Removal of Salt-and-Pepper Noise Using a High-Precision Frequency Analysis Approach

Masaya HASEGAWA, Kazuki SAKASHITA, Kousei UCHIKOSHI, Nonmembers, Shigeki HIROBAYASHI, ††, and Tadanobu MISAWA, †††, Members

SUMMARY A digital image is often deteriorated by impulse noise that may occur during processes such as transmission. An impulse noise converts the pixel data in the image into black (0) or white (255) values at a random frequency and is also called salt-and-pepper noise. In this paper, we identify the details of pixels that have been damaged by impulse noise by analyzing the frequency of the noisy image using non-harmonic analysis (NHA). From experimental results, we can confirm that this method shows superior performance compared to the recent PSNR denoising method. In addition, we show that the proposed method is particularly superior in eliminating impulse noise in images with high noise rates.

key words: salt-and-pepper noise, frequency analysis, macroblock analysis

1. Introduction

A digital image is often deteriorated by impulse noise that may occur during processes such as transmission. This is caused mainly by an error in a sensor or communication channel. In the case of a communication error, in particular, an impulse noise may significantly deteriorate a satellite photo image [1]. An impulse noise converts pixel data in the image into black (0) or white (255) values at a random frequency and is called salt-and-pepper noise because of this characteristic. The problem is that, because the brightness of the digital image is spoiled by this impulse noise, the precision of image processing such as edge detection and pattern recognition decreases [2]–[8].

Therefore, various filter technologies for impulse denoising have been suggested. One of the most common filters is the median filter (MF), which is known to show superior effects compared with impulse denoising in spite of its simple processing; however, MF has difficulty with removing high-density noise and reconstructing edges. A suggested solution is a two-step processing technique consisting of noise detection and denoising that does not require denoising on the whole image to an equal extent. This technique identifies the location of the noise and conducts denoising processing on noisy pixels only, but it is possible with this technique to inadvertently change the value of a pixel that is not affected by noise. Hence, various noise-removal methods based on the median filter with two-step processing techniques have been proposed. Adopting the two-step process, Sun et al. proposed the switching median filter (SMF), which combines the identification of the noise location with MF [9]. Chen et al. used a filter design that focuses on differences between the current pixel and center-weighted median filters [10]. In another method, Chan et al. proposed a method of performing noise removal after noise detection using an adaptive median filter [11]. Other techniques have been studied; some built weighted filters based on statistical data, adding weight based on differences between adjacent nodes; these include the directional weighted median (DWM) filter [12] and the switching-based adaptive weighted mean (SAWM) filter [13]. Other proposed methods include boundary discriminative noise detection (BDND), which is aimed at precision improvement of noise detection by incorporating a noise border distinction processing algorithm [14], and the simple adaptive median filter (SAMF) [15]. In recent years, highly precise noise-reduction techniques have been suggested, such as adaptive Gaussian filters (AGFs), which apply weighting using a Gaussian filter [16], and the method proposed by Chen et al., which is based on an efficient impulse detector and the edge-preserving total variation inpainting model [17].

In this paper, we abandoned the conventional mentality that supposes that the pixel information damaged by noise arises from a characteristic of an individual pixel’s numerical value, such as its brightness level. We identify the details of pixels that have been damaged by impulse noise by analyzing the frequency of the noisy image using non-harmonic analysis (NHA). It has been demonstrated that NHA’s excellent frequency-resolving power has produced outstanding research results in various fields [18]–[23]. We consider that NHA can construct a highly precise reconstruction image to eliminate impulse noise. The organization of this paper is as follows: in Sect. 2, we describe the proposed method’s algorithm; in Sect. 3, we summarize an experiment and its results; and finally, we present conclusions in Sect. 4.

2. Frequency Resolution in FFT, DCT, and NHA

In this section, we describe differences between NHA and
conventional frequency analysis methods. In order to remove noise with high accuracy in the frequency domain, the proposed method uses mask NHA, which is a high-resolution frequency analysis method. Mask NHA is based on NHA and is expanded to apply NHA to masking processing. NHA is different from general frequency analysis methods in engineering such as the discrete Fourier transform (DFT) in that it does not occur on side lobes, and there is little influence of the window length. The difference between the frequency resolution of 2D DFT and 2D NHA is shown in Fig. 1. Already, 2D NHA has been used in inpainting and motion estimation since 2D NHA achieves high performance [24], [25].

