A Novel Completion Algorithm for Color Images and Videos Based on Tensor Train Rank

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SUMMARY Due to the inevitable data missing problem during visual data acquisition, the recovery of color images and videos from limited useful information has become an important topic, for which tensor completion has been proved to be a promising solution in previous studies. In this paper, we propose a novel completion scheme, which can effectively recover missing entries in color images and videos represented by tensors. We first employ a modified tensor train (TT) decomposition as tensor approximation scheme in the concept of TT rank to generate better-constructed and more balanced tensors which preserve only relatively significant informative data in tensors of visual data. Afterwards, we further introduce a TT rank-based weight scheme which can define the value of weights adaptively in tensor completion problem. Finally, we combine the two schemes with Simple Low Rank Tensor Completion via Tensor Train (SiLRTC-TT) to construct our completion algorithm, Low Rank Approximated Tensor Completion via Adaptive Tensor Train (LRATC-ATT). Experimental results validate that the proposed approach outperforms typical tensor completion algorithms in recovering tensors of visual data even with high missing ratios.

key words: tensor train (TT), tensor train (TT) rank, tensor completion, visual data

1. Introduction

Multiway data analysis has received considerable attention in signal processing [1], [2], numerical linear algebra [3], machine learning [5]–[8], data mining [9] and computer vision [4]. As a generalization of scalars, vectors and matrices, tensors have become a popular way to represent multi-way data. However, the recovery of missing values from limited information has become an important topic due to the problems in acquisition process. Many scholars have studied the tensor completion problem and applied it to a wide range of real-world problems, such as image processing [10]–[12] and video decoding [13], [14].

Tensor completion for color images and videos is motivated by the success of low-rank matrix completion (LRMC) [15], [16]. The main method of solving the LRMC problem is converting the rank minimization problem into a nuclear norm minimization problem or other convex optimization forms because minimizing the rank is NP-hard [17]. Many matrix completion problems can be solved by relaxation [18], [19]. Considerable effort has been devoted to solving the low-rank tensor completion (LRTC) problem of visual data, which underlies the concept of LRMC. A reasonable manner is to formulate tensor completion of images or videos via Tucker rank [4]. However, a crucial drawback of this method is the components of Tucker ranks are matrix ranks constructed using an unbalanced matricization scheme, this technique may not be suitable for describing the global information of tensors especially strongly correlated tensors with high orders.

In this paper, we study the tensor completion problem and propose a novel completion scheme to estimate missing entries in tensors of color images or videos. We first introduce the novel concept of tensor train (TT) rank [20], which is different from Tucker rank and has become popular recently due to its effectiveness in capturing the global information of tensors. We then employ a modified TT decomposition [20] as tensor approximation scheme to generate better-constructed and more balanced tensors which preserve only relatively significant informative data in tensors of visual data considering the unbalanced information distribution in tensors has negative impact on completion results. We next introduce a TT rank-based adaptive weight scheme to define the values of weights adaptively in tensor completion problem. Finally, we combine the two schemes in simple low rank tensor completion via tensor train (SiLRTC-TT) [21] to present our completion scheme, low rank approximated tensor completion via adaptive tensor train (LRATC-ATT) algorithm, and employ this completion algorithm to reconstruct tensors from partially observed visual data, the framework of the proposed scheme is shown in Fig. 1. The proposed algorithm provides a detail steps to solve the tensor completion of color images and videos. Simulations show that the proposed model outper-
forms other algorithms in recovering missing values in tensors of color images and videos.

The rest of this paper is organized as follows. Section 2 reviews the related work. In Sect. 3, we introduce the basic notations and preliminaries of tensors. Section 4 describes the proposed scheme based on TT rank. In Sect. 5, we conduct the experiments of the proposed scheme against competing methods on synthetic data and real-world color images and videos. Finally, we provide conclusion and future research direction in Sect. 6.

2. Related Work

Many scholars have studied the completion problem of color images and videos, and they provided several approaches to solve the completion problem employing tensor, matrix and others.

