PAPER

Design and SNR Optimization for Multi-Relay Compress-and-Forward System Based on CEO Theory

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SUMMARY A novel compress-and-forward (CF) system based on multi-relay network is proposed. In this system, two networks are linked, wherein one is a sensor network connecting the analog source and the relays, and the other is a communication network between the relays and the destination. At several parallel relay nodes, the analog signals are transformed into digital signals after quantization and encoding and then the digital signals are transmitted to the destination. Based on the Chief Executive Officer (CEO) theory, we calculate the minimum transmission rate of every source-relay link and we propose a system model by combining sensor network with communication network according to Shannon channel capacity theory. Furthermore, we obtain the best possible system performance under system power constraint, which is measured by signal-to-noise ratio (SNR) rather than bit error rate (BER). Numerical simulation results show that the proposed CF outperforms the traditional amplify-and-forward (AF) system in the performance versus SNR.

key words: compress-and-forward, multi-relay, amplify-and-forward, SNR

1. Introduction

Multi-relay system is widely used in wireless communication network, which offers reliable transmission, decreases the transmitted power, promotes spectrum utilization, enhances wireless network coverage and improves communication quality [1].

Amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) are three kinds of fundamental relay transmission schemes. For AF, the relays simply amplify the received signal and retransmit it to the destination, where the noise is also amplified at the same time [2]–[5]. For DF, the relays demodulate and decode the received source information at first, then re-encode and send to the destination, it will cause error propagation if a decoding error occurs [6]. For the last one, the destination node simultaneously uses the original signal from the source and the compressed signals from relay nodes for joint decoding. However, a direct channel between the source and destination is required [7], [8].

Different from the traditional AF system, in which the

relays simply amplify the received signal and retransmit to the destination, and CF system, whose destination jointly utilizes the original signal and the compressed signals from relay nodes for decoding, in this paper, a CF system model based on multi-relay network is proposed. In the proposed system, the analog signal is transformed into digital signal at several parallel relay nodes as the link between the analog source and destination cannot be utilized directly. Here, the status of these relay nodes are assumed completely identical. The proposed system consists of two parts, where one part is wireless sensor network (WSN) between the source and the relays, and the other one is digital communication network which connects the relays and the destination. We study the system performance versus SNR rather than BER owing to the distortion as the received digital signals at destination are sampled from the initial analog signals at relays, which is similar to noise interference during analog signals transmission. Furthermore, we improve the system performance versus SNR by optimizing power allocation. The proposed system is applicable to the situation that the emitter can only generate analog signals and the relay nodes can finish the processing of source coding as well as channel encoding, such as signal processing and feature extraction of high-resolution radar echoes. Without loss of generality, what the paper proposed is a theoretical analysis framework and the problem with encoding is not taken into account.

The rest of the paper is organized as follows. In Sect. 2, sensor network and digital communication network are reviewed by combining CEO theory and Shannon channel capacity theory, and the proposed multi-relay CF system is presented. Section 3 gives the methods of getting maximal SNR in the destination of the CF system and conventional AF system under system power constraints. Comparisons of performance versus SNR between the proposed CF system and that of AF system are provided in Sect. 4. Section 5 concludes the paper and Sect. 6 is an acknowledgement.

2. Multi-Relay CF System

2.1 System Model

Figure 1 depicts the model of the proposed CF system, in which an analog source S communicates with a destination D with the assistance of L relay nodes $R_i (i = 1, 2, \dots, L)$ and there is no direct link between S and D . The analog signal transmitted from S is sampled, quantized, compressed and forwarded by all relay nodes and finally reaches the des-

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Fig. 1 The model of multi-relay CF system

destination D . The destination collects all signals forwarded from every relay node with equal weight and sends them to the decoder to reconstruct original information.