The Fourier transform (FT), which has been used for frequency analysis, can be represented as follows:

$$X(f) = \frac{1}{T} \int_{0}^{T} x(t)e^{-2\pi ft}dt$$  

(1)

where $T$ is the analysis window length. Equation (1) requires Fourier coefficients based on an integral equation. This is because the FT assumes that a complex periodic signal model is the sum of simple waves. Information about the whole signal is necessary for this volume calculation. In other words, the FT is not applicable when a part of the signal is missing. Therefore, when noise is added to the signal, the FT integrates based on the values changed by the noise. Therefore, it is difficult to estimate the amplitude accurately under the influence of noise. In particular, because salt-and-pepper noise is noise that involves black (0) or white (255), the amplitude of the analysis result fluctuates greatly.

On the other hand, mask NHA performs frequency estimation by solving using the least-squares method. The evaluation function is expressed in terms of the differences between the target signal and the model signal. Mask NHA’s model signal is expressed as follows:

$$\hat{I}(n_1, n_2) = \hat{A}\cos(2\pi(\frac{f_x}{f_{xs}}n_1 + \frac{f_y}{f_{ys}}n_2 + \phi))$$  

(2)

where $n_1$ and $n_2$ are the pixel numbers and $f_{xs}$ and $f_{ys}$ are the sampling frequencies, given as $f_{xs} = 1/x$ and $f_{ys} = 1/y$, respectively. Here, $x$ and $y$ are the two spatial dimensions.

To minimize the sum of the squares of the differences between the original signal $I$ and the sinusoidal model signal $\hat{I}$, the spatial frequencies $f_x$ and $f_y$, amplitude $\hat{A}$, and initial phase $\phi$ are calculated as follows:

$$F(\hat{A}, \hat{f}_x, \hat{f}_y, \phi) = \sum_{n_1=1}^{N_1-1} \sum_{n_2=1}^{N_2-1} w(n_1, n_2)(I(n_1, n_2) - \hat{I}(n_1, n_2))^2$$  

(3)

where $w(n_1, n_2)$ is a weighting factor constructed from binary information. The conditions of the weighting factor are as follows:

$$w(n_1, n_2) = \begin{cases} 0 & \text{(if noisy pixel)} \\ 1 & \text{(if NOT noisy pixel)} \end{cases}$$

Mask NHA performs a frequency estimate by error evaluation, not volume calculation. Accordingly, a frequency estimate is possible to the exclusion of a part of the signal. Then, we consider that it is possible to make a frequency estimate exactly by assuming that a noise pixel is a mask and excluding it from the evaluation function. Because salt-and-pepper noise is added in this study, the brightness level can easily distinguish pixels of black (0) or white (255) from noise. The fitting on the signal enabled exclusion of the influence of noise pixels by using this evaluation function. In other words, NHA can greatly control the change of the amplitude level caused by noise. Figure 2 shows differences between the integral calculus NHA and mask NHA for salt-and-pepper noise reduction. Note that we add a noise only in black (0) to facilitate explanation in Fig. 2. In Fig. 2, the integral calculus type of conventional method is used for the upper section to perform the analysis, including the black (0) pixels of the noise. Then, we perform noise reduction in the frequency domain and restore the signal. As a result, the integral value and the amplitude of the reconstructed wave are reduced. In contrast, mask NHA was used in the lower section to perform the analysis, excluding the black (0) pixels of the noise by masking. Thus, mask NHA can accurately remove the influence of noise alone.

Mask NHA has been applied to inpainting [24]. There, it was shown that mask-NHA-based inpainting can propagate information accurately from neighbor pixels. In this paper, we remove salt-and-pepper noise that has different characteristics from the missing region used in inpainting. Figure 3 shows the differences between the salt-and-pepper noise and missing region in inpainting. In Fig. 3, a 2D chirp signal whose frequency characteristics change spatially is damaged. In our study, Fig. 3 on the right is missing a region of wide range used in inpainting; the left shows salt-and-pepper noise to be removed in this paper. From Fig. 3 at left, in the case of inpainting, it is difficult to completely restore the center of the missing region. The image has the characteristic that the textures of neighboring regions are highly similar. Therefore, the missing region of wide range
loses its similarity to surrounding pixels closer to the center. Hence, if a signal that changes spatially, such as the chirp signal, is damaged over wide range, it is difficult to accurately restore it based on surrounding information. On the other hand, salt-and-pepper noise is impulse noise. Accordingly, an affected image is damaged at the pixel level, and the neighborhood pixels used in restoration remain original. In other words, salt-and-pepper noise is unlike inpainting: it can be restored for a signal that changes spatially by using information surrounding the damaged region.

