PAPER

Price-Based Power Control Algorithm in Cognitive Radio Networks via Branch and Bound

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SUMMARY Price-based power control problem is investigated in the spectrum sharing cognitive radio networks (CRNs) by Stackelberg game. Using backward induction, the revenue function of the primary user (PU) is expressed as a non-convex function of the transmit power of the secondary users (SUs). To solve the non-convex problem of the PU, a branch and bound based price-based power control algorithm is proposed. The proposed algorithm can be used to provide performance benchmarks for any other low complexity sub-optimal price-based power control algorithms based on Stackelberg game in CRNs.

key words: power control, cognitive radio, branch and bound, Stackelberg game

1. Introduction

With the development of wireless communication technology and increasing demand for wireless services, spectrum becomes more and more scarce, and it needs to be used efficiently. It is found that the utilization of the spectrum is low most of the time [1] by the Federal Communications Commission (FCC). Thus, the technology of cognitive radio networks (CRNs) [2] is proposed to improve spectrum efficiency and solve the problem of spectrum scarcity.

In underlay CRNs, the secondary users (SUs) can access the spectrum of the primary users (PUs) under the interference power constraint [3]. Some SUs in underlay CRNs may improve data rate by increasing their transmit power. However, this will cause more interference to the PUs and other SUs. Therefore, transmit power control is a crucial technique to optimize the performance of the underlay CRNs. There are basically two technical approaches to power control for underlay CRNs. One method is handled by optimization theory [4], other one is applied by game theory [5]. Power control in CRNs by optimization theory has been investigated in [6]–[10]. The authors in [6] studied optimal power control policies to maximize the achievable rates of underlay CRNs with arbitrary input distributions under interference power constraints for general fading distributions. An optimal power control algorithm are designed to maximize the energy efficiency for green CRNs in fading channels in [7]. The sum rate maximization problem of the SUs for wireless powered underlay CRNs is inves-

tigated in [8] under the interference power constraint. In [9], the authors propose two optimal power control schemes for cognitive satellite terrestrial networks to maximize the delay-limited capacity and outage capacity. A robust energy efficiency power allocation algorithm was proposed for underlay CRNs with channel uncertainty in [10]. Because the game theory can be used to model and analyze the interaction among SUs and PUs, it has been used to model the power control problem in CRNs [11], [12]. A fully distributed power control algorithm from the perspective of game theory is proposed in [11] to solve the energy efficiency maximization problem of CRNs. In [12], power control and relay selection algorithm are designed by game theory to optimize the capacity of SUs in a cognitive radio ad hoc network.

Because the priority level of the PUs is higher than the SUs, the PU can charge for SU's interference power with a suitable price to maximize its revenue. Price-based power control in CRNs by Stackelberg game was investigated in [13]–[18]. In [13], the authors investigated the pricing issue for the power control problem in code division multiple access (CDMA) based CRNs. A sub-optimal price-based power control algorithm was proposed to maximize the revenue of the base station (BS). In [14], a novel price-based power control algorithm was proposed to improve the revenue of both the BS and SUs compared with the algorithm proposed in [13]. In [15], we proposed a new price-based power control algorithm for a more general case of CDMA based CRNs system model compared with [13], [14]. However, the algorithms proposed in [13]–[15] are valid only for CDMA based CRNs, and they cannot handle the more general model such as Ad hoc based femtocell and CRNs [16]–[18]. Since the revenue of the PU is non-convex function, it is difficult to find the optimal pricing algorithm, and the algorithms proposed [13], [14], [18] are sub-optimal. For some special cases, we have proposed optimal price-based power control algorithms in CRNs [15]–[17]. However, the algorithm proposed in [15] can be not used for Ad hoc based CRNs. The algorithm proposed in [16] is optimal only if the minimum signal-to-interference and noise ratio (SINR) of SUs is larger than 0 dB. In [17], we proposed a novel price-based power algorithm based on the monotonic optimization. However, the proposed algorithm in [17], [18] does not consider the minimum SINR constraints of SUs. In this paper, we propose an optimal price-based power control algorithm based on branch and bound to maximize the revenue of the PU under the mini-

