Deblocking Artifact of Satellite Image Based on Adaptive Soft-Threshold Anisotropic Filter Using Wavelet

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SUMMARY New deblocking artifact, or blocking artifact reduction, algorithms based on nonlinear adaptive soft-threshold anisotropic filter in wavelet are proposed. Our deblocking algorithm uses soft-threshold, adaptive wavelet direction, adaptive anisotropic filter, and estimation. The novelties of this paper are an adaptive soft-threshold for deblocking artifact and an optimal intersection of confidence intervals (OICI) method in deblocking artifact estimation. The soft-threshold values are adaptable to different thresholds of flat area, texture area, and blocking artifact. The OICI is a reconstruction technique of estimated deblocking artifact which improves acceptable quality level of estimated deblocking artifact and reduces execution time of deblocking artifact estimation compared to the other methods. Our adaptive OICI method outperforms other adaptive deblocking artifact methods. Our estimated deblocking artifact algorithms have up to 98% of MSE improvement, up to 89% of RMSE improvement, and up to 99% of MAE improvement. We also got up to 77.98% reduction of computational time of deblocking artifact estimations, compared to other methods. We have estimated shift and add algorithms by using Euler \( \pm E \) and Runge-Kutta of order \( 4 \) \((RK4)\) algorithms which iterate one step an ordinary differential equation integration method. Experimental results showed that our \( E \) \( \pm \) and \( RK4 \) \( \pm \) algorithms could reduce computational time in terms of shift and add, and \( RK4 \) \( \pm \) algorithm is superior to \( E \) \( \pm \) algorithm.

key words: deblocking artifact, soft-threshold, reconstruction, estimation

1. Introduction

The discrete cosine transform (DCT) is the most popular image compression technique because its performance is known to be optimal in the mean squared error (MSE) term\(^{[1]–[4]}\). Our previous study\(^{[5]}\) and the block DCT (B-DCT) which used in JPEG\(^{[6]}, [7]\), H.264\(^{[8]}\), and MPEG\(^{[9]}\), show that DCT leaves blocking artifacts problem with linear filtering\(^{[2]–[5]}\). This is caused by the higher compression.

Various algorithms have been proposed for removing the blocking artifacts. The methods of\(^{[10]–[12]}\) used the projection onto convex sets (POCS) techniques. POCS uses a dynamic focus-plus context (DF+C)\(^{[13]}\) and the improved weighted projection onto convex sets (IWPOCS)\(^{[14]}\). The techniques make good effects, but the main disadvantages of DF+C and IWPOCS methods are non-unique solution, always slow convergence, high computational time\(^{[15]}\) and also unstable of numerical computation\(^{[16]}\) in reconstruction of the detailed structures of the satellite image. The other disadvantages of spatially adaptive techniques in DF+C and IWPOCS methods are much greater pre-computation and searching processes than wavelet transform, and also the limited reference of the satellite image.

In this paper, an adaptive deblocking artifact method based on the intersection of confidence intervals (ICI) rule\(^{[17]–[19]}\) is discussed. The ICI rule is an adaptive procedure for selecting the appropriate adaptive scale parameter in each pixel of blocking artifact to get deblocking artifact. Nevertheless the methods in\(^{[17]–[19]}\) tend to have a result of low quality and high computational time of deblocking artifact. To overcome these drawbacks, we propose deblocking artifact based on nonlinear adaptive soft-threshold anisotropic filter in wavelet.

Our adaptive soft-threshold removes blocking artifact significantly from both distorted and undistorted satellite images in different sub-bands of wavelet transform. It is adaptable to high transitions signal criteria of blocking artifact, flat area, and texture area. The other contribution of this paper is a reconstruction technique of deblocking artifact by using an optimal ICI (OICI) method. Our OICI improves quality of estimated deblocking artifact which is determined by soft-threshold in confidence intervals for every scale parameter. For large scale parameter, soft-threshold in confidence intervals and deblocking artifact estimation are more ideal. Our OICI also reduces execution time of deblocking artifact estimation. In order to further improve estimated deblocking artifact performance, OICI determines scale parameter from the left or right side kernel only, while original ICIs\(^{[17]–[19]}\) determine scale parameter from both the left and right side kernels, which have higher computational time.

In computational complexity, we propose \( E \) \( \pm \) and \( RK4 \) \( \pm \) algorithms for the computation of elementary functions like exponential, logarithm, trigonometric functions, hyperbolic functions, and their reciprocals in fixed precision, typically the computer single or double precision. Our proposed method combines shift and add algorithms and both classical methods, Euler and Runge-Kutta methods for the numerical integration of ordinary differential equations (ODEs).

The paper is organized as follows. Related work is given in Sect. 2. Section 3 provides two steps of our proposal. The first step is proposed for deblocking artifact of...
satellite image by using soft-threshold in wavelet, followed by an adaptive anisotropic filter. The second step is reconstruction and smoothing, followed by LPA estimation, OICI algorithm, and fusion of deblocking artifact as final estimation. Section 4 shows our experimental results and algorithm performances. Conclusions are found in Sect. 5.

2. Related Work

2.1 Deblocking Artifact

In [20], [21] methods for deblocking artifact of image were developed by using the wavelet for solving the drawbacks in [10]–[14]. The method of [20] applies a variety of wavelet-based multitemporal DInSAR algorithms to remove artifacts, such as spatially correlated and temporally uncorrelated components, the digital elevation model, and orbital errors of an image. However, in various images which have low bit rates, the method of [20] often removes some part of undistorted satellite image, like texture area and flat area that are assumed as distorted images. The method of [21] also can remove blocking artifacts by using orthogonal projections of wavelets from an upside down pyramid-shaped region in a multi-dimensional space. The method of [21] also has drawbacks, such as lack of shift invariance, lack of wavelet symmetry, and poor directionality, when projection-based wavelet [21] uses dyadic wavelet transform [22]–[24], logarithmic dyadic wavelet transform (LDWT) [25], and dyadic bounded mean oscillating (BMO) [26]. Blocking artifact removal methods in [22]–[26] and complete wavelet representation in [27], [28] are not suitable for various undistorted satellite images which have texture area and flat area, either. Discrete and continuous-time soft-threshold [29] and a linear expansion of thresholds (LET) [30] can remove blocking artifact and treat the drawbacks of [22]–[28].

Another deblocking artifact method [31] uses linear filtering and its estimation. Unfortunately, it does not work with adaptation. Sparse representation [31] removes blocking artifacts from a set of training images by using K-singular value decomposition (K-SVD) and the orthogonal matching pursuit (OMP) for the estimated error threshold. However, since K-SVD is a linear representation of the data, K-SVD has to face nonlinear distributions of the data. It often leads to poor performance, it lacks the capability to separate different classes and also dictionary learning redundant. OMP builds the solution by adding one vector of estimated error threshold in every distorted image criterion and also OMP results depend on the elementary signals, or atoms, in dictionary learning [31]. However, it produces a big number of iterations, which is difficult to determine an appropriate threshold value of different distorted images. For a large number of data and iterations, performing an atom decomposition can take high computational time. In addition, they [22]–[31] did not consider soft-threshold criteria for different images and their levels of distorted images.

