PAPER

Extended Personalized Individual Semantics with 2-Tuple Linguistic Preference for Supporting Consensus Decision Making

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SUMMARY Considering that different people are different in their linguistic preference and in order to determine the consensus state when using Computing with Words (CWW) for supporting consensus decision making, this paper first proposes an interval composite scale based 2-tuple linguistic model, which realizes the process of translation from word to interval numerical and the process of retranslation from interval numerical to word. Second, this paper proposes an interval composite scale based personalized individual semantics model (ICS-PISM), which can provide different linguistic representation models for different decision-makers. Finally, this paper proposes a consensus decision making model with ICS-PISM, which includes a semantic translation and retranslation phase during decision process and determines the consensus state of the whole decision process. These models proposed take into full consideration that human language contains vague expressions and usually real-world preferences are uncertain, and provide efficient computation models to support consensus decision making.

key words: *Computing with Words, consensus decision making, interval composite scale, 2-tuple linguistic model, linguistic preference*

1. Introduction

In reality, words are more frequently used than numbers which are centered on in traditional computation reasoning processes [1]. Computing with Words (CWW) is a concept proposed by Prof. Zadeh in 1996 to deal with some perceptual information [2]. It imitates the process how people reason, during which the premises and conclusions are both expressed in natural language. With the development of artificial intelligence technology, methods imitating human thinking habit such as CWW have come to attract more and more attention. Today, words are used in place of numbers in a wide variety of applications, especially with the use of CWW in consensus decision making.

At present, the application of CWW in decision-making is mainly proceeded in two directions: the use of interval type-2 fuzzy sets [3] and the use of multi-granular linguistic models [4]–[6]. These directions come from the assumption that different people have the same semantic understanding on the same word, and have equivalent numerical values translated by the word. However, in reality, different people have different semantics understandings, due to the fact that human language and real-world preferences

are usually vague [7].

Therefore, this paper will focus on the following two parts. One is how the decision makers express their preferences to the known information, and how to reflect the difference caused by language vagueness and preference uncertainty. Another is how to guide the entire decision process, to determine the consensus state among decision makers and deal with other issues if necessary. And we call this problem consensus decision making (CDM) problem with individual semantic understanding.

Specifically, when dealing with the linguistic preference information of the decision maker, we will study the translation and retranslation process in Yager's CWW scheme [8] and find some way that can translate the evaluation information expressed in linguistic term into the computable numerical information with loss of information as little as possible, and can retranslate the computable numerical information into the well-understood linguistic term. On this basis, considering the difference of individual semantic understanding, a personalized individual semantic model is further studied [1]. Finally, the method of determining the consensus state among decision makers is examined due to vagueness in human expression.

Based on the ideas listed above, Sect. 2 introduces some basic concepts, and additionally proposes an interval composite scale based 2-tuple linguistic model. Section 3 proposes an interval composite scale based personalized individual semantics model (ICS-PISM) to describe linguistic understanding of different people. Section 4 introduces a consensus decision making model with ICS-PISM for dealing with consensus decision making problems with different linguistic understanding. Section 5 concludes this paper.

2. Preliminaries

In this section, some basic concepts are introduced briefly.

2.1 Composite Scale Based 2-Tuple Linguistic Model

To solve the information distortion problem caused by traditional 2-tuple linguistic model with $0 \sim n$ scale, the composite scale based 2-tuple linguistic model was put forward [9], highly inspired by Herrera, F., & Martínez, L. [5]. This model took decision maker's psychophysical state into consideration on the basis of Weber-Fechner law [10]. In this model, with the composite scale, the linguistic preference information of the decision maker is still expressed by a

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2-tuple (s_i, ρ) .

Definition 1: Let $\mathbf{S} = \{s_i\}$ be a linguistic term set, where s_i is a linguistic term, $i \in [0, g]$, s_0 means the lower limit, s_g means the upper limit, $\mathbf{S}$ is ordered. The cardinality of $\mathbf{S}$ is $g + 1$.

Here, g is even. Generally, the cardinality of $\mathbf{S}$ is defined as $3 \sim 9$, and g takes $2 \sim 8$. In particular, depend on the particular case. For example, when the cardinality is 7, $g = 6$, $\mathbf{S} = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{HC, C, JC, YB, JH, H, HH\}$, where “ $HC, C, \dots, HH$ ”, respectively mean “*very poor, poor, a little poor, general, a little good, good, very good*”.

Definition 2: Let the linguistic information be represented by 2-tuple $(s_i, \rho) \in \mathbf{S}' = \mathbf{S} \times [-0.5, 0.5]$, where $s_i \in \mathbf{S}$ and $\rho \in [-0.5, 0.5]$. Let $v \in \mathbf{V} = [-1, 1]$ be a value representing the result of operation.

$s_i \in \mathbf{S}$ is a linguistic term, its equivalent 2-tuple form can be obtained as:

$$\theta(s_i) = (s_i, 0), \quad s_i \in \mathbf{S} \quad (1)$$

Definition 3: Let $v \in \mathbf{V} = [-1, 1]$ be a real. The 2-tuple that expresses the equivalent linguistic information to v is obtained by function $\Delta: \mathbf{V} \rightarrow \mathbf{S}'$ as:

$$\begin{aligned} \Delta: [-1, 1] &\rightarrow \mathbf{S} \times [-0.5, 0.5] \\ \Delta(v) &= (s_i, \rho) = \begin{cases} s_i \\ \rho \end{cases}, \\ i &= \begin{cases} g/2 + \text{round}(\log_a((a^{g/2} - 1)v + 1)), & 0 \leq v \leq 1 \\ g/2 - \text{round}(\log_a(|v|(a^{g/2} - 1) + 1)), & -1 \leq v < 0 \end{cases} \\ \rho &= \begin{cases} \log_a((a^{g/2} - 1)v + 1) + g/2 - i, & 0 \leq v \leq 1 \\ -\log_a(1 - (a^{g/2} - 1)v) + g/2 - i, & -1 \leq v < 0 \end{cases} \end{aligned} \quad (2)$$

where “round” is a rounding operation. a is the scale parameter, and the value can be acquired by some methods (see [11]). Function Δ is a one-to-one mapping.

Definition 4: Let (s_i, ρ) be a linguistic 2-tuple. The real v that expresses the equivalent numerical information to (s_i, ρ) is obtained by function $\Delta^{-1}: \mathbf{S}' \rightarrow \mathbf{V}$ as:

$$\begin{aligned} \Delta^{-1}: \mathbf{S} \times [-0.5, 0.5] &\rightarrow [-1, 1] \\ \Delta^{-1}(s_i, \rho) = v &= \begin{cases} \frac{a^{(i+\rho-g/2)} - 1}{a^{g/2} - 1}, & g/2 \leq i + \rho \leq g \\ \frac{-a^{(g/2-i-\rho)} + 1}{a^{g/2} - 1}, & 0 \leq i + \rho < g/2 \end{cases} \end{aligned} \quad (3)$$

Δ^{-1} is the inverse function of Δ . The value set to $\mathbf{S}$ is symmetrical and unbalanced.