A flowchart of the proposed method is shown in Fig. 4. First, the proposed method creates a mask and weighting factor. Next, it carries out NHA processing with a window length of $4 \times 4$ for a noisy image and analyzes the signal of the nonnoise pixels. However, when all the pixels in the analysis window are noise, the proposed method does not carry out NHA processing. The reason for setting a small analysis window length is to reduce the mixing of spectral analysis regions as much as possible, enabling the gathering of accurate information. When signals of different frequency are mixed, precision falls even if an excellent method of analysis is used. As an example, Fig. 5 shows the result from analyzing a signal in which 100 Hz and 200 Hz components were mixed using the fast Fourier transform (FFT). The red line is the result of analyzing a signal of only 100 Hz; the green line is the result of analyzing a signal of only 200 Hz; and the blue line is the result of analyzing the mixed-frequency signal. As can be seen from the blue line, when different frequencies are mixed, side lobes occur in large quantities, and the position of the peak widens. In short, it is not possible to extract features of the signal exactly. In order to solve this problem, it is necessary to set a small window length. Then, noise removal is performed by propagating image information around the noise pixel. Since the noise pixel information is excluded by the weighting factor, the proposed method can accurately estimate the frequency spectrum of the original image. Based on the extracted spectrum, this method eliminates noise pixels by image reconstruction with macroblocks. In frequency estimation, we converge Eq. (3) using steepest descent and Newton’s methods. The initial value for convergence is given by FFT (We refer the reader to [24] for the detailed convergence process of 2D Mask NHA). When the analysis of one macroblock is completed, the macroblock is shifted by the macroblock size. After analyzing entire images with the $4 \times 4$ macroblock, the proposed method checks whether noise pixels remain in the image. If so, the analysis window length is extended to $8 \times 8$ and noise removal is performed again. In this way, the proposed method can remove salt-and-pepper noise.
The proposed method analyzes an image in which salt-and-pepper noise has been added using a macroblock. The problem in the field of noise reduction is that precision decreases under the influence of multiple objects varying in frequency. Generally, an image has neighborhood pixels and high correlation. Therefore, it becomes important to reflect the characteristics of the neighborhood exactly in terms of noise; however, object division is difficult and, when performing noise reduction in consideration of the domains of all the images, the characteristics of multiple objects are reflected in the reconstructed image. To solve this problem, an image is analyzed locally, and it is necessary to extract neighboring characteristics exactly. Conventionally, FFT and DCT have been used for frequency analysis of images. Resolving power depends on macroblock size for these techniques. Consequently, when there are few characteristics of the original image in a macroblock, it is difficult to separate characteristics from the noise in a neighborhood of pixels; however, the frequency resolution of NHA does not depend on macroblock size, and NHA can extract characteristics in the domain of interest exactly. In addition, because the isolation of noise is significant, image expression using NHA assumes that accurate noise reduction is possible. Moreover, the precision of smoothing improves when increasing the macroblock size. From this fact, in noise reduction with a macroblock, we consider the quantity of extraction and the
noise reduction precision of a characteristic to be a trade-off; however, because NHA can express an image in few spectra effectively, even for a small macroblock, NHA can extract image characteristics without spoiling noise reduction precision.

3. Experiments

3.1 Experimental Outline

In order to evaluate the performance of the proposed method, we conducted experimental removal of salt-and-pepper noise. The standard images used for the experiment were Pepper and Lena, the size of which were each 512×512. As the noise image, we added impulse noise from 10% to 90% of the image. The noise rate was changed in increments of 10%. The reason we chose Lena and Pepper is that these images are commonly used in related studies [16], [26], allowing for comparison with many other methods. The original images of Lena and Pepper are shown in Fig. 6. In addition, in order to investigate the performance of the proposed method for various types of images, Barbara, House, Boat, and Airplane were also used for the experiment. As an additional method of noise removal, we converted the number of pixels at the noise rate into black (0) in the original images at random. In this experiment, since white (255) and black (0) are judged to be equivalent, noise pixels were converted to black (0). We applied the process of the proposed method to these noise images. The image at each processing stage when adding a 90% noise rate to the Lena image is shown in Fig. 7. As described in Sect. 2, the result of applying NHA with a window length of 4×4 to high-density noise images (such as at the 90% noise rate as shown in Fig. 7), noise pixels are still present as black. For this image, the result of applying NHA with a window length of 8×8 is shown in Fig. 7 (c), where it can be clearly seen that there are no noise pixels. In this result, using a two-step process of changing the analysis window length to remove all impulse noise, NHA makes it possible to restore the image with high accuracy.