Thus, removing blocking artifact has been left problem for linear filtering [20]–[22], [24]–[31] in satellite image processing analysis. The linear filter methods [20]–[22], [24]–[31] give blurring effect in edges and other fine satellite image details. It is difficult for image analysis of linear filter to reconstruct an undistorted image from a distorted image. This has been treated in [32]–[35]. They [20]–[22], [24]–[31] allow us to find an alternative method by using nonlinear filtering. In particular, median filtering tends to give good results. Median filter is quite popular for nonlinear filtering in image processing applications [36] because it is simple and can preserve edges. A variety of median filters, such as stack filters and adaptive stack filters [37]–[39], the mean absolute error (MAE) of stack filter [40], multilevel median and FIR-median hybrid filter [41], adaptive switching median filter [42], optimal weighted vector median [43], [44], generalized Gaussian median filter [45], and relaxed median filter [46] have been developed to treat these drawbacks of [20]–[22], [24]–[31].

Although median filter and its various methods [37]–[46] are useful as nonlinear filter for satellite image denoising and enhancement, yet they also have some drawbacks. The median filter removes both distorted and undistorted satellite images when it cannot detect the differences between them. In case of satellite image of small size, it has minimal effect on the value of the median. This effect problem is still filtered out. The crucial element of estimations in nonlinear filter methods is adaptation.

2.2 Soft-Threshold

The soft-threshold is one of the most popular threshold methods used for deblocking artifact [47]. The performances of these methods [47], [48] are close to an ideal coefficient selection method if the coefficients of the observed blocking artifact signal are known in advance. The soft-threshold is more efficient than hard-threshold in deblocking artifact. The authors of [47], [48] gave the mechanism for finding a universal threshold value, which is called VisuShrink. VisuShrink depends on the noise power and the number of samples in the satellite image, or signal size. VisuShrink is a single value of threshold which is applied to all the wavelet coefficients. This is derived by proving an approximation error in the limit of an arbitrary large signal size. This method consists of the magnitudes of signal which are less than the threshold and the magnitudes of noise which are more than the given threshold. These methods [47], [48] would cut off the parts of the true signal and also leave some noise in distorted satellite image. In this paper, we propose an adaptive soft-threshold based on the level of distorted satellite image to overcome the drawbacks in [47], [48]. Our soft-threshold values use a nonlinear method which is adaptive to various satellite images and their blocking artifacts.

The authors of [29], [30] proposed soft-threshold for removing blocking artifacts. In [29], the iterative soft-threshold algorithm (ISTA) was presented for performing a
discrete gradient step followed by a soft-threshold operation at each iteration. At the same time, this method would pay high computational cost resulting in the slow convergence rate in fusion of de-blocking artifact. The method of [29] would be difficult to analyze the noise properties in distorted satellite image. In [30], a linear expansion of thresholds (LET) is an excellent estimate of the noiseless image by decomposing the denoising process and optimizing the coefficients of this representation using an estimate of the Stein’s unbiased risk estimate (SURE). Unfortunately, they showed unstable and slow convergence performance, in particular for various satellite images.

Thus, we propose the accelerated gradient of convergence in fusion step for treating these drawbacks of [29] and [30]. The accelerated gradient of convergence is a function value of a kernel which improves convergence performance in estimation, particularly in the fusion of de-blocking artifact. The accelerated gradient of convergence which is demonstrated in the kernel builds iteration steps. The step taken at each iteration depends on the previous iterations, where the accelerated gradient of convergence grows from one iteration to the next iteration. When we take the current iteration as the new starting point, this erases the previous iterations and resets the accelerated gradient of convergence back to zero.

Many adaptation techniques have been proposed. They used wavelet shrinkage [48], wavelet shrinkage and adaptive elliptical kernel for image smoothing [49], and generalized shrinkage-threshold for deblurring images [50]. The Lepski’s methods like 1D estimator [51], minimax adaptive estimation [52], spatial adaptation inhomogeneous smoothness [53], and pointwise adaptive [54] are adopted in our paper. Our adaptation is proposed based on a nonlinear estimation of distorted satellite image with its edge recovering. A nonlinear estimator is applied, which is derived from a local polynomial approach of satellite image in a wavelet window. Generalized linear model [55], estimating regression [56], smoothing adaptation [57], sharp adaptive estimation [58], signal dependent noise (SDN) model [59], and an adaptive jump-preserving (AJP) estimation [60] discussed the kernel by using an adaptive estimation. Yet, they did not apply Lepski’s methods as we did. The further idea of [51] was proposed as SDN model [59] for single image noisy estimation by using the noise level function of signal-dependent noise which assumed the generalized signal-dependent noise model and the Poisson-Gaussian noise model. Moreover the adaptive methods [55]–[60] are difficult to analyze mathematically because they apply a diffusion process by using derivatives of the evolving image for smoothing and enhancing the important feature of distorted satellite image, such as edges and the details of fine satellite images. To deal with these drawbacks of [55]–[60], we propose an adaptive nonlinear diffusion filter by using anisotropic diffusion filter. Our anisotropic adaptation filter method finds a point of estimation. Our polynomial component of signal fits well with the entire distorted satellite image. Distorted satellite image estimations are calculated for a grid of window size. Each grid is compared to the other grids of window size in wavelet. The adaptation window size is defined as the largest window size in the grid.

2.3 Intersection of Confidence Intervals (ICI)

The ICI rule [17]–[19] is an automatic adaptive procedure for selecting the appropriate adaptive filter of scale parameter \( m \) for each pixel in each satellite image in order to obtain a de-blocking artifact with minimal estimation error and scale parameter \( m \) is non-negative vector. Let us consider distorted satellite image \( d(r,s) \) and undistorted satellite image \( u(r,s) \), where \( r \) and \( s \) are the pixel indices. The absolute estimation error \( e \) of them can be calculated as follows:

\[
|e(r, s, m)| = |u(r, s) - \hat{d}(r, s, m)|
\]

where \( |\hat{d}(r, s, m)| \) is the estimated distorted satellite image, obtained using the scale parameter \( m \). As mentioned in [18], the absolute estimation error is

\[
|e(r, s, m)| \leq |\hat{E}(r, s, m)| + |e^0(r, s, m)|
\]

where \( |\hat{E}(r, s, m)| \) is the maximum value of the estimation bias and \( |e^0(r, s, m)| \) is random error with probability density \( N(0, \sigma^2_d(r,s,m)) \).

It was shown in [18] the following inequality holds

\[
|e^0(r, s, m)| \leq G_{1-\frac{\alpha}{2}} \sigma_{\hat{d}_m}(r,s,m)
\]

where \( G_{1-\frac{\alpha}{2}} \) is \((1-\frac{\alpha}{2})\)-th quantile of the standard Gaussian distribution and \( \sigma_{\hat{d}_m}(r,s,m) \) is the standard deviation of distorted satellite image in orientation \( \alpha \). The following inequality can be derived.

\[
|e(r, s, m)| \leq |\hat{E}(r, s, m)| + G_{1-\frac{\alpha}{2}} \hat{d}_m(r,s,m)
\]

We define an abbreviated Eq. (4) to Eq. (5) as follows:

\[
|e(r, s, m)| \leq \delta \sigma_{\hat{d}_m}(r,s,m)
\]

for \( m \leq m'(r,s) \), where \( m'(r,s) \) is the estimated scale parameter \( m \) of the estimated distorted satellite image \( \hat{d}(r,s,m) \) and soft-threshold \( \delta \) is used in confidence interval.