Assume that the analog signal $X(t)$ transmitted from the source S follows Gaussian distribution $N(0, P_x)$. The received noise $N_{ri}(t)$ between every relay nodes and destination is additive white Gaussian noise (AWGN) with $N(0, \sigma_n^2)$, which is independently and identically distributed (i.i.d). Meanwhile, all the channels of source-relay and relay-destination are Gaussian channels in the real transmission environment of radar signal.

The signal $X(t)$ is generated from the source and the signal at the i^{th} relay node R_i as well as its power can be represented as

$$Y_{ri}(t) = h_{ri}X(t) + N_{ri}(t) = \sqrt{\alpha}X(t) + N_{ri}(t), \quad i = 1, 2, \dots, L \quad (1)$$

$$P_{ri} = \alpha P_x + \sigma_n^2, i = 1, 2, \dots, L \quad (2)$$

where $N_{ri}(t)$ denotes the Gaussian noise at the i^{th} relay node. h_{ri} is the channel coefficient between the source node and the i^{th} relay node. Without loss of generality, here what we considered is only propagation path loss in free-space, so

$$\alpha = \frac{G_T G_R c^2}{(4\pi R_{sr} f)^2} \quad (3)$$

where G_T and G_R are the transmitter and receiver antenna gains of source and relay node, respectively. c is velocity of light and f denotes the frequency of signal carrier. R_{sr} is the distance between source and relay node. For simplicity, we assume that all R_{sr} is identical.

When the received signal $Y_{ri}(t)$ is transformed into digital signal $Y'_{ri}(t)$ at each relay node, it is amplified and transmitted to the destination D with power P_{ri} . The received signal and its power of the destination from the i^{th} relay node are denoted by

$$Y_{di}(t) = h_{di}\beta Y'_{ri}(t) + N_{di}(t) = \sqrt{\mu}\beta Y'_{ri}(t) + N_{di}(t), i = 1, 2, \dots, L \quad (4)$$

$$P_{di} = \mu P_{ri} + \sigma_n^2, i = 1, 2, \dots, L \quad (5)$$

where $N_{di}(t)$ denotes the Gaussian noise at the destination and σ_n^2 is the power. β is the coefficient of power amplification and

$$\mu = \frac{G_R G_D c^2}{(4\pi R_{rd} f)^2} \quad (6)$$

is also the propagation path loss in free-space, where G_R and G_D are the antenna gains of relay and destination. R_{rd} is the distance between relay node and destination and we assume that R_{rd} is identical.

2.2 Theoretical Analysis

Since sensor networks links the source and relays, the signal cannot be directly observed and received by the destination unless the signal was compressed and encoded in advance at L relay nodes, which is similar to the CEO problem of source coding. The aim of CEO is to seek the tradeoff between two factors that one is the total rate the agents communicate with the CEO and the other is the distortion between the reconstructed information and the original data in the case that L tends to infinity. For the signal and noise that follow the Gaussian distribution, the CEO problem describes its rate-distortion region and the code rate after the coding process at the relay node under a certain distortion constraint [9]. In [10], in the case that every relay is identical, the expression of rate-distortion function about the quadratic Gaussian source is provided which is also applicable to the case of scalar Gaussian source. As a result, the compressed communication problem of analog Gaussian source in multi-relay network can be resolved in the similar way with the CEO problem of Gaussian source.

According to [11], if define the encoders functions $\varphi_i (i = 0, 1, \dots, L)$ by

$$\varphi_i : \chi_i \rightarrow \mathcal{M}_i = \{1, 2, \dots, M_i\} \quad (7)$$

for an arbitrary prescribed positive number δ , φ_i will satisfy rate constraints

$$\frac{1}{n} \log M_i \leq R_i + \delta \quad (8)$$

Simultaneously, the decoder function ψ is defined by

$$\psi : \mathcal{M}_0 \times \mathcal{M}_1 \times \dots \times \mathcal{M}_L \rightarrow \chi_0 \quad (9)$$