In [4], Liu et al. proposed an approach to estimate missing values in tensors of visual data. The authors extended the matrix case to tensor case by laying out the theoretical foundations to build the working algorithm. In their paper, they first proposed a definition for the tensor trace norm, that generalized the established definition of the matrix trace norm. And then, they formulated tensor completion as a convex optimization problem employing a relaxation technique. The experiments showed potential applications of the proposed algorithm and the quantitative evaluation indicated that their method was more accurate and robust than heuristic approaches.

Chen et al. [22] proposed an approach called simultaneous tensor decomposition and completion (STDC) that combines a rank minimization techniques with Tucker model decomposition. Moreover, as the model structure was implicitly included in the Tucker model, the authors used the factor Priors. The proposed method can leverage two classic schemes and accurately estimates the model factors and missing entries by exploiting the auxiliary information. The conducted experiments verified the convergence of the proposed algorithm on synthetic data and evaluated the effectiveness on various kinds of real-world data.

In [23], Zhao et al. formulated CP factorization using a hierarchical probabilistic model and employed a fully Bayesian treatment by incorporating a sparsity-inducing prior over multiple latent factors and the appropriate hyper-priors over all hyper-parameters, resulting in automatic rank determination. In order to learn the model, the authors developed an efficient deterministic Bayesian inference algorithm, which scaled linearly with data size. The proposed approach was characterized as a tuning parameter-free approach, which can effectively infer underlying multilinear factors with a low-rank constraint, while also providing predictive distributions over missing entries. The simulations on synthetic data showed the intrinsic entries were missing while the results from real-world applications, including image inpainting and facial image synthesis, demonstrated that the proposed method outperformed state-of-the-art approaches for both tensor factorization and tensor completion in terms of predictive performance.

The above algorithms are based on tucker rank or cp-rank while our completion algorithm is based on tensor train, that can capture the global information of tensors due to its construction by well-balanced matricization scheme.

Bengua et al. [25] proposed novel approaches to tensor completion to recover missing entries of color images or videos represented by tensors. The proposed algorithms were based on tensor train (TT) rank. Simulation results showed the clear advantage of the proposed methods over all other completion methods. In our paper, we utilize the proposed algorithms in SiLRTC-TT to strengthen algorithm performance.

In [6], Xu et al. proposed a new model to recover a low-rank tensor by simultaneously performing low-rank matrix factorizations to the all-mode matricizations of the underlying tensor. The authors also applied an alternating minimization algorithm and two adaptive rank-adjusting strategies to solve the proposed model. The simulation showed that the proposed algorithm performed consistently and gave better results than the compared methods though their model was non-convex. In addition, the subsequence convergence of their algorithm can be established in the sense that any limit point of the iterations satisfies the KKT (Karush-Kuhn-Tucker) conditions.

3. Notions and Preliminaries

In this paper, we adopt mathematical notations and terminologies of tensors in [17]. The tensor is a multidimensional array and its order stands for the dimensions. Scalars can be considered as zero-order tensors denoted by lowercase letters (x, y, z, . . .). Vectors and Matrices are considered as first-order and second-order tensors which can be denoted by boldface lowercase letters (X, Y, Z, . . .) and capital letters (X, Y, Z, . . .). An Nth-order tensor is denoted by a calligraphic letter X ∈ R1×2×···×IN where Ik, k = 1, . . . , N is the dimension corresponding to mode k, and X1,...,IN denotes the elements of X, where 1 ≤ i1 ≤ Ik, k = 1, . . . , N. Fibers are higher-order analogue of matrix rows and columns. A mode-n fiber of tensor X is defined by fixing every index but i_n and denoted as X_{i_1,i_3,...,i_{n-1},i_{n+1},...}. The inner product of two same sized tensors X, Y ∈ R1×2×···×IN is the sum of the product of their entires:

\begin{equation}
<X, Y> = \sum_{i_1,j_1} x_{i_1,...,i_N} y_{i_1,...,i_N}. \tag{1}
\end{equation}

Accordingly, the Frobenius norm of an Nth-order X ∈ R1×2×···×IN is defined as:

\begin{equation}
\|X\|_F = \sqrt{\sum_{i_1=1}^{I_1} \cdots \sum_{i_N=1}^{I_N} x_{i_1,...,i_N}^2}. \tag{2}
\end{equation}