The adaptive scale parameter \( m \) procedure based on the ICI rule [61] introduces a finite set of parameter sizes and calculates a sequence of confidence intervals limits of the biased estimates for each satellite image pixel separately and independently to its left or right hand side. The one side of confidence interval limits of our proposed algorithm can be defined as follows:

\[
U_i(r, s, m_i) = \hat{d}_{m_i}(r, s, m_i) + \sigma_\delta \tau_{\hat{d}_m}(r,s,m_i),
\]

\[
L_i(r, s, m_i) = \hat{d}_{m_i}(r, s, m_i) - \sigma_\delta \tau_{\hat{d}_m}(r,s,m_i),
\]

where \( U_i(r, s, m_i) \) and \( L_i(r, s, m_i) \) are the \( i \)-th upper and lower confidence interval limits, respectively. \( \hat{d}_{m_i}(r, s, m_i) \) is estimated de-blocking artifact of scale parameter \( m \). We calculate the intervals and their intersections which are being
tracked with the following Eq. (8).

\[
\max_{i=1,...,D-1} L_i(r, s, m_i) \leq \min_{i=1,...,D-1} U_i(r, s, m_i)
\]  

(8)

The largest \( i \) is the proper scale parameter \( m \) in the number of deblocking artifact \( D \). The left or right hand side of distorted satellite image \( d(r,s) \) represents as blocking artifact which is combined and also used in estimated deblocking artifact \( \hat{d}(r,s, m) \). Soft-threshold \( \delta \tau \) has important role for confidence intervals and estimated deblocking artifact efficiency. For small values of soft-threshold \( \delta \tau \), variance increase and bias estimation of scale parameter \( m \) decrease.

Deblocking artifact by using ICI based method [18, 19] has high computational time. To deal with this drawback, we propose an optimal ICI (OICI) method. OICI method improves a quality level of estimated deblocking artifact and reduces estimated deblocking artifact computational time compared to the ICI method. Our OICI method does not only calculate scale parameter for each estimated deblocking artifact pixel like the ICI method, but detects the appropriate time regions for each pixel in time as well. It calculates the first pixel value in the exacted blocking artifact time region of distorted satellite image. It uses a scale parameter in the detected time region for all estimated undistorted satellite images pixel values. Thus, the optimal time region is an important role in our estimated deblocking artifact quality. Cross-validation method [18], [61] and local adaptive transform [19] used the ICI method. They were proven to be a good estimation improvement. However, these previous studies [18], [19], [61] have higher computational time of deblocking artifact estimation compared to our OICI method. We describe the pros and cons comparison between conventional and non-conventional deblocking artifact methods in Table 1.

### 2.4 Ordinary Differential Equations (ODEs)

In this paper we propose fixed precision which is provided by the float type on any computer. The previous study [62] mentioned that some methods could be employed by using Padé approximations. However, since a fixed number of digits is required, pre-computed tables can be used. These tables have a fixed number of entries that are used in the same fixed storage. Thus, these methods [62] have been developed for fixed precision computations of elementary functions (exponential, logarithm, trigonometric functions, hyperbolic functions, and their reciprocals). Shift and add methods [62] belong computations of elementary functions. The methods decompose their arguments into a number of decomposition which is performed by means of additions only. Furthermore, shift and add algorithms are computed along with the decomposition and requires only additions and multiplications or divisions by 2, which are realized on a computer by shifts. Since additions and shifts are very efficiently performed, the estimated iteration is very efficient. Moreover, the number of the estimated iterations is small.

### 3. Deblocking Artifact and Reconstruction Algorithms

Deblocking artifact of our proposed algorithms starts with the detection of blocking artifact discussed in our previous study [5]. In this section, firstly, we propose new algorithms of deblocking artifact by using soft-threshold, adaptive wavelet direction, and adaptive anisotropic filter. Secondly, we propose new algorithms of deblocking artifact reconstruction and estimations by using LPA estimation, OICI estimation, and fusion estimate. Our proposed algorithms are illustrated in Fig. 1.
3.1 Deblocking Artifact

3.1.1 Soft-Threshold

In our proposed algorithms, we have a wavelet transform and adaptive filter for wavelet analysis. Our wavelet filter should keep a low computational time in soft-threshold. Thus, we need a symmetrical wavelet filter to keep spatial positions of blocking artifact in different scales. In the decomposition process, we have three high-pass filters, namely, horizontal high-pass or vertical low-pass (HL), horizontal low-pass or vertical high-pass (LH), and horizontal high-pass or vertical high-pass (HH). These are very important for the exacted blocking artifact position as we mentioned in [5].

We denote high-low, low-high, and high-high subband as \( \{HL_L(i), LH_F(i), HH_F(i)\} \), \( \{HL_L(i), LH_F(i), HH_L(i)\} \), and \( \{HL_R(i), LH_R(i), HH_R(i)\} \) for flat area, texture area, and blocking artifact, respectively. Blocking artifacts appear at the horizontal and vertical of edge directions. We have verified and compared a specific distorted satellite image with other compressed satellite image in our previous study [5].

Distorted satellite image is indicated as blocking artifact. Flat area, e.g., plain, plateau, steppe, tableland, tundra, and texture area, e.g., forest, wave of the sea, and mountain are indicated as an undistorted satellite image or normal compressed satellite image. However, some detections of blocking artifact detect flat area and/or texture area as blocking artifact.

To overcome this drawback, we find the thresholds of flat area, texture area, and blocking artifact, respectively, as described in our previous study [5]. We determine threshold criteria which depend on the exacted blocking artifact location in every decomposition level \( i \)-th, by comparing wavelet coefficient values of flat area to the flat area threshold \( \delta \), the entropy of texture area to the entropy threshold \( \epsilon \), and blocking artifact to the blocking artifact threshold \( B \). The thresholds are determined by Eqs. (9)–(11). We define the relations of them in Eqs. (9)–(11), respectively.

\[
F = \begin{cases} 
|C_{HL_L}(r,s)| & \leq |C_{HL_L}(r,s)| \\
|C_{LH_F}(r,s)| & \leq |C_{LH_F}(r,s)| \\
|C_{HH_F}(r,s)| & \leq |C_{HH_F}(r,s)|
\end{cases}
\]

(9)

\[
T = \begin{cases} 
|C_{HL_L}(r,s)| & > |C_{HL_L}(r,s)| \\
|C_{LH_F}(r,s)| & > |C_{LH_F}(r,s)| \\
|C_{HH_F}(r,s)| & > |C_{HH_F}(r,s)|
\end{cases}
\]

(10)

\[
B = \begin{cases} 
|C_{HL_R}(r,s)| & > |C_{HL_R}(r,s)| \\
|C_{LH_R}(r,s)| & > |C_{LH_R}(r,s)| \\
|C_{HH_R}(r,s)| & > |C_{HH_R}(r,s)|
\end{cases}
\]

(11)

where \( C_{\text{subband}}(r,s) \) indicates the wavelet coefficient in the position of subband coordinate \((r,s)\).

We can obtain the soft-threshold values of flat area \( \delta F \), texture area \( \delta T \), and blocking artifact \( \delta B \), as described in Eqs. (14)–(16), respectively, from the wavelet decomposition process. After the wavelet decomposition, blocking artifact appears as horizontal line-shaped edge in the \( \text{LH} \) subband and vertical line-shaped edge in the \( \text{HL} \) subband. For the whole satellite images containing blocking artifacts, we calculate the three level wavelet decompositions. According to the wavelet decomposition, we block the high-low subband of each satellite image block together from left to right and then top to bottom. We also block the high-low, low-high, and high-high subbands of \( i \)-th level decomposition of the whole satellite images.