For example, for $g = 6$, $a = 1.4$, with the composite scale based 2-tuple linguistic model, the mapping relationship of $\mathbf{S}$ and $\mathbf{V}$ is shown in Fig. 1. This non-linear representation fits the actual psychophysical state of the decision maker.

The improved composite scale based 2-tuple linguistic model provides the following operators as the traditional 2-tuple linguistic model [5]: comparison operator and aggregation operator. Using function Δ and Δ^{-1} , together with a

Fig. 1 Example of mapping relationship: $\mathbf{S}$ and $\mathbf{V}$.

specific aggregation operator such as weighted average operator and so on (see [12]), deal with the decision makers' linguistic 2-tuple information. And this is not the focus of this paper.

The composite scale used in this model can reflect the decision maker's psychological state well. However, in many actual cases, the linguistic preference information is fuzzy and uncertain. We cannot acquire the exact numerical value directly. So this paper further proposes a combination of interval numerical scale [2] and composite scale [9], we call this interval composite scale (ICS), which expresses the fuzziness of the decision-maker's thinking in the form of interval.

2.2 Interval Composite Scale Based 2-Tuple Linguistic Model

To better reflect deciders' psychological state, with interval numerical scale and composite scale, interval composite scale (ICS) is put forward [9]. Meanwhile, interval composite scale based 2-tuple linguistic model is presented inspired by Dong et al. [2], [13], [14]. This model can build a relationship between word and numerical interval, which can realize the process of translation from word to numerical interval and the process of retranslation from numerical interval to word. It restricts the uncertainty of decision maker's preference and the complexity of decision-making problem to a certain extent. And the interval composite scale is defined as follows.

Definition 5: Let $\mathbf{S} = \{s_i | i = 0, 1, 2, \dots, g\}$ be a linguistic a set, $\mathbf{M} = \{[A_L, A_R] | A_L, A_R \in [-1, 1], A_L \leq A_R\}$ a set of interval values in $[-1, 1]$. The function $ICS: \mathbf{S} \rightarrow \mathbf{M}$ is defined as the interval composite scale of $\mathbf{S}$, and $ICS(s_i)$ is called the interval composite value of s_i .

If $ICS(s_i) = [A_L^i, A_R^i]$, then the function ICS_L and ICS_R are defined as follows: $ICS_L(s_i) = A_L^i$ and $ICS_R(s_i) = A_R^i$. The interval composite scale ICS is ordered, if $ICS_L(s_i) < ICS_L(s_{i+1})$ and $ICS_R(s_i) < ICS_R(s_{i+1})$.

Next, the following is an introduction to the interval composite scale based 2-tuple linguistic model.

Definition 6: For $(s_i, \rho) \in \mathbf{S}'$, where $s_i \in \mathbf{S}$, $\rho \in [-0.5, 0.5]$. The interval composite scale of $\mathbf{S}'$, $ICS': \mathbf{S}' \rightarrow \mathbf{M}$, i.e. the mapping of linguistic 2-tuple to interval value, is defined as

$$ICS'((s_i, \rho)) = [A_L, A_R] \quad (4)$$

where

$$A_L = \begin{cases} ICS_L(s_i) + \rho \times (ICS_L(s_{i+1}) - ICS_L(s_i)), & \rho \geq 0 \\ ICS_L(s_i) + \rho \times (ICS_L(s_i) - ICS_L(s_{i-1})), & \rho < 0 \end{cases} \quad (5)$$

$$A_R = \begin{cases} ICS_R(s_i) + \rho \times (ICS_R(s_{i+1}) - ICS_R(s_i)), & \rho \geq 0 \\ ICS_R(s_i) + \rho \times (ICS_R(s_i) - ICS_R(s_{i-1})), & \rho < 0 \end{cases} \quad (6)$$

In regard to the interval composite scale, the followings can be proved similar to results in [13]. The process won't be covered again here.

Proposition 1: If ICS on $\mathbf{S}$ is ordered, then ICS' on $\mathbf{S}'$ is ordered.

Proposition 2: If ICS on $\mathbf{S}$ is ordered, then ICS'_L , ICS'_R , and ICS' are bijective (one-to-one mapping) functions.

From Proposition 2, if ICS on $\mathbf{S}$ is ordered, then the inverse operations of ICS'_L , ICS'_R and ICS' exist. We define them as ICS^{-1} , ICS_L^{-1} , and ICS_R^{-1} .

Let $\mathbf{I} = \{A | A = ICS(s), s \in \mathbf{S}'\}$ be the range of the interval composite scale ICS' . Corollary 1 can be obtained from Proposition 2.

Corollary 1: If ICS on $\mathbf{S}$ is ordered, then $ICS^{-1}(A) = ICS_L^{-1}(A_L) = ICS_R^{-1}(A_R)$, for any $A = [A_L, A_R] \in \mathbf{I}$.

In detail, we define the inverse operation of the ICS .

Definition 7: Let $\mathbf{S} = \{s_i | i = 0, 1, 2, \dots, g\}$ be a linguistic term set, ICS' be an ordered interval composite scale on $\mathbf{S}'$, and $\mathbf{M} = \{[A_L, A_R] | A_L, A_R \in [-1, 1], A_L \leq A_R\}$ be a set of interval values in $[-1, 1]$. The inverse operation ICS^{-1} of ICS' , i.e. the mapping of interval value to linguistic 2-tuple, is defined as:

$$ICS^{-1}: \mathbf{M} \rightarrow \mathbf{S}' \quad (7)$$

where for any $A \in \mathbf{M}$, $ICS^{-1}(A) = s$, and

$$\text{dis}(A, ICS'(s)) = \min_{x \in \mathbf{S}'} \text{dis}(A, ICS'(x)) \quad (8)$$

Here, dis is some distance function on interval values. In this paper, we will use the Euclidean distance, i.e. $\text{dis}([a, b], [c, d]) = \sqrt{(a - c)^2 + (b - d)^2}$, which is generally used in retranslation process of CWW [8].

2.3 Interval Fuzzy Preference Relation

Usually, decision makers express their preference to the evaluated objects in the following two ways: one is comparing a pair of objects and giving a higher score to one of them to indicate higher preference [2], [14]; another is directly expressing their own opinions according to their experience, knowledge one by one [9]. In this paper, the first way is introduced. Considering the uncertainty of the objective world, the vagueness in human expression and the complexity of the decision-making problem, this paper proposes the interval fuzzy preference relation.

The following describes the interval fuzzy preference relation.