Assume that encoding at relays and decoding at the receiver are both available for the average distortion

$$d^n(\mathbf{X}, \hat{\mathbf{X}}) = \frac{1}{n} \sum_{i=1}^n E d(X, \hat{X}) \quad (10)$$

where X and $\hat{X}$ are the information $X(t)$ that transmitted from the source and $\hat{X}(t)$ that is reconstructed version of $X(t)$ and attained by decoding at the destination. Here

$$d(X, \hat{X}) = (X - \hat{X})^2 \quad (11)$$

is defined as squared distortion. For a given $d > 0$ and any positive $\delta > 0$, if the code rate $R_i (i = 1, \dots, L)$ is admissible, there exists encoder functions $\varphi_i (i = 1, \dots, L)$ and decoder function ψ such that $d^n(\mathbf{X}, \hat{\mathbf{X}}) \leq d + \delta$. Let $R(d)$ denote rate-distortion function, which is the set of all the admissible rate.

According to [11], the optimal sum rate after source coding at relay nodes is given by the following:

$$R(d) = \frac{1}{2} \ln^+ \left(\frac{\alpha P_x}{d} \right) - \frac{L}{2} \ln \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{[\alpha P_x - d]^+}{Ld} \right) \quad (12)$$

where $[a]^+ \triangleq \max\{0, a\}$ and $\ln^+ a \triangleq \max\{\ln a, 0\}$, αP_x denotes the average signal power received at each relay node. Equation (12) is also known as the total minimum transmission rate of all relay nodes. If the signal power at each relay node is identical to those of other nodes, the rate-distortion function after quantization in each relay node is

$$\begin{aligned} R_l(d) &= \frac{R(d)}{L} \\ &= \frac{1}{2L} \ln^+ \left(\frac{\alpha P_x}{d} \right) - \frac{1}{2} \ln \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{[\alpha P_x - d]^+}{Ld} \right) \end{aligned} \quad (13)$$

Taking quantization of SNR $\gamma_D = \frac{\alpha P_x}{d} \geq 1$ into (13),

$$R_l(d) = \frac{1}{2L} \ln \gamma_D - \frac{1}{2} \ln \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{\gamma_D - 1}{L} \right) \quad (14)$$

where $R_l(d)$ is the minimum transmission rate of each source-relay link.

2.3 Joint Design

Here, the source-relay link is contained in analog sensor network, while the relay-destination link in this system is digital communication network. It means joint design for the minimum transmission rate $R_l(d)$ and channel capacity C_l is a fundamental issue. Meanwhile, the output of the destination is digital signal Y_r which is recovered from quantized analog signal $X(t)$, where the inevitable distortion in quantization resembles the noise interference on analog signal and the system performance can be assessed by quantization SNR γ_D . Obviously, the maximum SNR of decoded signal in destination can be achieved by maximizing γ_D .

To assure that the destination can recover the signal transmitted from relay nodes without distortion, the rate-distortion $R_l(d)$ should be smaller than the channel capacity C_l of relay-destination link, i.e.,

$$R_l(d) \leq C_l \quad (15)$$

According to (3) and Shannon channel capacity theory, we assume that the signal power of the output at each relay node is identical and the corresponding channel capacity C_l can be presented as

$$C_l = \frac{1}{2} \ln \left(1 + \frac{\mu P_{ri}}{\sigma_n^2} \right) \quad (16)$$

Combining (14), (15) and (16),

$$\frac{1}{2L} \ln \gamma_D - \frac{1}{2} \ln \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{\gamma_D - 1}{L} \right) \leq \frac{1}{2} \ln \left(1 + \frac{\mu P_{ri}}{\sigma_n^2} \right)$$

(17)

To fully utilize the channel resource, the above equation needs to follow

$$\gamma_D^{\frac{1}{L}} \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{\gamma_D - 1}{L} \right)^{-1} = 1 + \frac{\mu P_{ri}}{\sigma_n^2}, \gamma_D \geq 1 \quad (18)$$

Take the power constraint into account,

$$\begin{cases} P_x + L P_{ri} = P \\ P_x \geq 0 \\ P_{ri} \geq 0 \end{cases} \quad (19)$$