The high-low, low-high, and high-high subbands only appear on the block boundary in \( i \)-th level wavelet decomposition. The wavelet decomposition is a process downsampled by the length of the high-pass which equals to 3, and blocking artifacts appear on the block boundary.
The soft-threshold criteria of flat area $\delta F$, texture area $\delta T$, and blocking artifact $\delta B$ are determined by calculating difference of wavelet subbands coefficient between $HL_{Ft}(i)$ and $HL_{Ft}(i)$, $LH_{Ft}(i)$ and $LH_{Ft}(i)$, $HH_{Ft}(i)$ and $HH_{Ft}(i)$, $HL_{Ht}(i)$ and $HL_{Ht}(i)$, $LH_{Ht}(i)$ and $LH_{Ht}(i)$, $HH_{Ht}(i)$ and $HH_{Ht}(i)$, $HL_{Bt}(i)$ and $HL_{Bt}(i)$, $LB_{Bt}(i)$ and $LB_{Bt}(i)$, and the last one $HH_{Bt}(i)$ and $HH_{Bt}(i)$. Then, difference of wavelet subbands coefficient divided by $\{NHL_{Ft}(i), NHH_{Ft}(i), NHF_{Ft}(i), NHL_{Ft}(i), NHL_{Ht}(i), NHF_{Ht}(i), NHH_{Ht}(i), NLH_{Bt}(i), NHH_{Bt}(i), NHF_{Bt}(i), NHL_{Bt}(i), NHL_{Ht}(i), NHF_{Ht}(i), NHH_{Ht}(i)\}$, which stand for the numbers of the wavelet subbands coefficient of flat area, texture area, and blocking artifact, respectively.

We calculate flat area threshold $F_t$ in Eq. (14), entropy threshold $e_t$ in Eq. (15), and blocking artifact threshold $B_t$ in Eq. (16) by using probability density function (PDF) \[ f(\rho) = \frac{\beta^\gamma}{\Gamma(\gamma)} \rho^{\gamma-1} e^{-\beta \rho}; \rho > 0, \beta > 0, \gamma > 0 \] where the observed satellite image $\rho$ has a certain procedure consisting of $\gamma$ independent steps, and each step takes the number of exponential Gamma distributions $\beta$ per unit time. A complete Gamma function $\Gamma(\gamma)$ is defined as follows:

\[ \Gamma(\gamma) = \int_0^\infty \beta^\gamma \rho^{\gamma-1} e^{-\beta \rho} d\rho \]

Finally, to determine $F_t$, $e_t$, and $B_t$ in soft-threshold values of flat area, texture area, and blocking artifact in Eqs. (14)-(16), we use Eqs. (12) and (13). By using Eqs. (12) and (13), we got the distribution signals of $F_t = 0.25$, $e_t = 0.26$, and $B_t = 0.29$. Then, we define range values $0.23 < F_t < e_t < B_t < 0.31$ for the wavelet coefficient subband $C_{subband}(r,s)$ in the coordinate $(r,s)$.

The soft-threshold $\delta t$ in every decomposition level $i$-th of a satellite image can be defined as Eq. (17).

\[
\delta F(i) = \begin{cases} 
\delta HL_{F}(i) = F_t - \left( \| \sum_{cHL_{Ft}(i)(r,s) < cF_t} C_{HL_{Ft}(i)}(r,s) \| / N_{HL_{Ft}(i)} \right) \\
\delta LH_{F}(i) = F_t - \left( \| \sum_{cLH_{Ft}(i)(r,s) < cF_t} C_{LH_{Ft}(i)}(r,s) \| / N_{LH_{Ft}(i)} \right) \\
\delta HH_{F}(i) = F_t - \left( \| \sum_{cHH_{Ft}(i)(r,s) < cF_t} C_{HH_{Ft}(i)}(r,s) \| / N_{HH_{Ft}(i)} \right) 
\end{cases}
\]

\[
\delta T(i) = \begin{cases} 
\delta HL_{T}(i) = e_t + \left( \| \sum_{cHL_{Tt}(i)(r,s) > e_t} C_{HL_{Tt}(i)}(r,s) \| / N_{HL_{Tt}(i)} \right) \\
\delta LH_{T}(i) = e_t + \left( \| \sum_{cLH_{Tt}(i)(r,s) > e_t} C_{LH_{Tt}(i)}(r,s) \| / N_{LH_{Tt}(i)} \right) \\
\delta HH_{T}(i) = e_t + \left( \| \sum_{cHH_{Tt}(i)(r,s) > e_t} C_{HH_{Tt}(i)}(r,s) \| / N_{HH_{Tt}(i)} \right) 
\end{cases}
\]

\[
\delta B(i) = \begin{cases} 
\delta HL_{B}(i) = B_t + \left( \| \sum_{cHL_{Bt}(i)(r,s) > B_t} C_{HL_{Bt}(i)}(r,s) \| / N_{HL_{Bt}(i)} \right) \\
\delta LH_{B}(i) = B_t + \left( \| \sum_{cLH_{Bt}(i)(r,s) > B_t} C_{LH_{Bt}(i)}(r,s) \| / N_{LH_{Bt}(i)} \right) \\
\delta HH_{B}(i) = B_t + \left( \| \sum_{cHH_{Bt}(i)(r,s) > B_t} C_{HH_{Bt}(i)}(r,s) \| / N_{HH_{Bt}(i)} \right) 
\end{cases}
\]

3.1.2 Adaptive Wavelet Direction

Here, we propose 2D adaptive wavelet direction for the kernel which consists of two stages. First stage, or a 1D wavelet, is applied to blocking artifact followed by a vertical direction to obtain the vertical high-pass or horizontal low-pass subband (LH). The second stage, or the other 1D wavelet, is applied to blocking artifact followed by a horizontal direction to obtain the horizontal high-pass (HL) and horizontal or the vertical high-pass (HH). Each high-pass subband of the vertical or horizontal direction in wavelet can select a wavelet direction that detects blocking artifact. Blocking artifact can be represented in $d(x_0, x_i)$ coordinates of 2D distorted satellite image. We define an adaptive wavelet direction $\Psi$ as follows:

\[
\Psi_{m,\alpha}(x_0, x_i) = \frac{1}{\sqrt{m}} \left[ \frac{\alpha x_0 + x_i \sin \alpha}{m \delta t} \right]
\]

where $m$ is scale parameter or width of kernel in wavelet and non-negative vector. Scale parameter $m$ is represented as vectors $m_1$ and $m_2$. $\alpha$ is an orientation angle of kernel displacement. $x_0$ and $x_i$ are the first and $i$-th positions of kernel in wavelet direction, respectively, as shown in Fig. 2.

Scale parameter $m$ is determined by ridglet. Since the ridglet is constant along the ridge line $x_0 \cos \alpha + x_i \sin \alpha = 0$ and it is a wavelet along the orthogonal direction. Then, for small scale parameter $m$ which is identical with $m_2$, ridglet is quite clear and localized along the ridge line. For larger
scale parameter $m_1$, ridglet can be a smooth function with smooth waves. Thus, different scale parameter $m$ in ridglets can be used for directional approximations in smooth and straight-line edge area as we describe in Eqs. (19)–(22) as follows:

$m_1 = x_0 \cos \alpha + x_1 \sin \alpha$  
$m_2 = -x_0 \sin \alpha + x_1 \cos \alpha$  
$x_0 = m_1 \cos \alpha - m_2 \sin \alpha$  
$x_1 = m_1 \sin \alpha + m_2 \cos \alpha$

Figure 2 shows an adaptive wavelet which consists of two steps. The first step is a rotation of the window size function $w_m(x)$. The second step is the local polynomial approximation (LPA) design of the kernel with scale parameter $m$ as coordinate systems as we describe in Eqs. (19)–(22). Location of distorted pixels or blocking artifacts are inside of the rotation angle $\alpha$. The LPA in the variables $m_1$ and $m_2$ is used in order to obtain the accurate polynomial kernel for any discrete after rotation. The rotated kernels are produced with respect to the signals polynomial and also satisfy to the corresponding vanishing moment conditions of scale parameter $m$.