Definition 8: Suppose that $\tilde{\mathbf{V}} = (\tilde{v}_{ij})_{n \times n}$, where $\tilde{v}_{ij} = [v_{ij}^-, v_{ij}^+]$, is an interval fuzzy preference relation. If there is a fuzzy preference relation $\mathbf{F} = (f_{ij})_{n \times n}$, and $v_{ij}^- \leq f_{ij} \leq v_{ij}^+$ ($i, j = 1, 2, \dots, n$), $f_{ij} \leq f_{iz} + f_{zj} \leq f_{ij} + 1$ ($i, j, z = 1, 2, \dots, n$), then, $\tilde{\mathbf{V}}$ can be considered of acceptable consistency. Note that there is a set of alternatives

$\mathbf{X} = \{x_1, x_2, \dots, x_n\}$, here $\tilde{\mathbf{V}} = (\tilde{v}_{ij})_{n \times n}$, it is expressed in terms of the interval composite scale on $\mathbf{S}$, i.e. the interval fuzzy preference relation is composed of the interval composite values translated by the linguistic preference on the alternative pair (x_i, x_j) for the decision maker.

Proposition 3: The interval fuzzy preference relation $\tilde{\mathbf{V}} = (\tilde{v}_{ij})_{n \times n}$, where $\tilde{v}_{ij} = [v_{ij}^-, v_{ij}^+]$, is of acceptable consistency if and only if for $i, j, z = 1, 2, \dots, n$

$$v_{iz}^+ + v_{zj}^+ \geq v_{ij}^- \quad (9)$$

and

$$v_{iz}^- + v_{zj}^- \leq v_{ij}^+ + 1 \quad (10)$$

Proof:

Necessity: $\tilde{\mathbf{V}} = (\tilde{v}_{ij})_{n \times n}$ is an interval fuzzy preference relation of acceptable consistency, so exist a fuzzy preference relation $\mathbf{F} = (f_{ij})_{n \times n}$, and $v_{ij}^- \leq f_{ij} \leq v_{ij}^+$ ($i, j = 1, 2, \dots, n$), $f_{ij} \leq f_{iz} + f_{zj} \leq f_{ij} + 1$ ($i, j, z = 1, 2, \dots, n$). This means $v_{iz}^+ + v_{zj}^+ \geq f_{iz} + f_{zj} = f_{ij} \geq v_{ij}^-$ and $v_{iz}^- + v_{zj}^- \leq f_{iz} + f_{zj} \leq f_{ij} + 1 \leq v_{ij}^+ + 1$

So, we can get $v_{iz}^+ + v_{zj}^+ \geq v_{ij}^-$ and $v_{iz}^- + v_{zj}^- \leq v_{ij}^+ + 1$.

Sufficiency: Set $K = [v_{iz}^- + v_{zj}^-, v_{iz}^+ + v_{zj}^+]$, $Z = [v_{ij}^-, v_{ij}^+ + 1]$. Consider two cases:

Case 1: $v_{iz}^- + v_{zj}^- > v_{ij}^-$. In this case, since $v_{iz}^- + v_{zj}^- \leq v_{ij}^+ + 1$, there is $v_{iz}^- + v_{zj}^- \in [v_{ij}^-, v_{ij}^+ + 1] = Z$. Obviously, $v_{iz}^- + v_{zj}^- \in K$. Therefore, $K \cap Z \neq \emptyset$.

Case 2: $v_{iz}^- + v_{zj}^- \leq v_{ij}^-$. In this case, since $v_{iz}^+ + v_{zj}^+ \geq v_{ij}^+$, there is $v_{iz}^+ + v_{zj}^+ \in [v_{ij}^-, v_{ij}^+ + 1] = K$. Obviously, $v_{ij}^- \in Z$. Therefore, $K \cap Z \neq \emptyset$.

Summarizing cases 1 and 2, we can get $K \cap Z \neq \emptyset$, which implies that there is a fuzzy preference relation $\mathbf{F} = (f_{ij})_{n \times n}$, and $v_{ij}^- \leq f_{ij} \leq v_{ij}^+$ ($i, j = 1, 2, \dots, n$), $f_{ij} \leq f_{iz} + f_{zj} \leq f_{ij} + 1$. According to Definition 8, $\tilde{\mathbf{V}} = (\tilde{v}_{ij})_{n \times n}$ is of acceptable consistency. This completes the proof of Proposition 3.

3. Interval Composite Scale Based Personalized Individual Semantics Model (ICS-PISM)

As aforementioned, considering that different people have different semantics understandings, one word may mean different numerical meanings for different people, for this purpose, a personalized individual semantics model according to their personal differences is presented. Taking into account the uniqueness of each individual and the vagueness of individual thinking, based on the personalized individual semantics (PIS) model proposed by Herrera team [2], we do some further work and propose an interval composite scale based personalized individual semantics model (ICS-PISM) to express linguistic information of decision maker, which is usually utilized in the translation and retranslation problem of CWW.

3.1 Introduction of ICS-PISM

Let $\mathbf{S} = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set, $\mathbf{X} =$

$\{x_1, x_2, \dots, x_n\}$ be a set of alternatives. Let $\mathbf{E} = \{e_1, e_2, \dots, e_m\}$ be a set of decision makers, and let $\mathbf{C} = \{ICS^1, ICS^2, \dots, ICS^m\}$ be its corresponding set of ordered interval composite scales, where ICS^k is the ordered interval composite scale on $\mathbf{S}$ for decision maker e^k , $ICS^k(s_i) = [A_L^{i,k}, A_R^{i,k}]$ is the interval composite value of s_i for decision maker e^k . Let $\mathbf{P}^k = (p_{ij}^k)_{n \times n}$ be a linguistic preference relation of e^k , whose elements p_{ij}^k represents the linguistic preference on the alternative pair (x_i, x_j) for e^k . Assume that $\mathbf{P}^k$ is of acceptable consistency. The matrix $\tilde{\mathbf{V}}^k = (\tilde{v}_{ij}^k)_{n \times n}$ is the corresponding interval fuzzy preference relation of $\mathbf{P}^k$, where $\tilde{v}_{ij}^k = [v_{ij}^{k-}, v_{ij}^{k+}] = ICS^k(p_{ij}^k)$.

The width of the interval represents the uncertainty level of the interval judgment. In order to provide the most accurate judgment, it is desirable that the width of $ICS^k(s_i)$ be minimized, i.e. $\min_{ICS^k} \sum_{i=0}^g \text{wid}(ICS^k(s_i))$, where wid is a width function, can be transformed as follows:

$$\min \sum_{i=0}^g |A_R^{i,k} - A_L^{i,k}| \quad (11)$$

Meanwhile, the initial values $a_i \in ICS^k(s_i)$ are provided based on experience of decision maker and existing models, such as 2-tuple linguistic model with different scales and so on. We can get

$$-1 \leq A_L^{i,k} \leq a_i \leq A_R^{i,k} \leq 1, \quad i = 0, 1, \dots, g \quad (12)$$