The CF system model with maximum quantization SNR γ_D can be obtained by

$$\begin{aligned} \max : & \gamma_D \\ \text{s.t.} : & \gamma_D^{\frac{1}{L}} = \left[1 + \frac{\mu(P - P_x)}{L \sigma_n^2} \right] \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{\gamma_D - 1}{L} \right), \gamma_D \geq 1 \\ & 0 \leq P_x \leq P \end{aligned} \quad (20)$$

3. Maximum of SNR

In sensor network and wireless communication system, the power of source or each relay is dependent on battery that usually possesses limited power and energy. As a result, getting the best possible performance under power constraint is necessary.

3.1 Maximum of SNR in CF System

Here we give a complete solution to the optimization problem given by (20). We first consider an optimization problem related to the optimization problem.

For $\gamma_D \geq 1$ and $P_x \in [0, P]$, we set

$$\phi(P_x, \gamma_D) \triangleq \left[1 + \frac{\mu(P - P_x)}{L \sigma_n^2} \right] \left(1 - \frac{\sigma_n^2}{\alpha P_x} \frac{\gamma_D - 1}{L} \right) \quad (21)$$

Furthermore set

$$\phi(\gamma_D) = \max_{P_x \in [0, P]} \phi(P_x, \gamma_D) \quad (22)$$

Then we have the following lemma.

Lemma 1 Suppose that γ_D satisfies the following:

$$1 \leq \gamma_D \leq \frac{L \alpha \mu P^2}{\sigma_n^2 (L \sigma_n^2 + \mu P)} + 1 \quad (23)$$

then we have

$$\begin{aligned} \phi(\gamma_D) &= \max_{P_x \in [0, P]} \phi(P_x, \gamma_D) \\ &= \left(\sqrt{1 + \frac{\mu P}{L \sigma_n^2}} - \sqrt{\frac{\mu(\gamma_D - 1)}{L^2 \alpha}} \right)^2 \end{aligned} \quad (24)$$

The maximum $\phi(\gamma_D)$ is attained by

$$\frac{P_x}{P} = \sqrt{\frac{\sigma_n^2(L\sigma_n^2 + \mu P)}{L\alpha\mu P^2}}(\gamma_D - 1) \in [0, 1]. \quad (25)$$

We set

$$t = \frac{\mu(\gamma_D - 1)}{L^2\alpha} \Leftrightarrow \gamma_D = \gamma_D(t) = \frac{L^2\alpha}{\mu}t + 1 \quad (26)$$

so the condition (23) is equivalent to

$$\gamma_D = \frac{L^2\alpha}{\mu}t + 1, t \in [0, t_c], t_c \triangleq \frac{\mu^2 P^2}{L\sigma_n^2(L\sigma_n^2 + \mu P)} \quad (27)$$

Then we set

$$\begin{aligned} f(t) &\triangleq \gamma_D(t) - \phi^L(\gamma_D(t)) \\ &= \frac{L^2\alpha}{\mu}t + 1 - \left(\sqrt{1 + \frac{\mu P}{L\sigma_n^2}} - \sqrt{t} \right)^{2L} \end{aligned} \quad (28)$$

On the function $f(t)$, we have the following lemma.

Lemma 2 $f(t)$ is a monotone increasing function of t for $t \in [0, 1 + \frac{\mu P}{L\sigma_n^2}]$. Furthermore the equation $f(t) = 0$ has one unique root in $t \in [0, t_c]$

Let $(\gamma_D)_{\max}$ be the solution to the optimization problem of (20), which is related to the power allocation scheme of the CF system. The following proposition states that the solution $(\gamma_D)_{\max}$ is characterized with the root of $f(t) = 0$, which exists uniquely in $t \in [0, t_c]$.

Proposition 1 Let t_0 be the root of $f(t) = 0$, which exists uniquely in $t \in [0, t_c]$. Then we have

$$(\gamma_D)_{\max} = \frac{L^2\alpha}{\mu}t_0 + 1. \quad (29)$$

The details of the derivation of Lemma 1, Lemma 2 and Proposition 1 are attached in Appendix A, B and C.

