x)\omega^2(\eta - \eta_c) p(\tau - \frac{2R_{ij}(\eta)}{c})
\]  

where \(R_{ij}(\eta) = \sqrt{(x_i + h^2 + (y_j - \eta)^2)}\) denotes the distance between the radar and target. \(h\) is the height, \(v\) is the speed of the radar (associated with slow-time), \(\omega^2\) is the received signal strength, \(\eta_c\) denotes the time of the zero Doppler and the speed of propagation of the wave in the respective medium is denoted as \(c\) (speed of light, associated with fast-time). \(\tau\) and \(\eta\) denote the fast-time and the slow-time, respectively.

A CS framework would rearrange the 2D grids into a vector by row \([x_1, y_1, x_2, y_2, \ldots, x_n, y_n, \ldots, x_b, y_b, \ldots, x_b, y_b]^T\). The echo signal is discretely sampled and represented as

\[
\begin{bmatrix}
    x_{1,1} \\
    x_{1,2} \\
    \vdots \\
    x_{m,1} \\
    x_{m,2} \\
    \vdots \\
    x_{m,N} \\
\end{bmatrix} = \begin{bmatrix}
    \Theta_{1,1,1} & \cdots & \Theta_{1,1,i} & \cdots & \Theta_{1,1,J} & \cdots & \Theta_{1,J,1} \\
    \Theta_{1,2,1} & \cdots & \Theta_{1,2,i} & \cdots & \Theta_{1,2,J} & \cdots & \Theta_{1,J,2} \\
    \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \Theta_{M,1,1} & \cdots & \Theta_{M,1,i} & \cdots & \Theta_{M,1,J} & \cdots & \Theta_{M,J,1} \\
    \Theta_{M,2,1} & \cdots & \Theta_{M,2,i} & \cdots & \Theta_{M,2,J} & \cdots & \Theta_{M,J,2} \\
    \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \Theta_{M,N,1} & \cdots & \Theta_{M,N,i} & \cdots & \Theta_{M,N,J} & \cdots & \Theta_{M,J,N} \\
\end{bmatrix} \begin{bmatrix}
    r_{1,1} \\
    r_{1,2} \\
    \vdots \\
    r_{m,1} \\
    r_{m,2} \\
    \vdots \\
    r_{m,N} \\
\end{bmatrix}
\]  

where

\[
\Theta_{m,n,i,j} = \omega^2(\eta - \eta_c) p(\tau_m - \frac{2R_{ij}(\eta)}{c})
\]

In (5), \(s_{m,n}\) is a brief form of \(s(\tau_m, \eta_n)\). We rewrite (5) into the matrix expression as

\[
s = \Theta r
\]  

Here \(r = [r_{1,1}, \cdots, r_{b,1}, r_{1,2}, \cdots, r_{b,2}, \cdots, r_{b,b}]^T\) is the reflectivity of the scene of interest. The echo signal \(s\) regarded as the compressive sensing measurement, and \(\Theta\) is the MM of reflectivity \(r\). Assuming that \(r\) is sparse (most of the elements of \(r\) are zero or negligible), the estimate of the reflectivity \(\hat{r}\) can be reconstructed by a CS recovery method.

4. Weighted Overlapped Block-Based Compressive Sensing

In Sect. 3, we deduce that SAR imaging can be implemented by CS framework. To speed up the CS-based SAR imaging, block-based compressive sensing (BCS) was proposed in [8]. Though BCS speeds up the reconstruction, it also causes block artefacts in the same time. In this section, we propose a new method termed weighted overlapped block-based compressive sensing (WOBCS) to reduce the block artefacts in CS-based SAR imaging. It has two main characteristics: 1) the strategy of sensing small and recovering big (SSBR) and 2) adaptive weighting technique among overlapped blocks.