4. Methodology

The definition of tensor completion for color images and
videos recovery is fundamentally based on the well-known optimization problem [16] in LRMC, and it can be defined in Eq. (3):

$$\min_{X_{(k)}}: \sum_{k=1}^{N} \alpha_k \cdot \text{rank}(X_{(k)}) \quad s.t. \quad X_{\Omega} = T_{\Omega}$$

where $\alpha_k$ is a pre-specified weight that satisfies $\alpha_k \geq 0$ and $\sum_{k=1}^{N} \alpha_k = 1$. $X_{(k)} \in \mathbb{R}^{m \times n \times \cdots}$ $(m = l_k$ and $n = \prod_{k=1}^{N} l_k)$ is the matrix obtained by unfolding along the $k$-th mode on tensor $X$, $\Omega$ is the index set of the known entries in tensor $T$. The problem in Eq. (3) is usually formulated into Eq. (4) which employs the nuclear norm of tensor $X$, because Eq. (3) is a non-convex tensor minimization problem:

$$\min_{X_{(k)}}: \sum_{k=1}^{N} \alpha_k \|X_{(k)}\|_n \quad s.t. \quad X_{\Omega} = T_{\Omega}$$

where $\sum_{k=1}^{N} \alpha_k \|X_{(k)}\|_n$ is defined as Tucker nuclear norm of the tensor $X$. From the analysis in [21], we can see that the amount of correlation between elements of $X_{(k)}$ depends on the rank $r_k$ which is bounded by $m = l_k$. Hence, when the dimensions of modes are slightly different or the same: $l_1 \approx l_2 \approx \cdots \approx l_N \approx I$, the matrix $X_{(k)}$ is unbalanced due to $m << n$ when either $I$ or $N$ is large. Therefore, the limit of $r_k \in r = (r_1, r_2, \ldots, r_N)$ is too small to describe the correlation of the tensor with high order $(N \geq 4)$ or large dimension. To overcome this weakness, we introduce the concept of TT rank [20] which is defined by more balanced matrices. Underlying the concept of TT decomposition, the unfolding matrix $X_{(k)}$ can be defined as $X_{(k)} \in \mathbb{R}^{m \times n \times \cdots}$ $(m = \prod_{i=1}^{k} l_i$, $n = \prod_{i=k+1}^{N} l_i)$, since $X_{(k)}$ is obtained by metricizing along a few k modes rather than one single mode, its rank $r_k$ is bounded by $\min(m, n)$ thus is more appropriate to describe correlations in high order or large dimension tensors. Therefore, the LRRTC problem in Eq. (4) is formulated into:

$$\min_{X}\sum_{k=1}^{N-1} \alpha_k \|X_{(k)}\|_n \quad s.t. \quad X_{\Omega} = T_{\Omega}$$

where $X_{(k)} \in \mathbb{R}^{m \times n \times \cdots}$ $(m = \prod_{i=1}^{k} l_i$, $n = \prod_{i=k+1}^{N} l_i)$ is the unfolding matrix of the tensor.

In this section, we propose a novel scheme for Eq. (5) underlying the concept of TT rank. We employ a modified TT decomposition algorithm as tensor approximation scheme to obtain better-constructed and more balanced tensors which preserve only relatively significant informative data in tensors of visual data. And then we introduce an adaptive weight scheme in the concept of TT rank to define $\alpha_k$ adaptively. Finally, we present our completion scheme, LRARTC-ATT, to recover the tensors of color images and videos from limited information data.

4.1 Tensor Approximation Scheme Based on TT Rank

In tensor completion scheme, unbalanced information distribution has negative impact on obtaining satisfactory recovery results. Therefore, in this section, we introduce a tensor

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**Algorithm 1:** Tensor Approximation Scheme

**Input:** Tensor $A \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$, TT-rank $r \in (r_1, r_2, \ldots, r_N)$, $r_0 = 1$