3.2 Maximum of SNR in AF System

For the traditional multi-relay AF system, each source-relay-destination path is similar to a single-relay system and the destination will collect L signals from L relays simultaneously. If the power of signals received in relays are amplified β times and forwarded with power P_{ri} to destination, the signal at the destination is represented by

$$Y_d(t) = \sum_{i=1}^L \left\{ \sqrt{\mu}\beta \left[\sqrt{\alpha}X(t) + N_{ri}(t) \right] + N_{di}(t) \right\} \quad (30)$$

Actually, for such a multi-relay AF system, each of the sub-links of source-relay-destination is similar to a AF system with single one relay node. The corresponding SNR of every received signal at the destination that transmitted via each of the sub-links is as follow

$$\gamma_{Ai} = \frac{\alpha\mu P_{ri}P_x}{(\mu P_{ri} + \sigma_n^2 + \alpha P_x)\sigma_n^2} \quad (31)$$

After taking the power constraint (19) into account, the

maximization problem on the total SNR of all L signals at the destination is

$$\begin{aligned} \max : & \gamma_A \\ \text{s.t.} & \begin{cases} \gamma_A = \frac{L\alpha\mu(P-P_x)P_x}{[\mu(P-P_x) + L(\sigma_n^2 + \alpha P_x)]\sigma_n^2} \\ 0 < P_x < P \end{cases} \end{aligned} \quad (32)$$

Let $(\gamma_A)_{\max}$ be the optimal solution to the above optimization problem. We set $u \triangleq P_x/P$. For $u \in (0, 1)$, define

$$\varphi(u) \triangleq \frac{L(\sigma_n^2 + \alpha P)}{1 - u} + \frac{L\sigma_n^2 + \mu P}{u} \quad (33)$$

Then we have the following

$$\begin{aligned} (\gamma_A)_{\max} &= \max_{u \in (0,1)} \frac{L\mu\alpha P^2}{\sigma_n^2} [\varphi(u)]^{-1} \\ &= \frac{L\mu\alpha P^2}{\sigma_n^2} \left[\min_{u \in (0,1)} \varphi(u) \right]^{-1} \end{aligned} \quad (34)$$

By an elementary computation, we have

$$\min_{u \in (0,1)} \varphi(u) = \left[\sqrt{L(\sigma_n^2 + \alpha P)} + \sqrt{L\sigma_n^2 + \mu P} \right]^2 \quad (35)$$

The detail of the derivation is given in Appendix D. Hence $(\gamma_A)_{\max}$ is explicitly given by

$$(\gamma_A)_{\max} = \frac{L\mu\alpha P^2}{\sigma_n^2 \left[\sqrt{L(\sigma_n^2 + \alpha P)} + \sqrt{L\sigma_n^2 + \mu P} \right]^2} \quad (36)$$

4. Simulation Results

In this section, the maximum SNR γ_D and γ_A of CF and AF systems at the destinations, and the relevant power allocation between source and relay nodes are obtained by solving (28), (29) and (36), respectively. We assume that G_R , G_T and G_D are all equal to 1, carrier frequency is 2.5×10^9 Hz and noise power σ_n^2 is 1×10^{-9} W.

Figure 2 presents the performance plots of SNR γ_D and γ_A versus the relay nodes' number under the total power constraint, where R_{sr} and R_{rd} are equal to 250m. It is obviously shown that, with the relay nodes' number increasing, the SNR performance is improving for the case of P being

Fig. 2 SNR performance plots of CF and AF systems versus relay node's number

Fig. 3 SNR performance plots CF and AF systems versus the total power constraint

Fig. 4 SNR performance plots of CF and AF systems versus the distance

both 2W and 10W. Specifically, γ_D is always higher than γ_A with the same conditions and there exists a threshold for the performance improvement by increasing the number of relay nodes. As the channel noise stays constant, SNR of each sub-link decreases with the number of relay nodes beyond the threshold, which means the system performance cannot be substantially improved.