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imum SINR constraints of SUs and interference power constraint (IPC) at the receiver of the PU. The system model considered in [13]–[18] can be seen as a special case as we considered in this paper. To be our best knowledge, the proposed price-based power control algorithm based on branch and bound is first algorithm that can find the optimal prices for Ad hoc based CRNs under interference power constraint and SINR constraints. Therefore, the proposed algorithm can be also used to provide performance benchmarks for any other low complexity sub-optimal price-based power control algorithms in CRNs. Moreover, the proposed algorithm can be used to handle the price-based power control with minimum SINR constraints and interference power constraint for other types of wireless networks such as CDMA underlying a primary network [13]–[15], femtocell [18], device-to-device (D2D) communication underlying a cellular network [19], and non-orthogonal multiple access (NOMA) networks [20], [21], which can be regarded as a special case of the Ad hoc based secondary networks coexisting with a primary network.

2. System Model

We consider the uplink transmission for the CRNs as [16], [17]. The PU is licensed to transmit, and the n SUs need to pay the PU for their transmissions. Link gain between SU i and the PU is denoted by g_i ($i = 1, \dots, n$). Let h_{ij} denote the link gain from the j -th SU's transmitter to i -th SU's receiver. h_{ii} is the link gain from the i -th SU's transmitter to i -th SU's receiver. The IPC of SUs to the PU is T . The PU will charge the i -th SU λ_i per unit interference power.

We model the strategy between the PU and SUs as a Stackelberg game. The PU is the leader in this game. It chooses a price for each SU to maximize its own revenue under IPC. The SUs are the followers of the game. After the PU chooses the price for each SU, the SU will decide the transmit power to maximize its revenue based on non-cooperative power control game. The problem of the PU is as follows:

$$\text{maximize } u_p(\lambda_1, \dots, \lambda_n) = \sum_{i=1}^n \lambda_i g_i p_i \quad (1)$$

Fig. 1 Price-based power control model in CRNs

$$\text{subject to } \sum_{j=1}^n g_j p_j \leq T, \quad (2)$$

$$\gamma_i(\mathbf{p}) \geq \Gamma_i, i = 1, \dots, n, \quad (3)$$

where T is the IPC at the PU, λ_i is the price charged i -th SU per unit interference power. Constraint (2) means that the total interference power made by SUs should be below a given threshold T to ensure the SUs' transmission would not cause unendurable interference to the PU. Constraint (3) means that the SINR of the SU i should be larger than a threshold to guarantee its Quality of Service (QoS). The revenue of the i -th SU has two parts: one is the income from the transmit rate achieved at the SU when it transmits at a given power p_i , the other is the payment to the PU. The SINR at the receiver of the i -th SU is given by:

$$\gamma_i(\mathbf{p}) = \frac{h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \sigma_i^2} \quad (4)$$

where p_i is the transmit power of the i -th SU, $\mathbf{p} = (p_1, \dots, p_n)$ is the transmit power of all SUs and σ_i^2 is the interference caused by PU's transmissions plus the ambient noise at the i -th SU's receiver. σ_i^2 can be expressed as: $\sigma_i^2 = P l_i + n_i$, where P is the transmit power of the PU, l_i is the channel gain from the transmit of the PU to the receiver of SU i , n_i is the ambient noise at the SU i . For notational convenience, we use the notation σ_i^2 . Thus, the revenue of SU i is given by

$$u_i(\mathbf{p}, \lambda_i) = w_i \log(1 + \gamma_i(\mathbf{p})) - \lambda_i g_i p_i, \quad (5)$$

where w_i is the equivalent revenue per unit data rate valuation contributing to the i -th SU's revenue, which is predefined coefficient that transforms the i -th SU's transmission rate to a monetary utility. Therefore, the optimization problem for the i -th SU is as follows:

$$\begin{aligned} &\text{maximize } u_i(p_i, \mathbf{p}_{-i}, \lambda_i) \\ &\text{subject to } p_i \geq 0, \end{aligned} \quad (6)$$

where $\mathbf{p}_{-i}$ denotes the transmit power of the SUs except the i -th SU.