**Output:** The TT rank-approximated tensor $B$ with new cores $G_1, G_2, \ldots, G_n$

1) Temporary tensor $C = A$, $r_0 = 1$
2) for $k := 1 \to (n-1)$
3) begin
4) Compute SVD and threshold the number of singular values to be $r_k$, thus $A_k = U_k S_k V_k$
5) $G_k = \text{reshape}(U_k, [r_{k-1}, l_k, r_k])$
6) $C := S_k V_k$
7) end
8) endfor
9) $C = \text{reshape}(C, [r_{n-1}, l_n, r_n])$
10) $G_n = C$
11) return The TT rank-approximated tensor $B$ with new cores $G_1, G_2, \ldots, G_n$

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**Algorithm 2:** The Adaptive Weight Scheme

**Input:** Tensor $X \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$, threshold $T$

**Output:** The weights corresponding to each unfolding matrices

1) for $k := 1 \to (n-1)$
2) begin
3) $X_{(k)} = \text{reshape}(X, [\prod_{i=1}^{k} l_i, \prod_{i=k+1}^{N} l_i])$
4) Compute SVD of $X_{(k)}$ to get the singular values $\delta^{(k)}$ which in ascending order
5) Find the smallest $g_k$ according to Eq. (7) and $T$
6) Normalize $g_k$ with $n_k$ according to Eq. (8)
7) Compute the weight $\alpha_k$ according to Eq. (9)
8) end
9) return The weights corresponding to each unfolding matrices

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**Algorithm 3:** LRRTC-ATT Algorithm

**Input:** The observed data $T \in \mathbb{R}^{l_1 \times l_2 \times \cdots \times l_N}$, index set $\Omega$, the initial weight $\alpha_k$ where $k = 1, 2, \ldots, n - 1, \beta$, the TT rank $r_{r_{r_{1,2,\ldots,n-1}}}$

**Output:** The recovery tensor $X$

1) Obtain the TT rank-approximated tensor $T_{\beta}$ through Algorithm 1.
2) Initialization $X^0$, with $X^0_{\Omega} = (T_{\beta})_{\Omega}, l = 0$
3) while not converged do
4) for $k := 1 \to (n-1)$
5) begin
6) unfolding the tensor $X'$ to get $X'_{(k)}$
7) $M_{k+1}^{l_1} = D_{k+1}^l(X'_{(k)})$
8) end
9) endfor
10) update $X^{l+1}$ from $M_{l+1}^{l_1}$ by Eq. (13)
11) update the weights by Algorithm 2.
12) endwhile
13) return The recovery tensor $X$
approximation scheme in the concept of TT rank to convert tensors into better-constructed and more balanced tensors, which only preserve relatively important information, both considering the global information of original tensors.

The introduced scheme can be defined as a modified version of TT decomposition, which is originally introduced in [20], that can be described as:

\[ X = G_1 G_2 \ldots G_n \]  

where \( G_i \) is core tensor and \( r_k \in \mathbb{R} = (r_1, r_2, \ldots, r_{N-1}) \) is the TT rank of the tensor where \( r_0 = r_N = 1 \). Different with TT decomposition, the tensor \( X \) in our scheme is not necessarily a TT rank \( r \) tensor. In this part, Our scheme first computes SVD of unfolding matrix \( X_{[k]} \), which is under the concept of TT and generated by a well-balanced matricization scheme, then the introduced scheme employs the optimal TT rank \( r \) to threshold the number of singular values and reshape unitary matrix in \( r_{k-1} \times I_k \times r_k \). In this way, we can obtain the better-constructed and more-balanced tensor with cores \( G_1, G_2, \ldots, G_n \) which only preserve relatively significant information of original one. The pseudocode of tensor approximation scheme is exhibited in Scheme 1.

In Scheme 1, we use \( A \) to represent the original tensor which represents synthetic data or visual data, and then we compute the SVD of each unfolding matrix \( A_k \), next, \( r_k \) stands for TT rank to threshold the number of singular values and reshape unitary matrix \( U_k \) to generate new core tensor \( G_k \). Finally, we can obtain the generated tensor \( B \) and it is used in our completion scheme.

4.2 Adaptive Weight Scheme Based on TT Rank

From Eq. (5), we can see that \( \alpha_k \) stands for the contribution of nuclear norm in the tensor unfolding k-th dimension [24, 26], and different unfolding matrix causes different affect in completion process, the values of weight should be adaptive in order to well describe the effects of unfolding matrices in completion scheme.