And ICS^k on $\mathbf{S}$ is ordered, so exist

$$A_L^{i,k} < A_L^{i+1,k}, \quad i = 0, 1, \dots, g-1 \quad (13)$$

$$A_R^{i,k} < A_R^{i+1,k}, \quad i = 0, 1, \dots, g-1 \quad (14)$$

Let $ss \in \mathbf{S}'$, and $\varphi_p(ss)$ is defined as the position function of ss . For example, if $ss = (s_i, \rho)$, then $\varphi_p(ss) = i$, which can also be directly expressed as $\varphi_p(s_i) = i$. It is known that $\tilde{\mathbf{V}}^k$ ($k = 1, 2, \dots, m$) is of acceptable consistency, if $\mathbf{P}^k$ is of acceptable consistency. Based on Proposition 3, we can get

$$A_R^{\varphi_p(p_{iz}^k),k} + A_R^{\varphi_p(p_{zj}^k),k} \geq A_L^{\varphi_p(p_{ij}^k),k}, \quad p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null \quad (15)$$

and

$$A_L^{\varphi_p(p_{iz}^k),k} + A_L^{\varphi_p(p_{zj}^k),k} \leq A_R^{\varphi_p(p_{ij}^k),k} + 1, \quad p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null \quad (16)$$

Due to Eq. (12), there is $\min \sum_{i=0}^g |A_R^{i,k} - A_L^{i,k}| = \min \sum_{i=0}^g (A_R^{i,k} - A_L^{i,k})$. As a result, on the basis of Eqs. (11)–(16), an interval composite scale based personalized individual semantics model (ICS-PISM) is proposed, shown as the following linear programming model:

$$\begin{cases} \min \sum_{i=0}^g (A_R^{i,k} - A_L^{i,k}) \\ \text{s.t. } -1 \leq A_L^{i,k} \leq a_i \leq A_R^{i,k} \leq 1, \quad i = 0, 1, \dots, g \\ A_L^{i,k} < A_L^{i+1,k}, \quad i = 0, 1, \dots, g-1 \\ A_R^{i,k} < A_R^{i+1,k}, \quad i = 0, 1, \dots, g-1 \\ A_R^{\varphi_p(p_{iz}^k),k} + A_R^{\varphi_p(p_{zj}^k),k} \geq A_L^{\varphi_p(p_{ij}^k),k} \\ \quad (k = 1, 2, \dots, m; p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null) \\ A_L^{\varphi_p(p_{iz}^k),k} + A_L^{\varphi_p(p_{zj}^k),k} \leq A_R^{\varphi_p(p_{ij}^k),k} + 1 \\ \quad (k = 1, 2, \dots, m; p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null) \end{cases} \quad (17)$$

where $A_L^{i,k}, A_R^{i,k}$ ($i = 0, 1, \dots, g$) are decision variables. With model (17), we can obtain the interval composite value $ICS^k(s_i) = [A_L^{i,k}, A_R^{i,k}]$.

Moreover, the ICS-PISM is similar to the 2-tuple linguistic model proposed in Sect. 2.1, which can perform the following operations:

1) *Comparison operation*: Let r_j, r_k be the linguistic preference of decision makers e^j and e^k , then exist the following provisions:

(1) $r_j > r_k \Leftrightarrow INS^j(r_j) > INS^k(r_k)$;

(2) $r_j = r_k \Leftrightarrow INS^j(r_j) = INS^k(r_k)$;

(3) $r_j < r_k \Leftrightarrow INS^j(r_j) < INS^k(r_k)$.

In this paper, we do not give priority to the comparison method of the interval value, mainly use the comparison operator proposed in [15].

2) *Inverse operation*: $ICS^{k,-1}([A_L, A_R]) = (s_x, \rho)$

The detailed operation can be processed as Definition 7. With this inverse operation, we can realize the translation from the quantitative interval to a qualitative close linguistic term so that people can understand more easily.

3) *Aggregation operation*: In general, using ICS^k and $ICS^{k,-1}$ proposed, together with a specific aggregation operator, deal with linguistic preference information of different decision makers. And the weighted average operator is used in this paper.

3.2 Example of ICS-PISM

Let $\mathbf{E} = \{e_1, e_2, e_3, e_4\}$ be a set of decision makers, $\mathbf{X} = \{x_1, x_2, \dots, x_5\}$ be a set of alternatives. Let $\mathbf{S} = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{HC, C, JC, YB, JH, H, HH\}$ be a linguistic term set. Let $\mathbf{P}^k = (p_{ij}^k)_{n \times n}$ be a linguistic preference relation of e^k , where p_{ij}^k can be expressed by the linguistic term in $\mathbf{S}$. The specific linguistic preference relations are listed as follows:

$$\mathbf{P}^1 = \begin{pmatrix} - & s_3 & s_4 & s_3 & s_4 \\ - & - & s_3 & s_4 & s_3 \\ - & - & - & s_4 & s_4 \\ - & - & - & - & s_5 \\ - & - & - & - & - \end{pmatrix} \quad \mathbf{P}^2 = \begin{pmatrix} - & s_2 & s_4 & s_5 & s_4 \\ - & - & s_3 & s_4 & s_5 \\ - & - & - & s_3 & s_3 \\ - & - & - & - & s_2 \\ - & - & - & - & - \end{pmatrix}$$

$$\mathbf{P}^3 = \begin{pmatrix} - & s_2 & s_2 & s_0 & s_4 \\ - & - & s_3 & s_1 & s_5 \\ - & - & - & s_2 & s_6 \\ - & - & - & - & s_6 \\ - & - & - & - & - \end{pmatrix} \quad \mathbf{P}^4 = \begin{pmatrix} - & s_4 & s_4 & s_3 & s_5 \\ - & - & s_5 & s_4 & s_6 \\ - & - & - & s_2 & s_4 \\ - & - & - & - & s_6 \\ - & - & - & - & - \end{pmatrix}$$

In this example, decision makers correspond to different ordered interval composite scales $\mathbf{C} = \{ICS^1, ICS^2, ICS^3, ICS^4\}$, and according to the proposed 2-tuple linguistic model, the initial values of $\mathbf{S}$ are $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\} = \{-1, -0.55, -0.23, 0, 0.23, 0.55, 1\}$. According to Eq. (17), we can get the following ICS-PISM model to set the ordered interval composite scale of each decision maker:

$$\begin{cases} \min \sum_{i=0}^6 (A_R^{i,k} - A_L^{i,k}), & k = 1, 2, 3, 4 \\ s.t. & -1 \leq A_L^{i,k} \leq a_i \leq A_R^{i,k} \leq 1, \quad i = 0, 1, \dots, 6 \\ & A_L^{i,k} < A_L^{i+1,k}, \quad i = 0, 1, \dots, 5 \\ & A_R^{i,k} < A_R^{i+1,k}, \quad i = 0, 1, \dots, 5 \\ & A_R^{\varphi_p(p_{iz}^k),k} + A_R^{\varphi_p(p_{zj}^k),k} \geq A_L^{\varphi_p(p_{ij}^k),k} \\ & \quad (p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null) \\ & A_L^{\varphi_p(p_{iz}^k),k} + A_L^{\varphi_p(p_{zj}^k),k} \leq A_R^{\varphi_p(p_{ij}^k),k} + 1 \\ & \quad (p_{iz}^k, p_{zj}^k, p_{ij}^k \neq null) \end{cases}$$

where $A_R^{i,k}, A_L^{i,k}$ ($i = 0, 1, \dots, 6$) are the decision variables.