3.1.3 Adaptive Anisotropic Filter

We propose an adaptive anisotropic filter which uses an anisotrope measure of distorted satellite image level to control the shape of the kernel. The kernel $k$ is applied at each pixel of distorted satellite image $d$ with distorted level $\sigma$ in coordinate $(x_{0,d}, x_{1,d})$ which is defined as follows:

$$k(x_{0,d}, x_{1,d}) = w_m(x) \alpha(x_{1,d} - x_{0,d}) \cdot \exp(-\delta \tau) \cdot \left( \frac{(x_{1,d} - x_{0,d})m_1^2}{\sigma^2_{1,d} (x_{0,d})} + \frac{(x_{1,d} - x_{0,d})m_2^2}{\sigma^2_{2,d} (x_{0,d})} \right)$$  

where the window size function $w_m(x)$ of neighborhood $x$ in Eq. (23) is identical to that in Eq. (24) as follows:

$$w_m(x) = \frac{x}{m} = \frac{x}{m^2}$$

The symbol $\alpha(x_{1,d} - x_{0,d})$ in Eq. (21) represents a positive direction and the kernel rotation function where the condition $\alpha(x_i) = 1$ if $|x_i| \leq m_1$, and $m_1$ is the maximum scale parameter $m$. Distorted satellite image parameters of $\sigma^2_{1,d} (x_{0,d})$ and $\sigma^2_{2,d} (x_{0,d})$ are used to control the shape of kernel $k(x_{0,d}, x_{1,d})$.

3.2 Reconstruction

3.2.1 Local Polynomial Approximation (LPA) Estimation

Our idea of LPA estimation for deblocking artifact is to fit a polynomial to distorted satellite image in the neighborhood $x$ and use it to estimate the value of the blocking artifact at the considered point. The polynomial is developed locally, as a linear combination of the considered point and fitted using the weighted least squares (WLS) method. We define that the quantization noise in blocking artifact is the sum of undistorted satellite image and distorted satellite image which corresponds to standard deviation $\sigma$. The observed satellite image $p$ is expressed as follows:

$$p(r, s) = u(r, s) + \sigma d(r, s)$$

where $u$ is undistorted satellite image and $d$ is distorted satellite image or noise component of satellite image. Particularly, distorted satellite image is blocking artifact. $(r, s)$-th is the pixel coordinate in the block of satellite images and noise $\sigma$. Equation (25) and Taylor’s series in [59], [60] are used for an approximation of deblocking artifact function in distorted satellite image $d$ and neighborhood $x$, as follows:

$$d(r, s) \approx d(x) + d_1(x) \left( \frac{(x - (r, s))}{1!} \right) + d_2(x) \left( \frac{(x - (r, s))^2}{2!} \right) + d_3(x) \left( \frac{(x - (r, s))^3}{3!} \right) + \ldots$$

where $d$ is the function of distorted satellite image and $d_1$, $d_2$, and $d_3$ are the first, second, and third derivative functions of distorted satellite image, respectively. $d_{\hat{d}}$ is the $\hat{d}$-th of distorted satellite image $d$ estimation which can be given in the kernel $k$ of nonlinear filter as follows:

$$\hat{d}_{\hat{d}}(x) = \sum_n k(x) p_n; \quad \hat{d} = 1, 2, 3, \ldots, \infty$$

Thus, the directional LPA kernel $k_{m,\alpha}(x)$ is defined as $d_{\hat{m},\alpha}(x, m) = (k_{m,\alpha} * p)(x, m)$. Kernel $k_{m,\alpha}(x)$ and the observed satellite image $\rho$ are convolved (*) at direction $\alpha(x)$, neighborhood $x$, and scale parameter $m$. The derivative directional LPA kernel $k_{m,\alpha}$ of estimated scale parameter $m_{\hat{d}}$ in $\hat{d}$-th estimation can be defined as follows:

$$\hat{d}_{\hat{m},\alpha}(x, m_{\hat{d}}) = (k_{m,\alpha} * p)(x, m_{\hat{d}}); \quad \hat{d} = 1, 2, 3, \ldots, \infty$$
We also obtain estimated variance in $n$ number of deblocking artifacts which is computed as follows:

$$\sigma_{\hat{d}_{m,i\alpha}}^2(i,m_{\hat{d}}) = \sigma^2 \sum_{i \in \rho} [k_{m_{\hat{d}}(n)}]^2;$$  
(29)

$$\hat{d} = 1, 2, 3, \ldots, \infty$$

where $\rho^d$ is a set of $i$-th observed satellite images $\rho$ which is ranked by distorted satellite image $\hat{d}$. Thus, our derivative estimated variances of Eq. (29) do not depend on neighborhood $x$.

### 3.2.2 Optimal Intersection of Confidence Interval (OICI)

The adaptive scale parameter $m$ of each distorted satellite image can be seen in Eqs. (6)–(7). The intervals and their intersections have calculated in Eq. (8).

Our OICI method finds scale parameter $m$ by calculating the confidence intervals from the left or the right side only. However, the original ICI based method calculates them from both the left and the right sides. The same procedure is repeated for the left or the right side of our deblocking artifact in distorted satellite image pixel $d(r,s,m)$. The smaller values of the confidence interval soft-threshold $\delta r$ can increase variance $\sigma$ in scale parameter $m$. On the other hand, the larger values of the confidence interval soft-threshold $\delta r$ reduce variance in scale parameter $m$. Our OICI method does not only calculate scale parameter for each deblocking artifact pixel like ICI method, but detects the appropriate time regions for each pixel as well. Deblocking artifact pixel in time $t(r,s)$ of the confidence intervals is calculated by using our OICI method as follows:

$$U_i(r, s, m_i) = \hat{d}_m(r, s, m_i) + \frac{\sigma_m}{2\delta t^2}(r, s, m_i),$$  
(30)

$$L_i(r, s, m_i) = \hat{d}_m(r, s, m_i) - \frac{\sigma_m}{2\delta t^2}(r, s, m_i)$$  
(31)

Based on Eqs. (30) and (31), our adaptive scale parameter $m$ is calculated from the first to all pixel values in the detected time region of deblocking artifact from the left or the right side. Our OICI size ratio is calculated as follows:

$$OICI = \left( \frac{\min U_i - \max L_i}{U_i(x_i, m_i) - L_i(x_i, m_i)} \right)^i$$  
(32)

Our OICI based deblocking artifact algorithms which start by considering distorted satellite image pixel value in time which is denoted as $\hat{d}_m = (r, s, m_i)$, where $i = 1, 2, \ldots, D$, and $D$ is the number of deblocking artifacts. OICI calculates upper and lower confidence intervals of each $i$ by using the procedure in Eq. (8). If $m_i \neq D$, the procedure in Eq. (8) is repeated until $m_i = D$ as shown in Eq. (33).

$$\sum_{i=1}^{t(r,s)} m_i = D$$  
(33)

where $t(r,s)$ is the number of detected time regions for the deblocking artifact pixel. Our OICI algorithm is expressed as follows.