3. Price-Based Power Control Algorithm via Branch and Bound

In this section, we show that the revenue maximization problem of the PU is equivalent to maximize a non-convex function over the n -dimensional rectangle. Then, we propose an optimal price-based power control algorithm based on branch and bound method [22] to maximize non-convex function over n -dimensional rectangle. The relationship between the transmit power of SUs for the given price of λ_i ($i = 1, \dots, n$) is as follows:

Lemma 1: Let $(p_1, \dots, p_n)$ be the transmit power of the SUs when the PU charges the i -th SU for a given price λ_i such that λ_i is less than or equal to $\frac{w_i h_{ii}}{g_i \sigma_i^2}$ ($i = 1, \dots, n$), then

the transmit power $p_i (i = 1, \dots, n)$ of the SUs at the PU satisfies the following equations:

$$\begin{pmatrix} h_{11} & \dots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{1n} & \dots & h_{nn} \end{pmatrix} \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} w_1 h_{11} / (\lambda_1 g_1) - \sigma_1^2 \\ \vdots \\ w_n h_{nn} / (\lambda_n g_n) - \sigma_n^2 \end{pmatrix}. \quad (7)$$

Proof 1: Use the optimal best response condition for the i -th SU in (6), we have

$$\frac{\partial u_i(p_i, p_{-i})}{\partial p_i} = \frac{w_i h_{ii}}{\sum_{j=1}^n h_{ij} p_j + \sigma_i^2} - \lambda_i g_i = 0, \quad (8)$$

then we can get the following equations:

$$\sum_{j=1}^n h_{ij} p_j + \sigma_i^2 = \frac{w_i h_{ii}}{\lambda_i g_i} \quad (9)$$

for SU $i (i = 1, \dots, n)$. (7) is derived by rewriting (9) in the matrix form.

In the derivation lemma 1 by the optimal best response condition (8), we have assumed that the transmit power of each SU is non-negative. This is because the PU can always choose the prices to allow the SU's transmit power satisfies this condition. A more general expression for optimal condition in (8) for the best response condition of SU i can be expressed as $p_i = \frac{1}{h_{ii}} \max \left(0, \frac{w_i h_{ii}}{\lambda_i g_i} - \sigma_i - \sum_{j \neq i} h_{ij} p_j \right)$. If

$\frac{w_i h_{ii}}{\lambda_i g_i} - \sigma_i - \sum_{j \neq i} h_{ij} p_j \geq 0$ is hold, $p_i = \frac{w_i h_{ii}}{\lambda_i g_i} - \sigma_i - \sum_{j \neq i} h_{ij} p_j$ is satisfied. If $\frac{w_i h_{ii}}{\lambda_i g_i} - \sigma_i - \sum_{j \neq i} h_{ij} p_j < 0$, we have $p_i = 0$.

This mean that SU i will not transmit under price λ_i . Then, we can use a new price $\lambda'_i = \frac{w_i h_{ii}}{(\sigma_i + \sum_{j \neq i} h_{ij} p_j) g_i}$ instead of λ_i .

$p_i = \frac{w_i h_{ii}}{\lambda'_i g_i} - \sigma_i - \sum_{j \neq i} h_{ij} p_j = 0$ is held under the new price λ'_i . Moreover, the revenue of the PU will not change after we change the price for SU i from λ_i to λ'_i . Therefore, without loss of generality, we can also assume (7) is held.

From (9), we know that the transmit power of SU i is zero if the price for the i -th SU is larger than $\frac{w_i h_{ii}}{g_i \sigma_i^2}$. This means that the SU i will not buy the interference power when the price is larger than $\frac{w_i h_{ii}}{g_i \sigma_i^2}$. Therefore, we only need to find the optimal price for the PU in the price region such that λ_i is less than or equal to $\frac{w_i h_{ii}}{g_i \sigma_i^2}$.

By lemma 1, we have the following important identities:

$$\frac{w_i h_{ii} p_i}{\sum_{j=1}^n h_{ij} p_j + \sigma_i^2} = \lambda_i g_i p_i (i = 1, \dots, n). \quad (10)$$

It means that the revenue of the PU gets from the i -th SU can be expressed as: $\frac{w_i h_{ii} p_i}{\sum_{j=1}^n h_{ij} p_j + \sigma_i^2}$ such that $p_i (i = 1, \dots, n)$ satisfies