Figure 3 depicts γ_D and γ_A results versus P where R_{sr} and R_{rd} are equal to 250m. It shows clearly that when P is 1W, γ_D is larger than γ_A for at least 25dB. When P is increasing from 1W to 10W, the difference between γ_D and γ_A will witness a reduction of 9dB.

The SNR performance versus the distance between the source and destination is illustrated in Fig. 4, where P is 5W. It exhibits clearly that with the distance increasing from 200m to 800m, γ_D and γ_A both decrease. Specifically, γ_D converges to a lower bound with the distance increasing while γ_A is decreasing constantly, and γ_D is always higher than γ_A with the same conditions.

5. Conclusion

In this paper, the proposed system consists of two parts that one is the sensor network between the source and the relays, and the other is the communication network connecting the relays and the destination. We combine the CEO theory of source coding and Shannon channel capacity theory, and propose a CF system model. Moreover, we derive the OPA expression to maximize the SNR performance at the destination with fixed system power constraint. Numerical simulation results show that the proposed CF system outper-

forms the traditional AF system in SNR performance, which means that CF system can improve the SNR performance at the destination and the reliability in the transmitting process. Furthermore, the total system power is more fundamental to promote the system SNR performance than the relay nodes.

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Appendix A: Proof of Lemma 1

Proof: On upper bound of $\phi(P_x, \gamma_D)$, we have the following chain of inequalities:

$$\begin{aligned} \phi(P_x, \gamma_D) &= 1 + \frac{\mu P}{L\sigma_n^2} + \frac{\mu(\gamma_D - 1)}{L^2\alpha} \end{aligned}$$

$$\begin{aligned}
& - \left[\frac{\mu P}{L\sigma_n^2} \cdot \frac{P_x}{P} + \left(1 + \frac{\mu P}{L\sigma_n^2} \right) \frac{\sigma_n^2(\gamma_D - 1)}{L\alpha P} \cdot \frac{P}{P_x} \right] \\
& \stackrel{(a)}{\leq} 1 + \frac{\mu P}{L\sigma_n^2} + \frac{\mu(\gamma_D - 1)}{L^2\alpha} \\
& - 2 \sqrt{\frac{\mu P}{L\sigma_n^2} \cdot \frac{P_x}{P} \cdot \left(1 + \frac{\mu P}{L\sigma_n^2} \right) \frac{\sigma_n^2(\gamma_D - 1)}{L\alpha P} \cdot \frac{P}{P_x}} \\
& = 1 + \frac{\mu P}{L\sigma_n^2} + \frac{\mu(\gamma_D - 1)}{L^2\alpha} - 2 \sqrt{\left(1 + \frac{\mu P}{L\sigma_n^2} \right) \frac{\mu(\gamma_D - 1)}{L^2\alpha}} \\
& = \left(\sqrt{1 + \frac{\mu P}{L\sigma_n^2}} - \sqrt{\frac{\mu(\gamma_D - 1)}{L^2\alpha}} \right)^2. \tag{A.1}
\end{aligned}$$

Step (a) follows from the well-known inequality $a + b \geq 2\sqrt{ab}$ for $a, b \geq 0$. In the upper bound (A.1), the equality holds for

$$\frac{\mu P}{L\sigma_n^2} \cdot \frac{P_x}{P} = \left(1 + \frac{\mu P}{L\sigma_n^2} \right) \frac{\sigma_n^2(\gamma_D - 1)}{L\alpha P} \cdot \frac{P}{P_x} \tag{A.2}$$

or equivalent to

$$\frac{P_x}{P} = \sqrt{\frac{\sigma_n^2(L\sigma_n^2 + \mu P)}{L\alpha\mu P^2}(\gamma_D - 1)} \tag{A.3}$$

