Leakage-Resilient and Proactive Authenticated Key Exchange (LRP-AKE), Reconsidered

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SUMMARY In [31], Shin et al. proposed a Leakage-Resilient and Proactive Authenticated Key Exchange (LRP-AKE) protocol for credential services which provides not only a higher level of security against leakage of stored secrets but also secrecy of private key with respect to the involving server. In this paper, we discuss a problem in the security proof of the LRP-AKE protocol, and then propose a modified LRP-AKE protocol that has a simple and effective measure to the problem. Also, we formally prove its AKE security and mutual authentication for the entire modified LRP-AKE protocol. In addition, we describe several extensions of the (modified) LRP-AKE protocol including 1) synchronization issue between the client and server's stored secrets; 2) randomized ID for the provision of client's privacy; and 3) a solution to preventing server compromise-impersonation attacks. Finally, we evaluate the performance overhead of the LRP-AKE protocol and show its test vectors. From the performance evaluation, we can confirm that the LRP-AKE protocol has almost the same efficiency as the (plain) Diffie-Hellman protocol that does not provide authentication at all.

key words: password, two-factor authentication, key exchange, leakage of stored secrets, credential services, provable security

1. Introduction

An Authenticated Key Exchange (AKE) protocol is a fundamental cryptographic primitive by which involving parties can authenticate each other and generate secure session keys for protecting subsequent messages to be exchanged. For several decades, many different types of AKE protocols have been studied in the literature (c.f. [1]). In practice, AKE has been widely used in most real-world applications (e.g., online shopping, Internet banking, webmail, SNS, remote network access). A representative example is SSL/TLS, where one can find extensive AKE protocols in the TLS 1.2 cipher suits listed in the IETF RFC 5246 [5].

In [31], Shin et al. proposed a Leakage-Resilient and Proactive Authenticated Key Exchange (for short, LRP-AKE) protocol for credential services (e.g., [17]) where a roaming client has easy-to-be-lost/stolen devices (e.g., smartphones) with some memory capacity and wants to retrieve his/her private key with the help of a credential server equipped with its database, which might be insecure against possible attacks (e.g., viruses, hackers, insider attacks). The retrieved private key is used in the conventional PKI (Public-Key Infrastructures). However, the LRP-AKE protocol itself does not depend on PKI and TPM/TRM

(Tamper-Proof/Resistant Modules). Please, refer to [31] for more details.

Compared to the previous password-enabled PKI [12], [23], [28], [33] and two-factor authentication [6], [7], [16], [18], the LRP-AKE protocol provides not only a higher level of security against leakage of stored secrets but also secrecy of private key with respect to the involving server. Also, Shin et al. proved that the LRP-AKE protocol is secure in the random oracle model [3] under the computational Diffie-Hellman problem. Note that the existing widely-deployed two-factor authentication [6], [7], [16], [18] are insecure against password list attacks, Phishing attacks, and offline dictionary attacks after leakage of stored secrets from the server side because these are just a combination of password authentication and 2nd-factor (e.g., OTP/PIN/SMS) authentication. Very recently, the LRP-AKE protocol has been standardized in ISO/IEC 11770-4:2017/Amd 2:2021 [10].

1.1 Our Contributions

In this paper, we discuss what would be a problem in the security proof of the LRP-AKE protocol and its possible solutions. First, we show that the “simplified” security proof [31] for the LRP-AKE protocol cannot be applied to the entire protocol because an adversary can easily break the semantic security of session keys. Then, we propose a modified LRP-AKE protocol that has a simple and effective measure to the problem, and formally prove its AKE security (in Definition 1) and mutual authentication (in Definition 2) for the entire modified LRP-AKE protocol.

Also, we show several extensions of the (modified) LRP-AKE protocol including 1) synchronization issue between the client and server's stored secrets; 2) randomized ID for the provision of client's privacy; and 3) a solution to preventing server compromise-impersonation attacks. After describing our implementation with the elliptic curve domain parameters [25] and an efficiency-measurement environment, we evaluate the performance overhead of the LRP-AKE protocol and show its test vectors. For example, the average processing time of the LRP-AKE client with the elliptic curve domain parameter SECp256r1 [25] is 142.2 ms while that of the 3-pass Diffie-Hellman client is 129.4 ms (please, refer to Sect. 8 for details). From the performance evaluation, we can confirm that the LRP-AKE protocol has almost the same efficiency as the (plain) Diffie-Hellman protocol [4] that does not provide authentication at all.