3.2 Results and Discussion

The noise image and the restored image attained using the proposed method are shown in Fig. 8 (Lena) and Fig. 9 (Pepper), respectively. In each figure, (a) to (d) show the noise-affected images and (e) to (h) show the restored images. The noise rates were (a) 30%, (b) 50%, (c) 70%, and (d) 90%. From the results shown in Fig. 8 and Fig. 9, the proposed method can restore the image nearly to its original state in both the high-noise-rate and low-noise-rate cases. In the case of a low noise rate of less than 50%, because there is much information that remains around a noise pixel, fine textures such as hair or the calyx of a vegetable are restored exactly. In addition, edges are smoothly connected. In the case of a high noise rate of more than 50%, the quality of the restored image deteriorated compared to the case of low noise. In particular, smooth connection of the edges is impaired, and the contours of the objects are fuzzy at the highest noise rate of 90%. However, the regions composed of low frequencies were restored with high accuracy equivalent to the low-noise-rate cases. This is because the proposed method can accurately analyze and efficiently represent low-frequency spectra. Since most of the images are composed of low frequencies, overall restoration accuracy was increased by accurately restoring the low-frequency components.
components. In other words, the proposed approach is an effective denoising method.

In addition, in order to compare with other methods, we calculated the peak signal-to-noise ratio (PSNR) of the restored images. We then compared the PSNR values for DWM [12], SAWM [13], BDND [14], AGF [16], Chen's method [17], SAMF [15], ERMI [26], and cardinal B-splines [27]. These methods were chosen because they have been used in experiments on salt-and-pepper noise removal at rates from 10% to 90% with Lena and Pepper test images. Note that the PSNR matched the conditions of a model used in an experiment by Nasri et al. [16]. We summarize the comparison of PSNR values in Table 1 and Table 2. The PSNR of the proposed method was evaluated
Table 1  Comparison of different methods for Lena image noise reduction at different noise rates (PSNR: dB).

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<td>BDND [14]</td>
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<td>37.55</td>
<td>41.20</td>
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<td>42.43</td>
<td>43.41</td>
<td>52.62</td>
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<td>DWM [12]</td>
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<td>39.19</td>
<td>39.75</td>
<td></td>
<td>47.48</td>
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<td>SAMF [15]</td>
<td>36.50</td>
<td>33.51</td>
<td>35.22</td>
<td>37.25</td>
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<td>37.09</td>
<td>37.61</td>
<td>35.90</td>
<td>43.86</td>
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<td>Chen [17]</td>
<td>33.93</td>
<td>30.75</td>
<td>33.49</td>
<td>35.18</td>
<td>35.52</td>
<td>35.25</td>
<td>35.77</td>
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<tr>
<td>SAWM [13]</td>
<td>31.67</td>
<td>28.84</td>
<td>30.78</td>
<td>33.74</td>
<td>33.74</td>
<td>33.48</td>
<td>34.31</td>
<td>31.65</td>
<td>37.85</td>
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<tr>
<td>ERMI [26]</td>
<td>29.67</td>
<td>26.84</td>
<td>30.78</td>
<td>33.46</td>
<td>33.46</td>
<td>33.22</td>
<td>33.65</td>
<td>30.65</td>
<td>35.20</td>
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<tr>
<td>AGF [16]</td>
<td>28.19</td>
<td>23.05</td>
<td>29.22</td>
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<td>31.67</td>
<td>31.07</td>
<td>30.87</td>
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<td>CB-spline</td>
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<td>20.75</td>
<td>27.39</td>
<td>30.05</td>
<td>29.10</td>
<td>28.50</td>
<td>28.35</td>
<td></td>
<td></td>
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<tr>
<td>Mask NHA</td>
<td>24.53</td>
<td>18.25</td>
<td>25.30</td>
<td>27.50</td>
<td>26.45</td>
<td>25.90</td>
<td>25.48</td>
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Table 2  Comparison of different methods for Pepper image noise reduction at different noise rates (PSNR: dB).

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<td>DWM [12]</td>
<td>37.61</td>
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<td>SAMF [15]</td>
<td>34.83</td>
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<td>35.55</td>
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<td>34.41</td>
<td>42.64</td>
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<td>Chen [17]</td>
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<td>28.00</td>
<td>31.94</td>
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<td>33.53</td>
<td>33.48</td>
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<tr>
<td>SAWM [13]</td>
<td>30.40</td>
<td>25.18</td>
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<td>AGF [16]</td>
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<td>28.73</td>
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<tr>
<td>Mask NHA</td>
<td>24.53</td>
<td>18.25</td>
<td>24.45</td>
<td>24.00</td>
<td>25.46</td>
<td>24.71</td>
<td>25.85</td>
<td>24.65</td>
<td>28.41</td>
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Table 3  PSNR of noise removal at each noise rate for various images using the proposed method.