In completion problem, the weight \( \alpha_k \) measures the effect of nuclear norm of unfolding matrix in k-th dimension, and nuclear norm is capable of approximating the rank of the corresponding unfolding matrix. This indicates that \( \alpha_k \) depends on the rank of the unfolding matrix along the k-th dimension, that is the lower rank of unfolding matrix in k-th dimension, the smaller \( \alpha_k \) of corresponding unfolding matrix.

Furthermore, the low-rank property in k-th dimension can be measured through the distribution of singular values (the ordering of singular values is in descending order). Thus quicker decrease of the distribution of singular values in k-th unfolding matrix indicates the stronger low-rank property of the corresponding unfolding matrix. Therefore, we introduce the adaptive weight scheme, which is originally defined in [24], based on the distribution of the singular values of unfolding matrixes in TT structure.

In this method, we define the summation of all \( \alpha_k \) satisfy \( \sum_{k=1}^{N-1} \alpha_k = 1 \) and permute the singular values of k-th unfolding matrix in ascending order as: 

\[ \delta^{(k)} = [\delta^{(k)}_1, \delta^{(k)}_2, \ldots, \delta^{(k)}_{n_k}] \text{, where } k = 1, 2, \ldots, N - 1, n_k \text{ is the singular value number of } X_{[k]} \]

In this way, important singular components can then be retained.

To obtain the corresponding \( \alpha_k \) of \( X_{[k]} \), we first define a parameter \( \rho \) (0 < \( \rho \) < 1) to stand for the information proportion of the first g singular values to all singular values, which can be mathematically defined as:

\[ \rho = \frac{\sum_{i=1}^{g} \delta_i}{\sum_{i=1}^{n_k} \delta_i} \]

we then define a threshold \( T \) to find the smallest \( g \) when \( \rho \geq T \). Next, \( g_k \), which denotes the smallest \( g \) in different unfoldings, is normalized with singular value number \( n_k \) to guarantee obtaining the same scale of its value as:

\[ \tilde{g}_k = \frac{g_k}{n_k} \]

Finally, the weight \( \alpha_k \) can be obtained according to:

\[ \alpha_k = \frac{g_k}{\sum_{i=1}^{k} g_i}, \quad k = 1, 2, \ldots, N - 1 \]

where \( \eta \) is a constant parameter. The pseudocode of the proposed adaptive weight algorithm which is based on TT-rank is demonstrated in Scheme 2.

4.3 The Proposed Completion Scheme

In this paper, we define the task of tensor completion as the recovery of TT rank-approximated tensor \( T_p \) from missing values:

\[ \min_{X} : \sum_{k}^{N-1} \alpha_k \|X_{[k]}\|_*, \quad s.t. \quad X_{[k]} = (T_p)_{[k]} \]  

where \( T_p \) is the TT rank-approximated tensor generated by the introduced tensor approximation scheme.

In this section, we present a tensor completion algorithm based on SiLRTC-TT for Eq. (10), the proposed algorithm applies unfolding matrices \( M_1, M_2, \ldots, M_k \) to obtain the following equivalent formulation because the problem in Eq. (10) is difficult to solve directly.

\[ \min_{X, M_k} : \sum_{k}^{N-1} \alpha_k \|M_k\|_* \quad s.t. \quad X_{[k]} = M_k \quad for \quad k = 1, 2, \ldots, N - 1 \]

where \( \alpha_k \) is the weight determined by the introduced adaptive weight scheme.

The presented tensor completion algorithm then applies the block coordinate descent to alternatively optimize a group of variables while fixing the other groups. The variables are divided into unfolding matrices: \( M_1, M_2, \ldots, M_{N-1} \) and the tensor \( X \). With fixed \( X_{[k]} \), \( M_k \) can be obtained.
through
\[ M_k = D_{yk}(X_{[k]}) \] (12)
where \( y_k = \frac{\alpha_k}{\sigma_k} \) and \( D_{yk}(X_{[k]}) \) is the thresholding SVD of \( X_{[k]} \) [15]. After updating all \( M_k \) matrices, the optimal \( X \) can be obtained through
\[ X_{i_1\ldots i_N} = \left\{ \begin{array}{ll}
\left( \frac{\sum_{k=1}^{K} \rho_k \delta_k \left( M_k \right)}{\sum_{k=1}^{K} \rho_k} \right)_{i_1\ldots i_N} & (i_1\ldots i_N) \notin \Omega, \\
(1)_{i_1\ldots i_N} & (i_1\ldots i_N) \in \Omega
\end{array} \right. \] (13)
The pseudocode of LRATC-ATT algorithm is illustrated in Algorithm 3.