(7) for a given price $\lambda_i (i = 1, \dots, n)$. Then put (10) into (1), the revenue of the PU can be rewritten as follows:

$$\text{maximize } \sum_{i=1}^n \frac{w_i h_{ii} p_i}{\sum_{j=1}^n h_{ij} p_j + \sigma_i^2} \quad (11)$$

$$\text{subject to } \sum_{j=1}^n g_j p_j \leq T, \quad (12)$$

$$\gamma_j(\mathbf{p}) \geq \Gamma_j, p_j \geq 0, j = 1, \dots, n. \quad (13)$$

The optimization variables for (11)-(13) are $p_1, \dots, p_n$. Since the objective function (11) is not a concave function, (11)-(13) is non-convex optimization problem and thus cannot be solved globally optimally by the convex optimization algorithm. However, we propose an optimal pricing algorithm to find the global optimal solution to (11)-(13) based on branch and bound method.

Let $\gamma_i = \frac{h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \sigma_i^2} (i = 1, \dots, n)$, (11)-(13) is equivalent to the following problem:

$$\text{maximize } \sum_{i=1}^n \frac{w_i \gamma_i}{\gamma_i + 1} \quad (14)$$

$$\text{subject to } \sum_{j=1}^n g_j p_j \leq T, \quad (15)$$

$$\gamma_i = \frac{h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \sigma_i^2} \geq \Gamma_i, p_i \geq 0, i = 1, \dots, n. \quad (16)$$

The optimization variables for (14)-(16) are $p_1, \dots, p_n$ and $\gamma_1, \dots, \gamma_n$. Because (14) increases monotonically with γ_i , (14)-(16) is equivalent as follows:

$$\text{maximize } \sum_{i=1}^n \frac{w_i \gamma_i}{\gamma_i + 1} \quad (17)$$

$$\text{subject to } \sum_{j=1}^n g_j p_j \leq T, \quad (18)$$

$$\Gamma_i \leq \gamma_i \leq \frac{h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \sigma_i^2}, p_i \geq 0, i = 1, \dots, n. \quad (19)$$

Denote the objective function of problem (17) by $g(\gamma) = \sum_{i=1}^n \frac{w_i \gamma_i}{\gamma_i + 1}$ and the feasible SINR regions for variables $\gamma = (\gamma_1, \dots, \gamma_n)$ by S , where $S = \left\{ (\gamma_1, \dots, \gamma_n) \mid 0 \leq \sum_{i=1}^n g_i p_i \leq T, \Gamma_i \leq \gamma_i \leq \frac{h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \sigma_i^2} \right\}$. Let g^* be the optimal value of problem (17)-(19), we have

$$g^* = \sup_{\gamma \in S} g(\gamma). \quad (20)$$

Then, we define n -dimensional rectangle $Q_{\text{init}} = \{\gamma \mid \Gamma_i \leq \gamma_i \leq \frac{h_{ii} T}{g_i \sigma_i^2}, i = 1, \dots, n\}$ which encloses the set S , and define a new function $f: Q_{\text{init}} \rightarrow R$ as

$$f(\gamma) = \begin{cases} g(\gamma), & \text{if } \gamma \in S, \\ 0, & \text{otherwise,} \end{cases} \quad (21)$$

Because $g(\gamma) \geq 0$ for any $\gamma \in Q_{\text{init}}$, we have

$$g^* = \sup_{\gamma \in S} g(\gamma) = \sup_{\gamma \in Q_{\text{init}}} f(\gamma). \quad (22)$$

Thus, the non-convex problem (17)-(19) is equivalent to maximize the non-convex function f over the rectangle Q_{init} . Next, we show how to use branch and bound algorithm to maximize f over Q_{init} .

For a rectangle $Q \subseteq Q_{\text{init}}$, let us define

$$\phi_{\max}(Q) = \sup_{\gamma \in Q} f(\gamma). \quad (23)$$

We can see that

$$g^* = \phi_{\max}(Q_{\text{init}}) = \sup_{\gamma \in Q_{\text{init}}} f(\gamma). \quad (24)$$

The main idea of the branch and bound method is to find the upper bound and the lower bound to the of the problem (24) and then tightening the bounds by partition Q_{init} into k rectangles in the k -th iteration. The bounds should become tight as the number of rectangles increases. Therefore, the branch and bound method will use two functions $\phi_{\text{lb}}(Q)$ and $\phi_{\text{ub}}(Q)$ defined for any rectangle $Q \subseteq Q_{\text{init}}$ such that following two conditions are satisfied [22].