Since γ_D satisfies the condition (23), we have

$$0 \leq \frac{P_x}{P} \leq \sqrt{\frac{\sigma_n^2(L\sigma_n^2 + \mu P)}{L\alpha\mu P^2} \frac{L\alpha\mu P^2}{\sigma_n^2(L\sigma_n^2 + \mu P)}} = 1 \tag{A.4}$$

Hence the equality in (A.1) is attainable. $\square$

Appendix B: Proof of Lemma 2

Proof: By its formula it is obvious that $f(t)$ is a monotone increasing function of t for $t \in [0, 1 + \frac{\mu P}{L\sigma_n^2}]$. We next compute $f(0)$ and $f(t_c)$ to examine their signs. On $f(0)$, we have

$$f(0) = 1 - \left(1 + \frac{\mu P}{L\sigma_n^2} \right)^L < 0 \tag{A.5}$$

Computing $f(t_c)$, we have the following:

$$\begin{aligned}
f(t_c) &= \frac{L^2\alpha}{\mu} \cdot \frac{\mu^2 P^2}{L\sigma_n^2(L\sigma_n^2 + \mu P)} + 1 \\
& - \left(\sqrt{1 + \frac{\mu P}{L\sigma_n^2}} - \sqrt{\frac{\mu^2 P^2}{L\sigma_n^2(L\sigma_n^2 + \mu P)}} \right)^{2L} \\
&= \frac{\mu\alpha P^2}{\sigma_n^4} \cdot \frac{1}{1 + \frac{\mu P}{L\sigma_n^2}} + 1 - \left(1 + \frac{\mu P}{L\sigma_n^2} \right)^L \left(\frac{L\sigma_n^2}{L\sigma_n^2 + \mu P} \right)^{2L} \\
&= \frac{\mu\alpha P^2}{\sigma_n^4} \cdot \frac{1}{1 + \frac{\mu P}{L\sigma_n^2}} + 1 - \left(\frac{L\sigma_n^2}{L\sigma_n^2 + \mu P} \right)^L > 0 \tag{A.6}
\end{aligned}$$

From (A.5) and (A.6), we know that the equation $f(t) = 0$ has one unique root in $t \in [0, t_c]$. $\square$

Appendix C: Proof of Proposition 1

Proof: We set

$$\mathcal{D} \triangleq \left\{ \gamma_D : \gamma_D^{\frac{1}{L}} = \phi(P_x, \gamma_D) \text{ for some } P_x \in [0, P] \right\} \tag{A.7}$$

Then we have

$$(\gamma_D)_{\max} = \max_{\gamma_D \in \mathcal{D}} \gamma_D \tag{A.8}$$

We first prove that the quantity $(L^2\alpha/\mu)t_0 + 1$ serves as an upper bound of $\gamma_D \in \mathcal{D}$. We have the following:

$$\begin{aligned}
\gamma_D \in \mathcal{D} &\Leftrightarrow \gamma_D \geq 1, \gamma_D^{\frac{1}{L}} = \phi(P_x, \gamma_D) \\
&\text{for some } P_x \in [0, P] \stackrel{(a)}{\Rightarrow} \gamma_D \geq 1, \gamma_D^{\frac{1}{L}} \leq \phi(\gamma_D) \\
&\Leftrightarrow \gamma_D = \frac{L^2\alpha}{\mu}t + 1, t \geq 0, \gamma_D \leq \phi^L(\gamma_D) \\
&\Leftrightarrow \gamma_D = \frac{L^2\alpha}{\mu}t + 1, t \geq 0, \\
&\quad \frac{L^2\alpha}{\mu}t + 1 \leq \phi^L\left(\frac{L^2\alpha}{\mu}t + 1\right) \\
&\Leftrightarrow \gamma_D = \frac{L^2\alpha}{\mu}t + 1, t \geq 0, f(t) \leq 0 \\
&\stackrel{(b)}{\Rightarrow} \gamma_D = \frac{L^2\alpha}{\mu}t + 1, 0 \leq t \leq t_0 \leq t_c \\
&\Rightarrow 1 \leq \gamma_D \leq \frac{L^2\alpha}{\mu}t_0 + 1, t_0 < t_c. \tag{A.9}
\end{aligned}$$