### Algorithm 1 OICI

1: Initialize $\hat{d}_m, \sigma, \delta r, x, D, oici$
2: Declare $i : \text{integer}$ $\triangleright$ discrete value of deblocking artifact
3: Declare $x : \text{integer}$ $\triangleright$ number of deblocking artifacts
4: $D : \text{integer}$
5: while $x \notin D$ do
6:    $i \leftarrow 1$
7:    $\max Li \leftarrow -\infty$
8:    $\min U i \leftarrow +\infty$
9:    while $\max Li \leqslant \min U i$ do
10:        $U_i(r, s, m_i) \rightarrow \hat{d}_m(x) + \frac{\sigma_m}{2\delta t^2}(x, s, m_i)$
11:        $L_i(r, s, m_i) \rightarrow \hat{d}_m(x) - \frac{\sigma_m}{2\delta t^2}(x, s, m_i)$
12:        $\max Li \rightarrow \max[L_i(x), \max Li]$
13:        $\min U i \rightarrow \min[U_i(x), \min U i]$
14:    $oici \leftarrow \left\{ \frac{\min U i - \max Li}{\sum_{i=1}^{t(r,s)} m_i} \right\}$
15:    $i \rightarrow i + 1$
16: end while
17: $x \rightarrow x + 1$
18: end while
19: $\hat{d}(s) \rightarrow \hat{d}_m(x, m_i)$
20: $x \rightarrow x + 1$
21: end while
22: return $\hat{d}(s)$

Our OICI reduces computational time for $1 \leq t(r, s) \leq D$ at $t(r, s)$ time. Then, we have a directional estimated kernel of our OICI by using Eq. (28). Estimated variance of OICI is the adaptive scale which is calculated as follows:

$$\sigma_{\hat{d}_{m,i\alpha}}^2(x, m_{\hat{d}}) = \sigma^2 \sum_{i=1}^{m} k_{m_{\hat{d}}(n)}(x, m_{\hat{d}});$$  
(34)

$$\hat{d} = 1, 2, 3, \ldots, \infty$$

The adaptive scale estimate in Eq. (34) is the combined estimates which are exploited to obtain the final estimated deblocking artifact in the fusion process.

### 3.2.3 Fusion of Deblocling Artifact

Estimated fusion is a final estimation of estimated deblocking artifact which is computed as follows:

$$\hat{d}_{m,i\alpha}(x, m_{\hat{d}}) = (k_{m_{\hat{d}}(n)} \ast \delta r(m_{\hat{d}}^{m-1}))(x, m_{\hat{d}});$$  
(35)

$$\hat{d} = 1, 2, 3, \ldots, \infty$$

Finally, we calculate new estimated variance by using soft-threshold $\delta r$ in fusion of deblocking artifact as follows:

$$\sigma_{\hat{d}_{m,i\alpha}}^2(x, m_{\hat{d}}) = \delta r(k_{m_{\hat{d}}(n)} \ast \sigma_{\hat{d}_{m,i\alpha}}^2(m_{\hat{d}}^{m-1}))(x, m_{\hat{d}});$$  
(36)

Equation (36) is estimated fusion variance of OICI.
3.3 Integration of ODE

First, we use Euler and Runge-Kutta-4 (RK4) methods. They are a family of implicit and explicit iterative methods for the integration of an ODE. Second, we use shift and add methods which cover algorithms for exponential and logarithm as well as a CORDIC method for trigonometric functions. We propose an algorithm as well as a CORDIC method for trigonometric functions. They are a family of implicit and explicit iterative methods for the integration of an ODE. Second, we use shift and add methods which cover algorithms for exponential and logarithm as well as a CORDIC method for trigonometric functions.

We propose Euler + + (E + +) and Runge-Kutta of order 4++ (RK4++) methods which are combined with shift and add methods. In an ODE, y is a vector. y(x) = [y_1(x), y_2(x), ..., y_m(x)] is function. F is a vector valued function of y and its derivatives in n order of differential equation described as follows:

\[
y^{(n)} = F(x, y, y', y'', ..., y'^{(n-1)})
\]

or Eq. (38) can be written as follows:

\[
y' = F(t, y)
\]

where y' is the first derivative of y. y is a function of one variable t (one-dimensional) to real \( \mathbb{R}^p \). F is a given function which is defined on a domain \( \mathbb{R}^p \subset \mathbb{R} \times \mathbb{R}^p \) to \( \mathbb{R}^p \). Usually, an initial condition is written as \( y(t_0) = \eta \in \mathbb{R}^p \). The corresponding Cauchy problem [64] or initial value problem is as follows:

\[
\begin{align*}
\text{Cauchy} & : y' = F(t, y) \\
y(t_0) &= \eta
\end{align*}
\]

Let interval factor \( [\zeta; \theta] \subset \mathbb{R} \) with \( t_0 \in [\zeta; \theta] \), a \( C^1 \) function \( Y : t \in [\zeta; \theta] \rightarrow Y(t) \in \mathbb{R}^p \) is a solution if

\[
\begin{align*}
(t, Y(t)) & \in \mathcal{D}_F \\
y(t_0) &= \eta \\
Y(t) &= F(t, Y(t))
\end{align*}
\]

To integrate an ODE, numerical methods obtain approximate values of the solution at a set of points \( t_0 < t_1 < \cdots < t_n < \cdots < t_N \). The approximate value \( y_n \) of \( Y(t_n) \) is computed by using some of the values obtained from \( Y(t_{n-1}) \). Our explicit \( E + + \) and RK4 + + methods are considered in the initial value problem \( Y(t) \).

3.4 Explicit Euler and Runge-Kutta Methods

Explicit Euler method is a single step method of order 1 defined on \( [t_0; \xi] \) by

\[
\begin{align*}
y_{n+1} &= y_n + hF(t_n, y_n) \\
y(t_0) &= \eta
\end{align*}
\]

where \( h \) is the size of every step, \( \xi \) is a set of points in \( n \) number of point, \( \xi \) is the last range a set of points, and \( N \) is number of step. We define \( h = \frac{\xi - t_0}{N} \) and \( t_n = t_0 + nh \).

Runge-Kutta method aims to achieve a given order evaluating any derivatives of \( F \) and involves the evaluation of \( F \) at intermediary points. Among these methods, RK4 is 4 orders.

\[
\begin{align*}
\mathcal{K}_1 &= F(t_n, y_n) \\
\mathcal{K}_2 &= F(t_n + \frac{h}{2}, y_n + \frac{\mathcal{K}_1}{2}) \\
\mathcal{K}_3 &= F(t_n + \frac{h}{2}, y_n + \frac{\mathcal{K}_2}{2}) \\
\mathcal{K}_4 &= F(t_n + h, y_n + h\mathcal{K}_3)
\end{align*}
\]

(42)

We consider the computation of \( Y(t) \) for any \( t \in [t_0; \xi] \). \( E + + \) and RK4 + + methods compute \( Y(t_K) \) where \( t_K \) is an intermediary point used to reach \( t \). We consider only the error \( E = y_n - Y(t_n) \) in order to choose \( h \). For \( E + + \), \( h \) has to satisfy \( h \leq \frac{2E}{2^{24}y_j} \). The maximum of \( || \cdot ||_\infty \) is \( [t_0; \xi] \). For RK4 + +, \( h \) has to satisfy a more complex inequality \( h \leq \frac{\sqrt[24]{28800}}{2^{24}||y_j||_\infty} \). The sums of \( h \) for a given precision \( E = 2^{-24} \) correspond to the single precision (32 bits) and for a given precision \( E = 2^{-53} \) correspond to the double precision (64 bits) of each elementary function in our explicit \( E + + \) and RK4 + + methods.