4.1 SSBR Strategy

The size of the block in BCS affects the performance of reconstruction. To achieve a better quality and lower sensing cost in BCS is difficult. By sensing in small blocks but executing a big block reconstruction, this problem would be solved [9]. As shown in Fig. 2, we divied the scene into 9 small blocks. Each small block is of size \(a/2 \times a/2\). We use \(B(A, B, C, D)\) presents the big block including small blocks A, B, C and D. After that, we use \(r = [r_A, r_B, r_C, r_D]^T\) to represent the reflectivity of \(B(A, B, C, D)\) and use \(r_i(l = A, B, \ldots, )\) to represent the reflectivity of a small block. Then small blocks A, B, C and D are measured by a structural measurement matrix \(\Theta\) as

\[
s = \begin{bmatrix}
    s_A \\
    s_B \\
    s_C \\
    s_D \\
\end{bmatrix} = \begin{bmatrix}
    \Theta_A & \Theta_B & \Theta_C & \Theta_D \\
\end{bmatrix} \begin{bmatrix}
    r_A \\
    r_B \\
    r_C \\
    r_D \\
\end{bmatrix}
\]

\(\Theta = \text{diag}(\Theta_A, \Theta_B, \Theta_C, \Theta_D)\). In (8), small blocks are sensed respectively and then the estimate of \(\hat{r}\) is recovered from \(s\). We termed this strategy of sensing small and recovering big as SSBR strategy. We adopt SSBR strategy to implement WOBCS.

4.2 Weights of the Overlapped Recovered Points

In WOBCS, any CS recovery algorithm can be used to estimate the reflectivity \(\hat{r}\) of big block. Assuming that the reflectivities of 4 overlapped big blocks \(B(A, B, C, D), B(C, D, E, F), B(B, C, H, I)\) and \(B(C, F, G, H)\) are already recovered (see Fig. 2), so that a point in small block C that is covered by these overlapped big blocks, has
4 values. The key idea of WOBCS is to let multiple values help reduce the block artefacts while still have better performance. In this section, we would discuss the weights of each recovered points that are from big blocks.

Here, we assume that the point in the position \((x_i, y_j)\) of small block of interest has \(L\) values, where \(1 \leq L \leq 4\) due to that the margin area of the scene may not have 4 neighbors. We use \(\hat{r}_l\) to denote the recovered value from the \(l\)-th \((1 \leq l \leq L)\) big block and \(\hat{r}_{ij}\) to denote the comprehensive value that is obtained by the weighted averaging of those multiple values. And then we have

\[
\hat{r}_{ij} = \sum_{l=1}^{L} \omega_l \hat{r}_l \text{ s.t. } \sum_{l=1}^{L} \omega_l = 1 \tag{9}
\]

where \(\omega_l\) is a weight given to \(\hat{r}_l\). Let \(\sigma^2_{ij}\) and \(\sigma^2\) be variances of recovery errors of \((r_{ij} - \hat{r}_{ij})\) and \((r_{ij} - \hat{r})\), we have

\[
\sigma^2_{ij} = E[(r_{ij} - \hat{r}_{ij})^2] \tag{10}
\]

Supposing that the error \((r_{ij} - \hat{r}_{ij})\) is a random variable with zero mean and independent to the others \(\{(r_{ij} - \hat{r}_{ij}), l \neq l',\}\), we have

\[
\sigma^2_{ij} = E[(r_{ij} - \hat{r}_{ij})^2] = \sum_{l=1}^{L} \omega_l \sigma^2_{l} \tag{11}
\]

So that minimum-mean-square error (MMSE) estimation of the weights can be achieved by minimising \(\sigma^2\), and we have

\[
\omega_l = \sigma^{-2}_{l} / \sum_{l=1}^{L} \sigma^{-2}_{l}, l = 1, 2, \ldots, L \tag{12}
\]

From (12), a bigger \(\sigma^2\) would contributes a smaller weight to the comprehensive value \(\hat{r}_{ij}\).

A precise weights inside each big block should be further discussed. As we know that boundary points suffer more from recovery error than the centre points inside a block, so that the variance \(\sigma^2_{ij}\) depends not only on the quality of the \(l\)-th big block but also on the relative position of the point inside the block. Supposing \(\sigma^2_{l}\) are the average variance of errors of all points in the \(l\)-th big block, and \(\omega^\prime_{l}\) is the intra variance weight of each one point, we can write as

\[
\sigma^2_{ij} = \omega^\prime_{l} \sigma^2_{l} \tag{13}
\]

Substituting (13) into (12), we have

\[
\omega_l = \omega^{-1} \sum_{l=1}^{L} \omega^\prime_{l} \sigma^{-2}_{l}, l = 1, 2, \ldots, L \tag{14}
\]

Due to the piece-wise constant property of the image signal, a variance of small patches inside a block can estimate its variance of error well. To give the small intra variance weights to center points and the big intra variance weights to boundary points, we adopt a raised cosine model to generate value of \(\omega^\prime_{l}\).