Step (a) follows from Lemma 1. Step (b) follows from Lemma 2. Since $\gamma_D \in \mathcal{D}$ is arbitrary, we have

$$(\gamma_D)_{\max} \leq \frac{L^2\alpha}{\mu}t_0 + 1 \tag{A.10}$$

We next prove that the equality holds in (A.10). Suppose that $\gamma_D \in \mathcal{D}$. Then by (A.9), we have

$$1 \leq \gamma_D \leq \frac{L^2\alpha}{\mu}t_c + 1 = \frac{L\alpha\mu P^2}{L\sigma_n^2 + \mu P} + 1 \tag{A.11}$$

For such γ_D , we choose P_x so that

$$\frac{P_x}{P} = \sqrt{\frac{\sigma_n^2(L\sigma_n^2 + \mu P)}{L\alpha\mu P^2}(\gamma_D - 1)} \stackrel{(a)}{\in} [0, 1] \tag{A.12}$$

Setp (a) follows from (A.11). Then by Lemma 1 we have that for the choice of $P_x = P_x(\gamma_D)$ given in (A.12), we have $\phi(P_x, \gamma_D) = \phi(\gamma_D)$. Hence γ_D must satisfy the following:

$$\begin{aligned}
&\gamma_D^{\frac{1}{L}} = \phi(\gamma_D), 1 \leq \gamma_D \leq \frac{L^2\alpha}{\mu}t_c + 1 \\
&\Leftrightarrow \gamma_D = \phi^L(\gamma_D), 1 \leq \gamma_D \leq \frac{L^2\alpha}{\mu}t_c + 1 \tag{A.13}
\end{aligned}$$

From (A·13), we have the following:

$$\begin{aligned}\gamma_D &= \frac{L^2\alpha}{\mu}t + 1, t \in [0, t_c], \frac{L^2\alpha}{\mu}t + 1 = \phi^L\left(\frac{L^2\alpha}{\mu}t + 1\right) \\ \Leftrightarrow \gamma_D &= \frac{L^2\alpha}{\mu}t + 1, t \in [0, t_c], f(t) = 0 \\ \Leftrightarrow \gamma_D &= \frac{L^2\alpha}{\mu}t_0 + 1\end{aligned}\quad (\text{A} \cdot 14)$$

From (A·14), we know that the equality holds in (A·10). $\square$

Appendix D: Computation of $\min_{u \in [0,1]} \varphi(u)$

For simplicity of notation, we set $A = L(\sigma_n^2 + \alpha P)$, $B = L\sigma_n^2 + \mu P$. Since

$$\varphi''(u) = \frac{2A}{(1-u)^3} + \frac{2B}{u^3} > 0 \quad \text{for } u \in (0, 1) \quad (\text{A} \cdot 15)$$

The minimum of $\varphi(u)$ over $u \in (0, 1)$ is attained by some $u_0 \in (0, 1)$, satisfying

$$\begin{aligned}\varphi'(u_0) &= -\frac{A}{(1-u_0)^2} + \frac{B}{u_0^2} \\ &= \left(\frac{\sqrt{A}}{1-u_0} + \frac{\sqrt{B}}{u_0}\right) \left(-\frac{\sqrt{A}}{1-u_0} + \frac{\sqrt{B}}{u_0}\right) \\ &= 0\end{aligned}\quad (\text{A} \cdot 16)$$

Solving (A·16), we get

$$u_0 = \frac{\sqrt{B}}{\sqrt{A} + \sqrt{B}}, \quad \varphi(u_0) = \frac{A}{1-u_0} + \frac{B}{u_0} = (\sqrt{A} + \sqrt{B})^2 \quad (\text{A} \cdot 17)$$

Thus, we have (29).

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