Solving the above model, the decision makers' interval composite scales are:

Table 1 Interval composite scales.

ICS^k	s_0	s_1	s_2	s_3	s_4	s_5	s_6
e^1	-1	-0.55	-0.23	[0, 0.11]	0.23	0.55	1
e^2	-1	-0.55	[0.23, 0.08]	[0, 0.15]	0.23	[0.31, 0.55]	1
e^3	[-1, -0.84]	[-0.55, -0.51]	[-0.23, -0.01]	0	[0.16, 0.23]	[0.49, 0.55]	[0.99, 1]
e^4	-1	-0.55	-0.23	0	[0.23, 0.37]	[0.55, 0.59]	[0.96, 1]

3.3 Analysis and Comparison of ICS-PISM

In this section, we compare ICS-PISM with the composite scale based 2-tuple linguistic model in Sect. 2.1. Let r_1, r_2, r_3 , and r_4 be the linguistic preference of decision makers e^1, e^2, e^3 , and e^4 , which can also be expressed by the linguistic term in $\mathbf{S}$. Suppose $r_1 = s_4, r_2 = s_3, r_3 = s_5$, and $r_4 = s_5$. And the weights of the decision makers are given, $\mathbf{W} = \{1/4, 1/4, 1/4, 1/4\}$. The comparison results are shown in the following table:

In Table 2, using the composite scale based 2-tuple linguistic model to evaluate, we can get the comparison of decision makers' linguistic preference, $r_2 < r_1 < r_3 = r_4$, while using ICS-PISM, we can get the comparison, $r_2 < r_1 < r_3 < r_4$. It can be found that, because of the fact that different people have different semantic understandings of linguistic information, we get different rankings. Moreover,

Table 2 Comparison between models.

	Comparison	Aggregated Results
2-tuple linguistic model	$r_2 < r_1 < r_3 = r_4$	$(s_4, 0.3592)$
ICS-PISM	$r_2 < r_1 < r_3 < r_4$	$(s_4, 0.3711), (s_5, -0.4945), (s_4, 0.4731), (s_4, 0.1711)$

compared with the 2-tuple linguistic model, the ICS-PISM model is better to distinguish alternatives. In addition, the aggregated result by using the composite scale based 2-tuple linguistic model is $(s_4, 0.3592)$, while the aggregated result by using ICS-PISM indicates different meaning to each decision maker, that is to say, $(s_4, 0.3711)$ for e^1 , $(s_5, -0.4945)$ for e^2 , $(s_4, 0.4731)$ for e^3 , and $(s_4, 0.1711)$ for e^4 . From these results, we can find that for e^2 , his/her degree of linguistic preference differs from the others. Therefore, it can be found that the proposed ICS-PISM model takes into account the fuzzy characteristic of and diversification across the process of human thinking. In this paper, we try to model these features so as to facilitate the calculation, and find that alternatives can be distinguished effectively. Moreover, the model proposed can be applied to many actual decision making problems.

4. Consensus Decision Making Model with ICS-PISM

This section describes the role of ICS-PISM in the whole consensus decision making process, and the whole process to solve consensus decision making problem with individual semantic understanding, which takes CWW scheme into consideration, as shown in Fig. 2. In detail, the whole process mainly includes semantics translation process, aggregation process, semantics retranslation process, and consensus process. In allusion to the whole process, consensus decision making model with ICS-PISM is presented, which is on the basis of the fact that different people have different semantic understandings of linguistic information, i.e. the word means different numerical meanings to different people. And the whole consensus decision making process will be described as follows in detail.

4.1 Description of Consensus Decision Making Model with ICS-PISM

Let $\mathbf{S} = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set. Let $\mathbf{X} = \{x_1, x_2, \dots, x_n\}$ be a set of alternatives, and its associated weighting vector be $\mathbf{W}_x = \{w_x^1, w_x^2, \dots, w_x^n\}$, where $w_x^j \in [0, 1]$, $\sum_{j=1}^n w_x^j = 1$. Let $\mathbf{E} = \{e_1, e_2, \dots, e_m\}$ be a set of decision makers, and its weighting vector of decision makers be $\mathbf{W}_d = \{w_d^1, w_d^2, \dots, w_d^m\}$, where $w_d^k \in [0, 1]$, $\sum_{k=1}^m w_d^k = 1$. Let $\mathbf{P}^k = (p_{ij}^k)_{n \times n}$ be a linguistic preference relation of e^k , whose elements p_{ij}^k represents the linguistic preference on the alternative pair (x_i, x_j) for e^k .

1) Semantics translation process

According to ICS-PISM, generate the ordered interval com-

Fig. 2 Consistent decision process based on ICS-PISM.

positive scale ICS^k of $\mathbf{S}$ for the decision maker e^k . Then, use ICS^k to transform the linguistic preference relation $\mathbf{P}^k$, and get individual interval fuzzy preference relation $\tilde{\mathbf{V}}^k = (\tilde{v}_{ij}^k)_{n \times n}$, where $\tilde{v}_{ij}^k = [v_{ij}^{k-}, v_{ij}^{k+}] = ICS^k(p_{ij}^k)$ ($k = 1, 2, \dots, m$).

2) Aggregation process
Once get $\tilde{\mathbf{V}}^k = (\tilde{v}_{ij}^k)_{n \times n}$, we can gather the preference of decision makers by weighted average operator or other weight operators, and obtain the collective interval fuzzy preference relation $\tilde{\mathbf{V}}^c = (\tilde{v}_{ij}^c)_{n \times n}$. Here, the weighted average operator is used, and the operation is shown as follows:

$$\tilde{v}_{ij}^c = [v_{ij}^{c-}, v_{ij}^{c+}] = \left[\sum_{k=1}^m w_d^k \cdot v_{ij}^{k-}, \sum_{k=1}^m w_d^k \cdot v_{ij}^{k+} \right] \quad (18)$$

By the obtained $\tilde{\mathbf{V}}^c = (\tilde{v}_{ij}^c)_{n \times n}$ and the associated weighting vector $\mathbf{W}_x = \{w_x^1, w_x^2, \dots, w_x^n\}$, the collective alternative evaluation result $\mathbf{Z}^c = (z_1^c, z_2^c, \dots, z_n^c)^T$ is calculated as follows:

$$z_i^c = [z_i^{c-}, z_i^{c+}] = \left[\sum_{j=1}^n w_x^j \cdot v_{ij}^{c-}, \sum_{j=1}^n w_x^j \cdot v_{ij}^{c+} \right] \quad (19)$$

According to the collective result, the value of z_i^c can be used to rank the alternatives.