4. Experimental Results

We used 200 reference distorted satellite images which had blocking artifacts and each size was 32x32, 64x64, 128x128, 256x256, 512x512, and 1024x1024 pixels [65] in our experiment.

We demonstrate deblocking artifact algorithms performance five blocking artifact satellite images to represent 200 distorted satellite images [65]. The size of each satellite image is 512x512 pixels. Deblocking artifacts show that blocking artifacts were removed. The estimated deblocking artifacts show the resulting of deblocking artifacts which are estimated and reconstructed by our LPA and OICI. We calculate our deblocking artifact and estimated deblocking artifact performance by using root-mean-square error (RMSE), signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR), improvement in signal-to-noise ratio (ISNR), mean absolute error (MAE), and maximum MAE (MaxMAE).

We compare our proposed algorithms to conventional methods [14], [28] which are no estimation nor adaptation, and also to non-conventional methods [18], [19], [30], [59], [64] which use adaptive filtering and estimation. We use non-conventional methods of an adaptive soft-threshold which deals for deblocking artifact. Our OICI is adaptable to a crucial element of nonlinear filtering in reconstruction and estimation. The other non-conventional methods [18], [19], [30], [59], [64] have left larger distorted effect and also have removed finer signals of estimated deblocking artifact in reconstruction and estimation.

In Fig. 3, we show blocking artifacts which consist of detected blocking artifacts (a.1-e.1) denoted by red regions, deblocking artifacts (a.2-e.2) denoted by yellow regions, and final estimated deblocking artifacts (a.3-e.3) denoted by green regions. Based on Fig. 3 (a.2-e.2), we calculate SNR,
PSNR, MSE, and standard deviations $\sigma$ which start from deblocking artifact level as we show in Tables 2 and 3. Final estimated deblocking artifact evaluation of Fig. 3 (a.3-e.3) are shown in Tables 4–8. Tables 2 and 3 show that our proposed deblocking artifact algorithms have higher SNR and PSNR values and lower MSE and standard deviation values than other methods.

Our proposed deblocking artifact algorithms have finer signal of the deblocking artifacts. Table 4 shows SNR (dB) and PSNR (dB) values which have a significant increasing SNR and the PSNR values compared to our deblocking artifact in Table 2 and also other methods in Table 4. Table 5 shows MSE and standard deviation $\sigma$ values of our final estimated deblocking artifact algorithms Fig. 3 (a.3-e.3) which have decreased MSE values compared to our deblocking artifact in Table 3 and also other methods in Table 5. We
Table 4 SNR (dB) and PSNR (dB) values of estimated deblocking artifact

<table>
<thead>
<tr>
<th>Method</th>
<th>SNR</th>
<th>PSNR</th>
<th>SNR</th>
<th>PSNR</th>
<th>SNR</th>
<th>PSNR</th>
<th>SNR</th>
<th>PSNR</th>
<th>SNR</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>53.1913</td>
<td>54.1965</td>
<td>48.4288</td>
<td>49.4319</td>
<td>54.5921</td>
<td>55.5964</td>
<td>49.7015</td>
<td>50.7068</td>
<td>49.9274</td>
<td>50.9327</td>
</tr>
<tr>
<td>LET [30]</td>
<td>37.2339</td>
<td>37.9376</td>
<td>33.9001</td>
<td>34.6023</td>
<td>38.2145</td>
<td>38.9175</td>
<td>34.7911</td>
<td>35.4948</td>
<td>34.9492</td>
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</tr>
<tr>
<td>LPA-ICI [18], [19]</td>
<td>42.5530</td>
<td>43.3572</td>
<td>39.7430</td>
<td>39.5455</td>
<td>43.6737</td>
<td>44.4771</td>
<td>39.7612</td>
<td>40.5654</td>
<td>39.9419</td>
<td>40.7462</td>
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</table>

Table 5 MSE and standard deviation ($\sigma$) values of estimated deblocking artifact

<table>
<thead>
<tr>
<th>Method</th>
<th>Fig. 3.a.3</th>
<th>Fig. 3.b.3</th>
<th>Fig. 3.c.3</th>
<th>Fig. 3.d.3</th>
<th>Fig. 3.e.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>1.1131</td>
<td>0.9086</td>
<td>1.8255</td>
<td>0.4428</td>
<td>0.9526</td>
</tr>
<tr>
<td>LET [30]</td>
<td>80.9297</td>
<td>785.6270</td>
<td>140.8200</td>
<td>63.2526</td>
<td>128.5639</td>
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<tr>
<td>LPA-ICI [18], [19]</td>
<td>41.1003</td>
<td>638.4020</td>
<td>71.5191</td>
<td>31.6971</td>
<td>64.4259</td>
</tr>
<tr>
<td>SDN [59]</td>
<td>40.5535</td>
<td>393.6929</td>
<td>70.5676</td>
<td>31.6971</td>
<td>64.4259</td>
</tr>
<tr>
<td>MWD [66]</td>
<td>68.2413</td>
<td>662.4865</td>
<td>118.7476</td>
<td>31.6971</td>
<td>64.4259</td>
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</table>

Table 6 RMSE values of estimated deblocking artifact

<table>
<thead>
<tr>
<th>Method</th>
<th>Fig. 3.a.3</th>
<th>Fig. 3.b.3</th>
<th>Fig. 3.c.3</th>
<th>Fig. 3.d.3</th>
<th>Fig. 3.e.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>0.0735</td>
<td>2.1337</td>
<td>0.0506</td>
<td>1.2481</td>
<td>1.2481</td>
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<tr>
<td>LET [30]</td>
<td>2.1317</td>
<td>61.8764</td>
<td>1.4683</td>
<td>2.0037</td>
<td>2.0037</td>
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<td>LPA-ICI [18], [19]</td>
<td>1.9847</td>
<td>57.6091</td>
<td>1.8655</td>
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</tr>
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</table>

Table 7 MAE and MaxMAE values of estimated deblocking artifact

<table>
<thead>
<tr>
<th>Method</th>
<th>Fig. 3.a.3</th>
<th>Fig. 3.b.3</th>
<th>Fig. 3.c.3</th>
<th>Fig. 3.d.3</th>
<th>Fig. 3.e.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>0.0735</td>
<td>2.1337</td>
<td>0.0506</td>
<td>1.2481</td>
<td>1.2481</td>
</tr>
<tr>
<td>LET [30]</td>
<td>2.1317</td>
<td>61.8764</td>
<td>1.4683</td>
<td>2.0037</td>
<td>2.0037</td>
</tr>
<tr>
<td>LPA-ICI [18], [19]</td>
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<td>57.6091</td>
<td>1.8655</td>
<td>1.8655</td>
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</table>

Table 8 Computational time (CT) in second and ISNR (dB) values of estimated deblocking artifact