\[
\omega^\prime_{l} = \frac{1}{4} [1 - \cos (2\pi \frac{\alpha + 0.5}{a})][1 - \cos (2\pi \frac{\beta + 0.5}{a})] \tag{15}
\]

where the coordinate \((\alpha, \beta)\), with \(1 \leq \alpha \leq a\) and \(1 \leq \beta \leq b\), is the relative position of point \(r_{ij}\) in the \(l\)-th big block. (15) makes the comprehensive weight \(\omega_l\) become smaller as going to block boundaries. It additionally enhances smooth transition at the boundary of overlapped blocks.

5. Experiments

To evaluate the effectiveness of proposed method in CS-based SAR imaging, the well-known CS recovery schemes of OMP [10] and BCS-SPL [11] were employed to implement WOBCS. Experiments were conducted for different subrate \(rc = \frac{a}{a} = 0.125, 0.25, 0.375\) and 0.5, where \(a = 32\) is the side length of big block. The performance analysis is characterized by peak signal to noise ratio (PSNR). We chose a common used scene of ships from Canadian RADARSAT-2 as the test image.

Objective and subjective experimental results are arranged in Table 1 and Fig. 3, respectively. Figure 3 shows the reconstructed SAR images at subrate of 0.5 through different CS recovery algorithms. Two districts of scene of ships is zoomed in for visual effects and clarity in each subfigure of Fig. 3. Several summaries can be obtained accordingly:

1. WOBCS improves the PSNRs obtained from traditional CS reconstruction. That’s to say, WOBCS-OMP performs better than OMP, and WOBCS-OMP performs better than BCS-SPL.
2. The best improvement is 2.86 dB (OMP vs. WOBCS-OMP, at the subrate of 0.125), and the least improvement is 2.08 dB (BCS-SPL vs. WOBCS-SPL, at the subrate of 0.5).
3. WOBCS achieves better performance at small subrate than that at large subrate. That’s because most of the information of original reflectivity are missed at small subrate and the results of WOBCS are brought from 4 blocks, so that more information would be reconstructed at small subrate.
4. The average improvement of WOBCS-OMP (2.6975 dB) is larger than WOBCS-SPL (2.2675 dB). That’s because BCS-SPL also can reduce block artefacts itself as being a block-based CS recovery algorithm, but there is no smooth mechanism when applying OMP to CS-based SAR imaging, so that the performance improvement on OMP brought by WOBCS is better than that on BCS-SPL.

<table>
<thead>
<tr>
<th>Subrate (rc)</th>
<th>0.125</th>
<th>0.25</th>
<th>0.375</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMP</td>
<td>17.35</td>
<td>23.61</td>
<td>28.56</td>
<td>31.49</td>
</tr>
<tr>
<td>WOBCS-OMP</td>
<td>20.21</td>
<td>26.31</td>
<td>31.17</td>
<td>34.11</td>
</tr>
<tr>
<td>BCS-SPL</td>
<td>21.16</td>
<td>28.12</td>
<td>32.98</td>
<td>37.06</td>
</tr>
<tr>
<td>WOBCS-SPL</td>
<td>23.63</td>
<td>30.46</td>
<td>35.16</td>
<td>39.14</td>
</tr>
</tbody>
</table>
5. We can see apparent block artefacts around ships in zoomed districts from Fig. 3-(a), and those artefacts are smoothed by WOBCS in Fig. 3-(b).

6. Though BCS-SPL has smooth mechanism, WOBCS is also able to bring certain improvements to BCS-SPL according to the visual impress of zoomed districts in Fig. 3-(c) and Fig. 3-(d).

7. The PSNRs in Table 1 are in agreement with the subjective results.

Hence, we can conclude that WOBCS performs better than traditional schemes both in terms of subjective and objective qualities.

6. Conclusion

In this paper, we proposed a new method termed WOBCS to CS-based SAR imaging for block artefacts reduction. Compared to traditional CS recovery schemes, WOBCS improves both subjective and objective qualities of SAR imaging especially when subrate is small. The best improvement is 2.86 dB and the least improvement is 2.08 dB. It can be concluded that WOBCS is a promising CS-based SAR imaging method. Since we only have the preliminary results, further researches should be done in the future.

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References