3) Semantics retranslation process

According to the inverse operation of interval composite scale, we can get the individual linguistic understanding of e^k on the obtained $\mathbf{Z}^c$, $\mathbf{Z}_k^{c,-1} = \{ICS^{k,-1}(z_1^c), ICS^{k,-1}(z_2^c), \dots, ICS^{k,-1}(z_n^c)\}$, where $ICS^{k,-1}(z_i^c) = (s_{i,k}^c, \rho_{i,k}^c)$ represents the individual linguistic understanding of e^k on z_i^c for x_i . According to the above-mentioned position function φ_p , we can get the position set on $\mathbf{S}$ of each result value $\mathbf{Z}_k^{c,-1}$, i.e. $\mathbf{L}_k^c = \{\varphi_p(s_{1,k}^c), \varphi_p(s_{2,k}^c), \dots, \varphi_p(s_{n,k}^c)\} = \{l_{1,k}^c, l_{2,k}^c, \dots, l_{n,k}^c\}$.

4) Consensus process

In this process, include three steps: calculate the consensus degree, control consensus state, and provide feedback mechanism.

(1) Calculate the consensus degree

The above collective alternative evaluation result $\mathbf{Z}^c = (z_1^c, z_2^c, \dots, z_n^c)^T$ is known, next, the individual alternative evaluation result $\mathbf{Z}^k = (z_1^k, z_2^k, \dots, z_n^k)^T$, is obtained as follows:

low:

$$z_i^k = [z_i^{k-}, z_i^{k+}] = \left[\sum_{j=1}^n w_x^j \cdot v_{ij}^{k-}, \sum_{j=1}^n w_x^j \cdot v_{ij}^{k+} \right] \quad (20)$$

According to the inverse operation of interval composite scale, we can get the individual linguistic understanding of e^k on the obtained $\mathbf{Z}^k$, $\mathbf{Z}_k^{k,-1} = \{ICS^{k,-1}(z_1^k), ICS^{k,-1}(z_2^k), \dots, ICS^{k,-1}(z_n^k)\}$, where $ICS^{k,-1}(z_i^k) = (s_{i,k}^k, \rho_{i,k}^k)$ represents the individual linguistic understanding of e^k on z_i^k for x_i . According to the above-mentioned position function φ_p , we can get the position set of each result value $\mathbf{Z}_k^{k,-1}$, i.e. $\mathbf{L}^k = \{\varphi_p(s_{1,k}^k), \varphi_p(s_{2,k}^k), \dots, \varphi_p(s_{n,k}^k)\} = \{l_1^k, l_2^k, \dots, l_n^k\}$.

In the consensus decision making process, define the following three consensus degree.

Definition 9: The consensus degree $\delta_{i,k}$ on x_i between individual alternative evaluation of e^k and collective alternative evaluation, is defined as follows:

$$\delta_{i,k} = 1 - \frac{|l_{i,k}^c - l_i^k|}{g} \quad (21)$$

where $\delta_{i,k} \in [0, 1]$. The higher $\delta_{i,k}$, the more consensus on x_i between individual alternative evaluation of e^k and collective alternative evaluation.

The consensus degree δ_k between individual alternative evaluation of e^k and collective alternative evaluation, is defined as follows:

$$\delta_k = \frac{1}{n} \sum_{i=1}^n \delta_{i,k} \quad (22)$$

where $\delta_k \in [0, 1]$. The higher δ_k , the more consensus between individual alternative evaluation of e^k and collective alternative evaluation.

The consensus degree of whole decision process δ_{con} , which is regarded as the consensus degree of all decision makers on all alternatives, is given by:

$$\delta_{con} = \frac{1}{m} \sum_{k=1}^m \delta_k \quad (23)$$

where $\delta_{con} \in [0, 1]$. The higher δ_{con} , the more consensus among decision makers on all alternatives.

(2) Control consensus state

If the consensus degree δ_{con} satisfies the minimum consensus threshold δ_{thrd} , i.e. $\delta_{con} \geq \delta_{thrd}$, the consensus process finishes, and we can choose the alternatives by ranking $\mathbf{Z}^c$. Otherwise, if $\delta_{con} < \delta_{thrd}$, the feedback mechanism will be provided and relevant recommendations will be made available to decision makers. Meanwhile, decision makers make some adjustments based on the new information, and start a new round of decision making process, and so forth. Until the consensus degree δ_{con} satisfies the minimum consensus threshold δ_{thrd} , or the round number reaches the maximum number of rounds $MaxRound$, the consensus process finishes. Here $MaxRound$ is a parameter proposed to avoid endless rounds of the consensus process. δ_{thrd} and

MaxRound are given according to views of domain experts and practical experience.

(3) Feedback mechanism

This section mainly provides a feedback mechanism to give some advice when decision makers don't reach a consensus, and to find the conflict among decision makers.

First, determine the set of decision makers that need to be adjusted **ED**:

$$\mathbf{ED} = \{e^k | \delta_k < \delta_{thrd}\} \quad (24)$$

The collective interval fuzzy preference relation is obtained as $\tilde{\mathbf{V}}^c = (\tilde{v}_{ij}^c)_{n \times n}$, where $\tilde{v}_{ij}^c = [\tilde{v}_{ij}^{c-}, \tilde{v}_{ij}^{c+}]$. According to the inverse operation of interval composite scale, we can get the individual linguistic understanding of e^k on the obtained $\tilde{\mathbf{V}}^c$, $\mathbf{P}^{k*} = (p_{ij}^{k*})_{n \times n}$, where $p_{ij}^{k*} = ICS^{k,-1}(\tilde{v}_{ij}^c)$.

Second, determine the set of linguistic preference that e^k need to adjust **EDX_k**:

$$\mathbf{EDX}_k = \{p_{ij}^k | e^k \in \mathbf{ED} \wedge i < j \wedge \delta_{i,k} < \delta_{thrd} \wedge \varphi_p(p_{ij}^k) \neq \varphi_p(p_{ij}^{k*})\} \quad (25)$$

Then, for each **EDX_k**, provide some specific recommendations for decision makers:

(i) If $\varphi_p(p_{ij}^k) > \varphi_p(p_{ij}^{k*})$, e^k may decrease the linguistic preference information p_{ij}^k on alternative pair (x_i, x_j) .

(ii) If $\varphi_p(p_{ij}^k) < \varphi_p(p_{ij}^{k*})$, e^k may increase the linguistic preference information p_{ij}^k on alternative pair (x_i, x_j) .

Finally, make some adjustments to obtain new individual linguistic preference relation, and start a new round of decision making process.

4.2 Example of Consensus Decision Making Model with ICS-PISM

Let **E**, **X**, **S** and **P^k** be as Sect. 3.2. In addition, the weighting vector of decision makers be $\mathbf{W}_d = \{1/4, 1/4, 1/4, 1/4\}$, and the associated weighting vector of alternatives be $\mathbf{W}_x = \{1/5, 1/5, 1/5, 1/5, 1/5\}$. In the following, we illustrate the consensus decision making model with ICS-PISM in detail.