C1: The function $\phi_{\text{lb}}(Q)$ and $\phi_{\text{ub}}(Q)$ are lower and upper bounds on $\phi_{\max}(Q)$, respectively: for any $Q \subseteq Q_{\text{init}}$, we have $\phi_{\text{lb}}(Q) \leq \phi_{\max}(Q) \leq \phi_{\text{ub}}(Q)$.

C2: As the maximum half length the sides of Q , denoted by $\text{size}(Q)$ goes to zero, the difference between the upper and lower bounds uniformly converges to zero, i.e., $\forall \varepsilon > 0, \exists \delta, \forall Q \subseteq Q_{\text{init}}, \text{size}(Q) \leq \delta \Rightarrow \phi_{\text{ub}}(Q) - \phi_{\text{lb}}(Q) \leq \varepsilon$.

For the sake of clarity, we give the definition and computation of ϕ_{lb} and ϕ_{ub} in next section. We will present details of the branch and bound algorithm in this section. Let ε be an specified tolerance. The algorithm is started by computing $L_1 = \phi_{\text{lb}}(Q_{\text{init}})$ and $U_1 = \phi_{\text{ub}}(Q_{\text{init}})$, which are lower and upper bounds on f^* . If $U_1 - L_1 \leq \varepsilon$, the algorithm terminates and we have an upper bound $\phi_{\text{ub}}(Q_{\text{init}})$ which is at most ε -away from the optimal value T^* . Otherwise, we partition Q_{init} into small rectangles. At the k -th partition, Q_{init} is split into k rectangles such that $Q_{\text{init}} = \bigcup_{i=1}^k Q_i$. Then, the lower L_k and upper bounds U_k on f^* are updated as follows:

$$L_k = \max_{i=1, \dots, k} \phi_{\text{lb}}(Q_i), U_k = \max_{i=1, \dots, k} \phi_{\text{ub}}(Q_i). \quad (25)$$

If $U_k - L_k \leq \varepsilon$, the algorithm terminates. Otherwise, we continue to split the rectangle. One standard method for choosing the rectangle in the current partition to be split is to choose a rectangle that satisfies $\phi_{\text{lb}}(Q) = L_k$. Once we choose the rectangle to split, we split it along one of its longest edges. The condition C2 ensures that $U_k - L_k$ will become less than ε for some finite k . The proposed algorithm based on branch and bound method is given in algorithm 1. The convergence of algorithm 1 is proven in [23] by the following theorem.

Theorem 1: If for any $Q \subseteq Q_{\text{init}}$ such that $Q = \{\gamma | \gamma_{i,\min} \leq \gamma_i \leq \gamma_{i,\max}, i = 1, \dots, n\}$, the function $\phi_{\text{ub}}(Q)$ and $\phi_{\text{lb}}(Q)$ satisfy the conditions C1 and C2, then algorithm 1 will be convergent after a finite number of iterations to the optimal value t^* .

From the algorithm 1, the key point is destining a cheaply computable function $\phi_{\text{ub}}(Q)$ and $\phi_{\text{lb}}(Q)$ such that the condition C1 and C2 are satisfied. Next, we give a lower and upper bound functions and prove they satisfy the condition C1 and C2.

Algorithm 1 Price-based Power Control Algorithm via Branch and Bound Method

Initialization: Set $Q_{\text{init}} = \{\gamma | \Gamma_i \leq \gamma_i \leq \frac{h_{ii}T}{g_i\sigma_i^2}, i = 1, \dots, n\}$, given tolerance $\epsilon > 0$ and set $k = 1, \Omega_1 = \{Q_{\text{init}}\}, U_1 = \phi_{\text{ub}}(Q_{\text{init}})$, and $L_1 = \phi_{\text{lb}}(Q_{\text{init}})$.

repeat

Branching:

1) Pick $Q \in \Omega_k$ for which $\phi_{\text{lb}}(Q) = L_k$;