<table>
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<tr>
<th>Image</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
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<td>Barbara</td>
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<td>33.85</td>
<td>32.24</td>
<td>30.62</td>
<td>29.01</td>
<td>27.42</td>
<td>25.83</td>
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<td>House</td>
<td>42.44</td>
<td>40.82</td>
<td>39.23</td>
<td>37.61</td>
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<td>32.78</td>
<td>31.19</td>
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<tr>
<td>Boat</td>
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<td>36.86</td>
<td>35.14</td>
<td>33.62</td>
<td>32.02</td>
<td>28.50</td>
<td>26.90</td>
<td>25.31</td>
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<td>Airplane</td>
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<td>38.65</td>
<td>36.96</td>
<td>35.34</td>
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<td>30.22</td>
<td>28.62</td>
<td>27.03</td>
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</table>

The proposed method offers the highest PSNR values in all experimental results. Further, the PSNR of the proposed method was improved by more than 2 dB compared with the conventional method when the noise rate was 90%. The PSNR of the proposed method for the five images other than Lena and Pepper are shown in Table 3. These images were not used as the main comparison images for the conventional method, as most papers use Lena and Pepper as the main comparison images. In many cases, experimental results are often shown with Lena, Pepper, and images peculiar to each article. Therefore, the five images used in this experiment cannot be compared unconditionally with various methods. However, there are also studies that perform noise removal experiments with a specific noise density using the same images as this study. Therefore, for images other than Lena and Pepper, the PSNR was compared only with these methods. From Table 3, the PSNR of the proposed method for Boat at noise densities of 40%, 50%, and 60% were 33.29 dB, 31.35 dB, and 29.52 dB, respectively. DWM [12] was also tested under similar conditions, and its PSNRs for Boat were 27.03 dB, 25.75 dB, and 24.01 dB for the same densities. Cardinal B-splines [27] also tested noise removal for Boat. For cardinal B-splines [27], the experimental conditions were slightly different from DWM [12]; the noise densities were 30%, 50%, 70% and 90%. The PSNRs of these noise densities were 31.4695 dB, 28.4766 dB, 24.8949 dB, and 22.3284 dB. With these experimental conditions, the PSNR of the proposed method was higher than that of the conventional methods at each noise density. With high-density noise (90%), the PSNR of the proposed method improved by about 1.9 dB compared with the cardinal B-splines. Further, for House, the PSNRs of the cardinal B-splines were 30% = 33.2812 dB, 50% = 30.2354 dB, 70% = 26.1280 dB, and 90% = 23.5488 dB. For the House image, the proposed method also has a higher PSNR than that of the conventional method. Finally, we measured the calculation time of the proposed method, implemented in MATLAB and run on an Intel Xeon CPU with 32 GB RAM. For the Lena image...
with 50% noise density, analysis of the entire image was completed in about 165 seconds with a 4×4 analysis window, and in about 64 seconds with an 8×8 analysis window. Therefore, the time required for analysis using the proposed method is 229 seconds. In the conventional method compared in [16], the average CPU time for denoising the Lena image with 50% noise density was about 8.4 seconds. Accordingly, the proposed method requires about 27 times the average CPU time of the conventional method. However, the CPU time of the proposed method per macroblock is about 0.01 seconds with a 4×4 analysis window and about 0.015 seconds with an 8×8 analysis window. In recent years, high-speed calculation by parallel calculation using GPUs has been proposed. Therefore, it is possible that the CPU time of the proposed method can be dramatically accelerated by performing parallel calculation on macroblocks.

Based on the above result, we think that the proposed method is an effective noise removal method for high-density impulse noise.

4. Conclusion

The removal of impulse noise is an image processing problem that has been studied frequently. Conventionally, noise removal in the image domain using a filter has been the mainstream approach. In this paper, using mask NHA, which is a highly precise frequency analysis method, we performed impulse denoising from a new viewpoint: frequency analysis of the noisy image. In integral-type frequency analysis methods such as FFT and DCT, it is difficult to eliminate the influence of noise in the frequency domain. The proposed method excludes the influence of noise by masking and numerical computation. From the experimental results, we confirmed that this method shows superior performance compared with the recent PSNR denoising method. In particular, the PSNR of the proposed method improved by more than 2 dB compared with conventional methods when the noise rate was 90%. The CPU time of the proposed method is longer than that of the conventional methods. However, it may be possible to solve the problem of CPU time using parallel computation, as with a GPU. Accordingly, we showed that the proposed method is particularly superior in eliminating impulse noise in images with high noise rates.

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References

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