1) Semantics translation process

As shown in Sect. 3.2, generate the ordered interval composite scale ICS^k and individual interval fuzzy preference relation $\tilde{\mathbf{V}}^k$. When it comes to the concrete calculation of the interval fuzzy preference relation matrix, each matrix considers the symmetry and complements the preference relation matrix. That is to say, the interval fuzzy preference relation matrix has the following properties: $\tilde{v}_{ij}^{k-} = \tilde{v}_{ji}^{k+}$ ($i \neq j$) and $\tilde{v}_{ii}^k = [0, 0]$. Then the interval fuzzy preference relation $\tilde{\mathbf{V}}^k$ is:

$$\tilde{\mathbf{V}}^1 = \begin{bmatrix} [0, 0] & [0, 0.11] & [0.23, 0.23] & [0, 0.11] & [0.23, 0.23] \\ [-0.11, 0] & [0, 0] & [0, 0.11] & [0.23, 0.23] & [0, 0.11] \\ [-0.23, -0.23] & [-0.11, 0] & [0, 0] & [0.23, 0.23] & [0.23, 0.23] \\ [-0.11, 0] & [-0.23, -0.23] & [-0.23, -0.23] & [0, 0] & [0.55, 0.55] \\ [-0.23, -0.23] & [-0.11, 0] & [-0.23, -0.23] & [-0.55, -0.55] & [0, 0] \end{bmatrix}$$

Table 3 Individual linguistic understanding of e^k on the obtained $\mathbf{Z}^c$.

e^k	Individual Linguistic Understanding $\mathbf{Z}_i^{c-1}$	Position Set $\mathbf{L}_i$
e^1	$\{(s_3, 0.15), (s_3, 0.44), (s_3, -0.18), (s_3, 0.47), (s_2, -0.40)\}$	$\{3, 3, 3, 3, 2\}$
e^2	$\{(s_3, 0.10), (s_3, 0.40), (s_3, -0.21), (s_3, 0.44), (s_1, 0.38)\}$	$\{3, 3, 3, 3, 1\}$
e^3	$\{(s_3, 0.43), (s_4, -0.30), (s_3, -0.06), (s_4, -0.28), (s_1, 0.40)\}$	$\{3, 4, 3, 4, 1\}$
e^4	$\{(s_3, 0.28), (s_4, 0.45), (s_3, 0.04), (s_4, 0.46), (s_2, -0.40)\}$	$\{3, 4, 3, 4, 2\}$

$$\tilde{\mathbf{V}}^2 = \begin{bmatrix} [0, 0] & [-0.23, 0.08] & [0.23, 0.23] & [0.31, 0.55] & [0.23, 0.23] \\ [-0.08, 0.23] & [0, 0] & [0, 0] & [0.23, 0.23] & [0.31, 0.55] \\ [-0.23, -0.23] & [0, 0] & [0, 0] & [0, 0] & [0, 0] \\ [-0.55, -0.31] & [-0.23, -0.23] & [0, 0] & [0, 0] & [-0.23, 0.08] \\ [-0.23, -0.23] & [-0.55, -0.31] & [0, 0] & [-0.08, 0.23] & [0, 0] \end{bmatrix}$$

$$\tilde{\mathbf{V}}^3 = \begin{bmatrix} [0, 0] & [-0.23, -0.01] & [-0.23, -0.01] & [-1, -0.84] & [0.16, 0.23] \\ [0.01, 0.23] & [0, 0] & [0, 0] & [-0.55, -0.51] & [0.49, 0.55] \\ [0.01, 0.23] & [0, 0] & [0, 0] & [-0.23, -0.01] & [0.99, 1] \\ [0.84, 1] & [0.51, 0.55] & [0.01, 0.23] & [0, 0] & [0.99, 1] \\ [-0.23, -0.16] & [-0.55, -0.49] & [-1, -0.99] & [-1, -0.99] & [0, 0] \end{bmatrix}$$

$$\tilde{\mathbf{V}}^4 = \begin{bmatrix} [0, 0] & [0.23, 0.37] & [0.23, 0.37] & [0, 0] & [0.55, 0.59] \\ [-0.37, -0.23] & [0, 0] & [0.55, 0.59] & [0.23, 0.37] & [0.96, 1] \\ [-0.37, -0.23] & [-0.59, -0.55] & [0, 0] & [-0.23, -0.23] & [0.23, 0.37] \\ [0, 0] & [-0.37, -0.23] & [0.23, 0.23] & [0, 0] & [0.96, 1] \\ [-0.59, -0.55] & [-1, -0.96] & [-0.37, -0.23] & [-1, -0.96] & [0, 0] \end{bmatrix}$$

2) Aggregation process

According to Eq. (18), the collective interval fuzzy preference relation $\tilde{\mathbf{V}}^c$ is obtained as follows:

$$\tilde{\mathbf{V}}^c = \begin{bmatrix} [0, 0] & [-0.0575, 0.1375] & [0.115, 0.205] & [-0.1725, 0.045] & [0.2925, 0.32] \\ [-0.1375, 0.0575] & [0, 0] & [0.1375, 0.175] & [0.035, 0.08] & [0.44, 0.5525] \\ [-0.205, 0.115] & [-0.175, -0.1375] & [0, 0] & [-0.0575, -0.0025] & [0.3625, 0.4] \\ [0.045, 0.1725] & [-0.08, -0.035] & [0.0025, 0.0575] & [0, 0] & [0.5675, 0.6575] \\ [-0.32, -0.2925] & [-0.5525, -0.44] & [-0.4, -0.3625] & [-0.6575, -0.5675] & [0, 0] \end{bmatrix}$$

According to Eq. (19), we calculate the collective alternative evaluation result $\mathbf{Z}^c$:

$$\mathbf{Z}^c = \begin{bmatrix} [0.0355, 0.1235] \\ [0.095, 0.173] \\ [-0.015, 0.029] \\ [0.107, 0.1705] \\ [-0.386, -0.3325] \end{bmatrix}$$

The obtained collective ranking of alternatives is $x_4 > x_2 > x_1 > x_3 > x_5$.