2) split Q along one of its longest edges into Q_I and Q_{II}

3) form Ω_{k+1} from Ω_k by removing Q_k and adding Q_I and Q_{II}

Bounding: compute lower bound and upper bound as follows $L_{k+1} = \max_{Q \in \Omega_{k+1}} \phi_{\text{lb}}(Q), U_{k+1} = \max_{Q \in \Omega_{k+1}} \phi_{\text{ub}}(Q)$,

4) $k = k + 1$;

until $U_k - L_k \leq \epsilon$

Output:

Compute the optimal transmit power $(p_1, \dots, p_n)$ of SUs by solving $\gamma_i = \frac{h_{ii}p_i}{\sum_{j \neq i} h_{ij}p_j + \sigma_i^2} (i = 1, \dots, n)$ where γ_i is the i -th element of

$\gamma = \arg \max_{Q \in \Omega} \phi_{\text{lb}}(Q)$. The optimal price λ_i for the i -th SU is given by

$$\lambda_i = \frac{w_i h_{ii}}{g_i \left(\sum_{j=1}^N h_{ij} p_j + \sigma_i^2 \right)}.$$

4. Lower and Upper Bound

In this section, we first give the definition of $\phi_{\text{lb}}(Q)$ and $\phi_{\text{ub}}(Q)$ for Algorithm 1. Second, we prove that the conditions C1 and C2 are satisfied. Finally, we proposed an efficient method to compute $\phi_{\text{lb}}(Q)$ and $\phi_{\text{ub}}(Q)$. For the rectangle $Q = \{\gamma | \gamma_{i,\min} \leq \gamma_i \leq \gamma_{i,\max}, i = 1, \dots, n\}$, we define $\phi_{\text{lb}}(Q)$ and $\phi_{\text{ub}}(Q)$ as follows:

$$\phi_{\text{lb}}(Q) = \begin{cases} g(\gamma_{\min}), & \text{if } \gamma_{\min} \in S, \\ 0, & \text{otherwise,} \end{cases} \quad (26)$$

$$\phi_{\text{ub}}(Q) = \begin{cases} g(\gamma_{\max}), & \text{if } \gamma_{\max} \in S, \\ 0, & \text{otherwise,} \end{cases} \quad (27)$$

We prove the conditions C1 and C2 are satisfied for functions ϕ_{lb} and ϕ_{ub} . Using lemma in [24], we only need to verify the function $g(\gamma)$ is monotonically increasing in each variable and Lipschitz continuous on R_+^n . Because $\frac{dg}{d\gamma_i} = \frac{w_i}{(\gamma_i + 1)^2}$, $g(\gamma)$ is monotonically increasing with γ_i for $i = 1, \dots, n$.

$$\begin{aligned}
\phi_{ub}(Q) - \phi_{lb}(Q) &\leq g(\gamma_{\max}) - g(\gamma_{\min}) \\
&= \sum_{i=1}^n \frac{w_i(\gamma_{i,\max} - \gamma_{i,\min})}{(1 + \gamma_{i,\min})(1 + \gamma_{i,\max})} \\
&\leq \max_{1 \leq i \leq n} \{w_i\} \sum_{i=1}^n (\gamma_{i,\max} - \gamma_{i,\min}) \\
&\leq n \max_{1 \leq i \leq n} \{w_i\} \max_{1 \leq i \leq n} (\gamma_{i,\max} - \gamma_{i,\min}) \\
&= 2n \max_{1 \leq i \leq n} \{w_i\} \text{size}(Q) \quad (28)
\end{aligned}$$

Therefore, for any given $\varepsilon > 0$, the condition C2 is satisfied if we choose $\delta \leq \frac{\varepsilon}{2n \max_{1 \leq i \leq n} \{w_i\}}$. From the definition of $\phi_{lb}(Q)$ and $\phi_{ub}(Q)$, the evaluation $\phi_{lb}(Q)$ and $\phi_{ub}(Q)$ only depends on check whether $\gamma_{\min}$ in the region S or not. The condition $\gamma_{\min} \in S$ can be checked by the feasible of following linear programming:

$$\text{minimize } \sum_{i=1}^N p_i \quad (29)$$

$$\text{subject to } \frac{h_{ii}p_i}{\sum_{j \neq i} h_{ij}p_j + \sigma_i^2} = \gamma_{i,\min}, i = 1, \dots, n, \quad (30)$$

$$\sum_{i=1}^n g_i p_i \leq T, p_i \geq 0, i = 1, \dots, n. \quad (31)$$

5. Simulation Results

5.1 Convergence of the Proposed Algorithm

The convergence of the proposed algorithms is shown for three SUs in Fig. 2. The interference power constraint $T = 1$, the channel gain matrix H among the SUs is

$$H = \begin{bmatrix} 0.6892 & 0.0008 & 0.0015 \\ 0.0075 & 0.7290 & 0.0083 \\ 0.0045 & 0.0091 & 0.5383 \end{bmatrix}.$$