5. Experiment

The current section summarizes the results obtained from LRATC-ATT simulations and its comparisons with HaLRTC [4], Simple Low Rank Tensor Completion (SiLRTC) [4], Twist Tensor Nuclear Norm (t-TNN) [27], SiLRTC-TT [4], and TMac-TT [21]. The results are divided into five subsections: parameter initialization, synthetic data completion, color images, color videos and runtime.

5.1 Parameter Initialization

In simulations, we employ synthetic and real-world visual data to evaluate the performance of our proposed scheme with respect to different missing ratios (mr):
\[ mr = \frac{P}{\prod_{k=1}^{N} l_k} \] (14)
where \( P \) is the number of missing entries, which are chosen randomly from a tensor \( T \) using a uniform distribution.

In these experiments, we simply choose the weight \( \alpha_k \) for HaLRTC, SiLRTC as follows:
\[ \alpha_k = \frac{l_k}{\sum_{k=1}^{N} l_k} \] (15)
In SiLRTC-TT algorithm, we define the weights as:
\[ \alpha_k = \frac{\sigma_k}{\sum_{k=1}^{N} \sigma_k} \quad \text{with} \quad \sigma_k = \min \left( \prod_{i=1}^{k} l_i, \prod_{i=k+1}^{N} l_i \right) \] (16)
where \( k = 1, \ldots, N - 1 \). For our completion algorithm, we initialize weights as:
\[ \alpha_k = \frac{r_k}{\sum_{k=1}^{N} r_k} \] (17)
where \( k = 1, \ldots, N - 1 \) and \( r_k \) is the optimal TT rank, which are identified empirically, and \( \alpha_k \) is updated by Algorithm 2 in each iteration. Moreover, in the proposed adaptive weight algorithm, the threshold \( T \) is [0.85, 0.98] and \( \eta \) set to 500.

The positive parameters in our completion algorithm are chosen by \( \beta_k = f \alpha_k \) where \( f \) is empirically selected as 1.05. The convergence criterion in completion algorithms is defined as:
\[ \varepsilon = \frac{\| X^{l+1} - X^l \|_F}{\| X^l \|_F} \leq \text{eps} \] (18)
where \( \| X^{l+1} - X^l \|_F \) is the relative error of tensor \( X \) and \( \text{eps} = 10^{-4} \), The maximum number of iterations is 2000 in this paper.

The experiments are implemented under a MATLAB environment with an Interl(R) Core(TM) i5 2400 processor with 4GB RAM.

5.2 Synthetic Data Completion

This section demonstrates a completion experiments of two types of synthetic data, one is: 4D tensor \((20 \times 20 \times 20 \times 20)\), another type is: 5D tensor \((30 \times 5 \times 5 \times 5 \times 5)\), 6D tensor \((30 \times 5 \times 5 \times 5 \times 5 \times 5)\) and 7D tensor \((30 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)\). The experimental tensors are generated randomly with respect to the independent standard norm distribution. We compare our algorithm with HaLRTC, SiLRTC-TT and TMac-TT. The experimental results are exhibited in Fig. 2.

Figure 2 depicts the RSE values of tested algorithms in different missing ratios, and the RSE curve of LRATC-ATT displayed in this figure indicates the highly competitive of our scheme in recovering synthetic data, and the performance is getting better with the increasement of order, but still exists a gap with TMac-TT which performs the best in the completion experiments of synthetic data.

5.3 Color Images

In this section, we evaluate the proposed algorithm with
color images of “Lena”, “Baboon” and “Peppers”, and make comparisons with HaLRTC, SiLRTC, SiLRTC-TT and TMac-TT. The images are initially represented by 3D tensors with the same size of $512 \times 512 \times 3$ and $256 \times 256 \times 3$, the missing entries of each image are chosen randomly based on a uniform distribution.