3) Semantics retranslation process

According to the inverse operation of interval composite scale presented in Sect. 2.2, we can get the individual linguistic understanding of e^k on the obtained $\mathbf{Z}^c$:

4) Consensus process

(1) Calculate the consensus degree

First, according to the Eq. (20), the individual alternative evaluation result $\mathbf{Z}^k$ is obtained. And we can calculate the individual linguistic understanding of e^k on $\mathbf{Z}^k$, and get its corresponding position set $\mathbf{L}^k$ on **S**, and the individual ranking of alternatives for e^k (see Table 4):

Then, calculate the consensus degree of whole decision process δ_{con} . In detail, the first step is to calculate the consensus degree $\delta_{i,k}$ on x_i between individual alternative evaluation of e^k and collective alternative evaluation:

$$\{\delta_{1,1}, \delta_{2,1}, \delta_{3,1}, \delta_{4,1}, \delta_{5,1}\} = \{1, 1, 1, 1, 1\}$$

$$\{\delta_{1,2}, \delta_{2,2}, \delta_{3,2}, \delta_{4,2}, \delta_{5,2}\} = \{0.83, 1, 1, 0.83, 0.83\}$$

Table 4 Individual linguistic understanding of e^k on the obtained $\mathbf{Z}^k$.

e^k	Position Set	$\mathbf{L}^k$	Individual Ranking
e^1	{3,3,3,3,2}		$x_1 \succ x_2 \succ x_3 \succ x_4 \succ x_5$
e^2	{4,3,3,2,2}		$x_1 \succ x_2 \succ x_3 \succ x_5 \succ x_4$
e^3	{2,3,4,5,1}		$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5$
e^4	{4,4,2,4,1}		$x_2 \succ x_1 \succ x_4 \succ x_3 \succ x_5$

$$\{\delta_{1,3}, \delta_{2,3}, \delta_{3,3}, \delta_{4,3}, \delta_{5,3}\} = \{0.83, 0.83, 0.83, 0.83, 1\}$$

$$\{\delta_{1,4}, \delta_{2,4}, \delta_{3,4}, \delta_{4,4}, \delta_{5,4}\} = \{0.83, 1, 0.83, 1, 0.83\}$$

The second step is to calculate the consensus degree δ_k between individual alternative evaluation of e^k and collective alternative evaluation:

$$\{\delta_1, \delta_2, \delta_3, \delta_4\} = \{1, 0.9, 0.87, 0.9\}$$

Finally, calculate the consensus degree of whole decision process δ_{con} : $\delta_{con} = 0.9167$

(2) Control consensus state

In this example, if the minimum consensus threshold is $\delta_{thrd} = 0.85$, the decision process finishes, and the final ranking of alternatives is $x_4 > x_2 > x_1 > x_3 > x_5$.

If the minimum consensus threshold is $\delta_{thrd} = 0.95$, then $\delta_{con} < \delta_{thrd}$. So the feedback mechanism is provided in the following, and some recommendations will be made available to decision makers.

(3) Feedback mechanism

First, determine the set of decision makers that need to be adjusted **ED**: $\mathbf{ED} = \{e^2, e^3, e^4\}$

The collective interval fuzzy preference relation $\tilde{\mathbf{V}}^c$ is obtained. According to the inverse operation of interval composite scale, we can get the individual linguistic understanding of e^k on $\tilde{\mathbf{V}}^c$, $\mathbf{P}^{k*}$:

$$\mathbf{P}^{2*} = \begin{pmatrix} - & (s_3, -0.2439) & (s_4, -0.4798) & (s_2, 0.1864) & (s_4, 0.3107) \\ - & - & (s_4, -0.434) & (s_3, 0.0413) & (s_5, 0.1338) \\ - & - & - & (s_3, -0.4135) & (s_5, -0.4026) \\ - & - & - & - & (s_5, 0.3331) \\ - & - & - & - & - \end{pmatrix}$$

$$\mathbf{P}^{3*} = \begin{pmatrix} - & (s_3, -0.2236) & (s_4, -0.165) & (s_2, 0.2599) & (s_4, 0.3432) \\ - & - & (s_4, -0.207) & (s_3, 0.3057) & (s_5, -0.0743) \\ - & - & - & (s_3, -0.25) & (s_5, -0.4263) \\ - & - & - & - & (s_5, 0.1925) \\ - & - & - & - & - \end{pmatrix}$$

$$\mathbf{P}^{4*} = \begin{pmatrix} - & (s_3, 0.1984) & (s_4, -0.461) & (s_3, -0.2772) & (s_4, 0.0597) \\ - & - & (s_4, -0.4922) & (s_3, 0.1984) & (s_5, -0.2881) \\ - & - & - & (s_3, -0.0027) & (s_4, 0.3249) \\ - & - & - & - & (s_5, 0.1037) \\ - & - & - & - & - \end{pmatrix}$$

Second, determine the set of linguistic preference that e^k need to adjust $\mathbf{EDX}_k$:

$$\mathbf{EDX}_2 = \{p_{12}^2, p_{14}^2, p_{45}^2\}$$

$$\mathbf{EDX}_3 = \{p_{12}^3, p_{13}^3, p_{14}^3, p_{23}^3, p_{24}^3, p_{34}^3, p_{35}^3, p_{45}^3\}$$

$$\mathbf{EDX}_4 = \{p_{12}^4, p_{15}^4, p_{34}^4\}$$

Then, for each $\mathbf{EDX}_k$, provide some specific recommendations for decision makers. The following is one of these:

$$\mathbf{P}^{2'} = \begin{pmatrix} - & s_3 & s_4 & s_4 & s_4 \\ - & - & s_3 & s_4 & s_5 \\ - & - & - & s_3 & s_3 \\ - & - & - & - & s_3 \\ - & - & - & - & - \end{pmatrix} \quad \mathbf{P}^{3'} = \begin{pmatrix} - & s_3 & s_3 & s_1 & s_4 \\ - & - & s_4 & s_2 & s_5 \\ - & - & - & s_3 & s_5 \\ - & - & - & - & s_5 \\ - & - & - & - & - \end{pmatrix}$$

$$\mathbf{P}^{4'} = \begin{pmatrix} - & s_3 & s_4 & s_3 & s_4 \\ - & - & s_5 & s_4 & s_6 \\ - & - & - & s_3 & s_4 \\ - & - & - & - & s_6 \\ - & - & - & - & - \end{pmatrix}$$

Finally, acquire some new information and make some adjustments after a serious thought. Then get the new individual linguistic preference relation, and start a new round of decision making process, including semantics translation process, aggregation process, semantics retranslation process, and consensus process. Until the consensus degree δ_{con} satisfies the minimum consensus threshold δ_{thrd} , or the round number reaches the maximum number of rounds *MaxRound*, the consensus process finishes. In many cases, deciders make some decisions without abundant intercourse. And the proposed method may avoid this and can help deciders exchange and understand their unique decision information. And this feedback gives a direction about how to modify so as to improve the final group decision quality.

5. Conclusions

Different people have different semantics understandings, because of the human cognition difference and the fuzziness of thinking expression. By studying the use of CWW on consensus decision making, this paper first proposes an interval composite scale based 2-tuple linguistic model, which takes into account the psychophysical state of the human being. It realizes the process of translation from word to interval numerical and the process of retranslation from interval numerical to word, and can reflect the fuzziness of human thinking as well as the complexity and uncertainty in decision making problems to some extent. Second, with the consideration of individual differences in semantic understanding, this paper proposes an interval composite scale based personalized individual semantics Model (ICS-PISM), which can provide different linguistic representation models according to individual decision-makers. At last, because decision-makers use different linguistic representation models, this paper proposes a consensus decision making model with ICS-PISM, which emphasizes semantic translation process and semantic retranslation process of decision process. This model can judge the consensus state of the whole decision process, and provide a feedback mechanism to give some advice to the decision-maker for readjusting the linguistic preference information so as to reduce conflicts.

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