<table>
<thead>
<tr>
<th>Method</th>
<th>CT</th>
<th>ISNR</th>
<th>CT</th>
<th>ISNR</th>
<th>CT</th>
<th>ISNR</th>
<th>CT</th>
<th>ISNR</th>
<th>CT</th>
<th>ISNR</th>
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<tbody>
<tr>
<td>Proposed Method</td>
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<td>38.8914</td>
<td>0.5948</td>
<td>33.0773</td>
<td>0.5593</td>
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<td>0.5525</td>
<td>29.9380</td>
<td>0.5773</td>
<td>31.3851</td>
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<tr>
<td>LET [30]</td>
<td>42.8919</td>
<td>30.3699</td>
<td>44.2291</td>
<td>26.5314</td>
<td>41.6058</td>
<td>27.7328</td>
<td>41.1003</td>
<td>25.3046</td>
<td>42.9326</td>
<td>26.0489</td>
</tr>
<tr>
<td>LPA-ICI [18], [19]</td>
<td>42.6773</td>
<td>31.1131</td>
<td>44.0145</td>
<td>26.4618</td>
<td>41.3912</td>
<td>23.3690</td>
<td>40.8857</td>
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<tr>
<td>SDN [59]</td>
<td>43.9897</td>
<td>11.3617</td>
<td>45.3269</td>
<td>12.1972</td>
<td>42.7036</td>
<td>17.8019</td>
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<td>15.7027</td>
<td>44.0304</td>
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<tr>
<td>MWD [66]</td>
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<td>2.0435</td>
<td>46.9138</td>
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<td>7.3235</td>
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<td>0.3543</td>
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</tbody>
</table>

achieved 98.65% (Fig. 3.a.3), 98.72% (Fig. 3.b.3), 99.31% (Fig. 3.c.3), 99.27% (Fig. 3.d.3), and 98.72% (Fig. 3.e.3) MSE improvement. Thus, Tables 4 and 5 show that our estimated deblocking artifact algorithms have lower error and higher quality level than the other methods.

Table 6 shows RMSE values of our estimated deblocking artifact which compared to other methods. We achieved 88.36% (Fig. 3.a.3), 88.70% (Fig. 3.b.3), 91.70% (Fig. 3.c.3), 91.46% (Fig. 3.d.3), and 88.70% (Fig. 3.e.3) RMSE improvement. Thus, our proposed algorithms have lower RMSE values than the other methods. This means that our proposed algorithms have lower error than other methods. Table 7 shows MAE and MaxMAE values which achieve improvement over 90% of lower MSEs and standard deviations $\sigma$ of deblocking artifact as shown in Table 3 compared with both conventional [14], [28] and non-conventional methods [18], [19], [30], [59], [64].

Our deblocking artifact outperforms other methods which have many errors in classifying and removing of blocking...
artifact. Our proposed method successfully outperforms in terms of reducing MSEs, standard deviations, RMSE values, MAE and MaxMAE values of estimated deblocking artifact as shown in Table 5, Table 6, and Table 7, respectively, compared to other non-conventional methods [18], [19], [30], [59], [64]. We use non-conventional methods of an adaptive soft-threshold which deals for deblocking artifact in various satellite images, both undistorted and distorted satellite images.

Furthermore, computational times of estimated deblocking artifacts are shown in Table 8. We used a personal computer which had the specifications of Processor Intel
### Table 9
The values of $I$ correspond to the single and double floating-point numbers for various elementary functions by using $E + +$ and RK4 $+ +$.

<table>
<thead>
<tr>
<th>Method</th>
<th>Function</th>
<th>$\exp$</th>
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<tr>
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<tr>
<td>$\text{LET}_{[30]}$</td>
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</table>

### Table 10
Elementary functions times ($\mu$ second) of $E + +$ and RK4 $+ +$ algorithms.

<table>
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<tr>
<th>Method</th>
<th>Function</th>
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<tr>
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<td>$0.1$</td>
<td>$0.2$</td>
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<td>$0.5$</td>
<td>$0.6$</td>
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<td>$0.8$</td>
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<td>$1.0$</td>
</tr>
<tr>
<td>$\text{LET}_{[30]}$</td>
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<td>$0.2$</td>
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### Equation
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\]
Core i7-2600K@3.40 GHz and 8 GB DDR3 RAM. Our proposed algorithms show smaller computational time than the other methods. We achieved 77.95% (Fig. 3.a.3), 77.83% (Fig. 3.b.3), 78.08% (Fig. 3.c.3), 78.12% (Fig. 3.d.3), and 77.94% (Fig. 3.e.3) computational time improvement compared to the other methods. Our OICI estimates scale parameter m from one way, from the left or right side kernel only. In contrast, original ICIs [17–19] and other methods [30], [59], [64] have a big number of iterations in estimation as shown in Fig. 5. Also, the ISNR values of our proposal are positive and higher values of SNR up to 30 dB than the other methods.

In Table 9, \( Y(t) \) was computed with the required accuracy \( 2^{-n} \). In each step \( \mathcal{K} \) of the shift and add algorithms, the absolute error on \( t \) is \( |t - t_K| \leq 2^{-K+1} \) with \( t_K = \sum_{j=0}^{K} c_K \) and the value computed at step \( \mathcal{K} \), \( y_K \), was exactly equal to \( Y(t_K) \). It was determined the step \( h \) of the numerical integration method corresponding to a method error less than \( 2^n \). Let \( I \) be the smallest integer for \( h > 2^{I+1} \). Shift and add algorithms are stopped after the \( I-th \) iteration by giving \( t_I \) and \( y_I \). If an iteration of the numerical integration method is performed by using a step \( h' = t - t_I \), the error is bounded by \( |y_{I+1} - Y(t)| \leq 2^{-n} \).

We have determined the value of \( I \) which corresponded to the single (32 bits) and double (64 bits) IEEE floating-point numbers, for the various elementary functions when the numerical integration method is either explicit \( E++ \) or \( RK4++ \). The error has to be less than \( 2^{-24} \) for the single precision and less than \( 2^{-53} \) for the double precision. Table 9 shows our \( E++ \) and \( RK4++ \) algorithms which compute floating-point numbers for various elementary functions.

We have estimated the number of clock cycles, \( E++ \), \( RK4++ \), and shift and add methods were performed in one clock cycle, whereas a division was ten times longer. We observe that this is in good performance with the experimental times. Our \( E++ \) and \( RK4++ \) algorithms took over shift and add ones. Furthermore, our \( E++ \) and \( RK4++ \) algorithms were computational time of the shift and add ones. Furthermore, \( RK4++ \) algorithm is superior to \( E++ \) algorithm as described in Table 10.

5. Conclusions

We proposed new deblocking artifact algorithms by using adaptive soft-threshold anisotropic filter values in wavelet. Our deblocking artifact algorithms outperform other methods, both conventional and non-conventional methods. Our deblocking artifact algorithms are adaptable to different blocking artifact in distorted and undistorted satellite image. In reconstruction and estimation, we proposed OICI for estimated deblocking artifact. Our OICI improves MSE of estimated deblocking artifact up to 98%, RMSE up to 89%, and MAE up to 99%. Computational time was reduced up to 77.98% compared to the other methods.

We have accelerated shift and add algorithms by substituting some iterations \( E++ \) and \( RK4++ \) algorithms. These combine the advantages of the two kinds of methods, without suffering their drawbacks. Our \( E++ \) and \( RK4++ \) algorithms have performed less steps, but shift and add methods have performed more steps. We have compared the number of clock cycles required by each method. On a processor, our \( E++ \) and \( RK4++ \) algorithms have improved and reduced computational time in terms of the shift and add. \( RK4++ \) algorithm is superior to \( E++ \) algorithm.

References


[34] C. Xie, W. Zhong, and K. Mueller, “A Visual analytics approach for categorical joint distribution reconstruction from marginal projec-


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