The compared performance results of “Lena” in $512 \times 512 \times 3$ are exhibited in Fig. 3. The analysis of the experimental results indicates that the performance of the proposed scheme prevails against others. Specifically, SiLRTC and HaLRTC algorithms only give simple structures of the original ones, especially when $mr \geq 80\%$. SiLRTC-TT algorithm performs better than SiLRTC and HaLRTC. The proposed algorithm obtains clearer recovered images and gives the best RSE result: 0.096 with $mr = 70\%$, which demonstrates the efficiency of our scheme in recovering color images.

Figure 4 presents the recovered images of “Baboon” in $512 \times 512 \times 3$. The recovered images of SiLRTC have low similarity with original ones, and the images of HaLRTC are better. SiLRTC-TT and the proposed scheme produce high resemblance to the original images, while our scheme, LRATC-ATT obtain enhanced recovered images in textures and colors. Furthermore, the proposed scheme obtain the best result RSE = 0.160.

The compared performance of “Peppers” in $512 \times 512 \times 3$ are displayed in Fig. 5. Experimental results reveal that SiLRTC and HaLRTC have relatively poor performance when compared with SiLRTC-TT and LRATC-ATT. The best RSE result obtained by our scheme demonstrates the superiority of the proposed model though there exist few obvious differences in recovered images of SiLRTC-TT and the proposed algorithm.

Table 1 exhibits the RSE results of “Lena”, “Baboon” and “Peppers” in $256 \times 256 \times 3$ with $70\%$ missing ratio.

In summary, it is obvious that algorithms based on the concept of TT perform better than others. And the proposed tensor completion algorithm, LRATC-ATT, performs much efficiently in recovering images with more vibrant colors and textures.

5.4 Color Video

We apply the color videos dataset (Coastguard, Hall Monitor, Salesman, Highway, Carphone, Container) [28] to eval-
evaluate the proposed algorithm.

In this section, all the videos are converted into RGB format from YUV and represented by fourth-order tensors with the same size of $176 \times 144 \times 3 \times 50$ (frame_row $\times$ frame_column $\times$ RGB $\times$ frames). Subsequently, we resize the fourth-order tensors into tensors with a size of $8800 \times 144 \times 3$ ((frame_row $\times$ frames)$\times$ frame_column$\times$RGB) for t-TNN considering the difficulty of computing the fourth-order tensors directly. The recovered images with 98%, 95% and 90% missing ratios are exhibited in Figs. 6, 7 and 8, and RSE results are presented in Fig. 9.

From the experimental results, it is obviously that the proposed algorithm outperforms others even with high missing ratios, i.e. $mr \geq 95\%$.

For “Coastguard”, which is a video with complex textures areas. From its recovered video images displayed in Figs. 6, 7 and 8, we can see that: the t-TNN cannot produce any resemblance to the original video images. HaLRTC, SiLRTC-TT generate only a simple structure of the original ones, and TMac-TT obtains the best RSE result when $mr = 98\%$, but the proposed algorithm achieves the best recovered frame images, especially when $mr \geq 95\%$. The RSE results are presented in Fig. 9 (a), that confirm the superiority of LRATC-ATT.

For “Highway”, which is a video with uniform motion object. The analysis of experimental results indicates that t-TNN cannot recover the samples with $mr \geq 95\%$, HaLRTC and SiLRTC-TT obtain only basic structure of original frame images, TMac-TT algorithm shows the competitive results in recovered images and RSE results when $mr = 98\%$, but our completion scheme exhibits the best performances in detail. The RSE results shown in Fig. 9 (b) demonstrate the overall advantage of LRATC-ATT.

For “Hall Monitor”, which is a video with near static background. From the recovered video images exhibited in Figs. 6, 7 and 8, we can see that t-TNN, HaLRTC and TMac-TT can hardly produce any resemblance of original video frame images when $mr \geq 95\%$. SiLRTC-TT generates the video frame images with messy textures. In contrast, the proposed completion algorithm obtain much clearer frame images. The RSE results displayed in Fig. 9 (c) also evaluate the superiority of LRATC-ATT.