The channel gain g from the SUs to PU is given by $g = [0.078 \ 0.144 \ 0.136]$, $\sigma_i^2 = 1$, $w_i = 1$ for all the SUs. The tolerance ϵ for algorithm is set to be 0.1. Figure 2 shows the revenue of PU versus iteration. The lower bound and upper bound converges to 1.918 and 2.0174 after about 146 iterations. The optimal revenue of PU obtained by exhaustive search is 1.9244. Therefore, the proposed algorithm finds the global optimal solution within allowable error $1.9244 - 1.918 = 0.0064$, which is less than the given tolerance $\epsilon = 0.1$.

5.2 Benchmark for Other Algorithm

Because the proposed is an optimal algorithm for all the SINR requirement region, it can be used to as a benchmark for other algorithm. Next, we use the proposed algorithm to evaluate the performance of the price-based power control algorithm with QoS constraints (PPCAQC) algorithm

Fig. 2 Lower bound and upper bound versus iteration

Fig. 3 Revenue of the PU versus INR

proposed in [16]. The PPCAQC algorithm is proved to be optimal when the minimum SINR requirement for the SUs is at least 0dB. However, whether the PPCAQC algorithm is an optimal or a sub-optimal is still unknown when the minimum SINR requirement for the SUs is less than 0dB. Therefore, we can use the proposed to evaluate the performance of the PPCAQC algorithm for the case that the minimum SINR requirement for the SUs is less than 0dB. The simulation parameters are given by: $\sigma_i^2 = 1$, $w_i = 1$ for all the SUs, the number of SUs is 4, $\sigma_i^2 = 1$, g_i and h_{ii} are uniform distribution in $[0, 1]$, $h_{ij}(j \neq i)$ is a uniform distribution in $[0, 0.01]$. The minimum SINR for each SU is 0.5. For both algorithms, We assume that the SU with lower preference factor is removed when the SINR constraint is not satisfied. The interference-to-noise ratio (INR) which is defined as T/σ_i^2 changes from -20 dB to 40 dB. All simulations are averaged by 10^3 independent channel realizations.

Figure 3 shows the revenue of the PU obtained by two algorithms versus the INR. The revenue of the PU obtained by the proposed algorithm is always better than or equal to PPCAQC algorithm. When the INR equals 0 dB, the PP-

CAQC algorithm can obtain 91.83% sum revenue compared to the proposed algorithm. Therefore, the PPCAQC algorithm is not an optimal algorithm but a near-optimal algorithm even at the low INR when the minimum SINR requirement of each SU is less than 0 dB. Moreover, the performance gap between two algorithms will trend to be zero as the INR increase.

6. Conclusion

In this paper, price-based power control in CRNs is investigated from the Stackelberg game. We propose an optimal price-based power control algorithm based on branch and bound to obtain the global optimal solution. The proposed algorithm can be used to handle a class of the price-based power control algorithms for various other network topologies which can be viewed as a specially case of the Ad hoc based CR such as CDMA [13]–[15], femtocell [18], D2D [19], and NOMA [20], [21]. Given its generality, the proposed algorithm can be used as a benchmark for other algorithms, which are lower complexity sub-optimal algorithms used in practice.

We have assumed that power control algorithm is convergent in the time scale that the network is static and channel condition is not changed. When the time scale of the SUs coming in and leaving out the networks is much faster than the convergence of the algorithm, the utility function for PU and SUs will be changed with the consideration of the average power constraint, outage probability and delay of CRNs. The proposed work may not be applicable. The price-based power control in the CRNs under dynamic of SUs will be part of our future work. Stackelberg differential game may be used to model the interaction between the PU and the SUs.

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