For “Container”, which is a video with object intrusion. The recovered images depicted in Figs. 6, 7 and 8 indicate that t-TNN cannot complete video images, HaLRTC and SiLRTC-TT generate only basic structure of original ones, TMac-TT exhibits a better performance but worse than the proposed algorithm which obtains relatively high quality recovered frame images. The RSE results represented in Fig. 9 (d) confirm the overall superiority of our completion scheme though TMac-TT obtain the best RSE result when $mr = 98\%$.

For “Salesman” and “Carphone”, which are videos with nearly static backgrounds. It is obviously to find that the performance of our completion algorithm is not as good as it in other videos such as “Coastguard”. LRATC-ATT cannot recover the facial features clearly. However, the proposed scheme still outperforms others in completing pro-

![Fig. 6](Image)

The recovered frame images with 98% missing ratio.
Fig. 7  The recovered frame images with 95% missing ratio.

Fig. 8  The recovered frame images with 90% missing ratio.
The superiority can be easily confirmed by the recovered images shown in Figs. 6, 7 and 8, and RSE results demonstrated in Figs. 9 (e) and 9 (f).

In addition, we also make a comparison of the proposed completion algorithm with and without Scheme 1 on six videos to evaluate the effect of the presented tensor approximation scheme, and the experimental results are shown in Table 2. From this table, we can see that the introduced Scheme 1 has positive impact on obtaining satisfactory recovery results.

### Table 2
Average RSE of color videos recovered by the proposed algorithm with and without Scheme 1 (S1 is Scheme 1).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>98%</th>
<th>95%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRATC-ATT</td>
<td>0.3151</td>
<td>0.1567</td>
<td>0.1074</td>
</tr>
<tr>
<td>LRATC-ATT no S1</td>
<td>0.3190</td>
<td>0.1593</td>
<td>0.1092</td>
</tr>
</tbody>
</table>

5.5 Runtime

In this section, we evaluate the computation time in recovering color images and videos represented by tensors from limited information with different missing ratios, and here we mainly depict the results in processing color videos. We display the average computation time of tested algorithm in Table 3 and time histograms of “Coastguard” and “Hall Monitor” in Fig. 10, from which we can see that the computation time of LRATC-ATT does not increase significantly because of the weights update during each iteration. Moreover, we do not demonstrate the computation time of processing color images because it cannot exhibit the performance of the proposed adaptive weight algorithm.
In summary, the RSE results of the proposed algorithm indicate a better performance in recovering missing values in tensors of synthetic and real-world images and videos when in comparison with other completion algorithm namely, SiLRTC, HaLRTC, SiLRTC-TT, t-TNN and TMac-TT. The recovered results of images with vibrant color and videos with complex textures demonstrate the superiority of the proposed scheme in completion problem with high missing ratios. Moreover, the time of LRATC-ATT in computation time experiments does not increase significantly although adopting measures to improve the recovery accuracy.

6. Conclusions

This paper presents a novel completion scheme based on TT rank for the recovery of color images and videos, which can be represented by three or high-order tensors. To build this scheme, we first employ a modified TT decomposition algorithm as tensor approximation scheme in the concept of TT rank to generate better-constructed and more balanced tensors which preserve only relatively significant informative data in tensors. We then introduce an adaptive weight algorithm, which is also based on TT rank, to define the value of weights adaptively in tensor completion problem. Finally, we present a LRATC-ATT based on the two schemes and SiLRTC-TT to estimate missing entries of color images or videos represented by tensors. We evaluate the proposed scheme with synthetic and real-world visual data, simulation results show that our scheme outperforms state-of-the-art approaches namely, HaLRTC, SiLRTC, SiLRTC-TT, t-TNN and TMac-TT, in tensor completion of visual data. However, the RSE results of LRATC-ATT decrease minimally when used in videos with vibrant colors. Thus, we aim to explore a solution to further improve the proposed algorithm in our next work.

In addition, the proposed scheme is efficient in reconstructing the background of color videos and can be used in other areas, such as background modeling, motion object detection and video indexing.

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References


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