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2. Notation

Here, we give some notation to be used throughout this paper. Let $\mathbb{G}$ be a finite, cyclic group of prime order q and g be a generator of $\mathbb{G}$ (quadratic residues modulo p where $p = aq + 1$) where the Diffie-Hellman problem is hard. Let h be another generator of $\mathbb{G}$ so that its discrete logarithm problem with g (i.e., computing $b = \log_g h$) should be hard. Both g and h are given as public parameters [13]. In the aftermath, all the subsequent arithmetic operations are performed in modulo p unless otherwise stated.

Let k and l denote the security parameters for hash functions and public-key cryptosystems, respectively. Also, let N be a dictionary size of passwords. Let $\{0, 1\}^*$ denote the set of finite binary strings and $\{0, 1\}^k$ the set of binary strings of length k . If D is a set, then $d \xleftarrow{R} D$ indicates the process of selecting d at random and uniformly over D . Let $\parallel$ denote the concatenation of bit strings in $\{0, 1\}^*$. Let $\oplus$ denote the exclusive-OR operation of bit strings. The hash functions are denoted by $H_j : \{0, 1\}^* \rightarrow \{0, 1\}^{k_j}$ for $j = 1, 2, 3, 4, 5$ and 6 where H_j are distinct secure one-way hash functions (e.g., SHA-2/3 [20], [21]) one another. And we denote by AE a secure authentication encryption [2] (e.g., AES-GCM [29]). Let C and S be the identities of client and server, respectively, with each $\text{ID} \in \{0, 1\}^*$. Let (K', Cert_C) be a credential where the former is a partial secret for client's private key K associated with a public key, which is included in the latter (i.e., the client's certificate Cert_C) with other information.

3. The LRP-AKE Protocol

In this section, we describe the LRP-AKE protocol [31] which consists of the initialization phase and i -th ($i \geq 1$) protocol execution phase.

3.1 Initialization

In this phase, client C registers two kinds of secrets to server S where one is used for authentication and generation of session keys, and the other is used for recovering the client's private key. First, the client chooses two random numbers $s_1 \xleftarrow{R} \mathbb{Z}_q^*$ and $K_{11} \xleftarrow{R} \{0, 1\}^l$ where K_{11} is a partial secret and has the same size of his/her private key K . The client generates a verification data $W_1 \equiv h^{v_1}$, which is computed with secret value s_1 and his/her password pw (i.e., $v_1 \equiv s_1 + pw \pmod{q}$), and the other partial secret $K_{12} = K_{11} \oplus K$ from K_{11} and the private key K . Then, client C registers securely W_1 and K_{12} along with his/her certificate Cert_C .[†] At the end of this phase, the client stores secret value s_1 and one partial secret key K_{11} on devices that may happen to leak these secrets, and remembers his/her password pw in mind.

[†]Note that all the currently-used password-based protocols require users to register their passwords securely in the initialization phase.

On the other hand, the server stores W_1 and (K_{12}, Cert_C) on its database. Finally, they set a counter i as 1. Of course, all the other intermediate values should be deleted from the devices. This initialization is securely done only once.

3.2 i -th ($i \geq 1$) Protocol Execution

3.2.1 Masked Diffie-Hellman Protocol

When client C wants to share a session key securely with server S in the i -th ($i \geq 1$) protocol execution phase, he/she should recover v_i by combining his/her password pw with secret value s_i stored on devices: $v_i \equiv s_i + pw \pmod{q}$. The client chooses a random number $x \xleftarrow{R} \mathbb{Z}_q^*$ and computes its Diffie-Hellman public value $X \equiv g^x$. The latter is masked in a way of the product of the public value with the verification data $W_i \equiv h^{v_i}$: $X^* \equiv X \times W_i$. Then, the X^* is sent to the server together with the client's identity C and the counter i . If i is not a correct counter, server S terminates the protocol.

Otherwise, the server chooses a random number $y \xleftarrow{R} \mathbb{Z}_q^*$ and computes its Diffie-Hellman public value $Y \equiv g^y$. The server also computes X by dividing the received masked public value with the verification data: $X \equiv X^*/W_i$. From the resultant value X , the Diffie-Hellman key $\text{DHK} \equiv X^y$ is derived. With the W_i and DHK , server S easily generates an authenticator $V_S = H_1(\text{SID} \parallel \text{DHK})$, where a session identifier $\text{SID} = C \parallel S \parallel i \parallel X^* \parallel Y \parallel W_i$, and sends its Diffie-Hellman public value Y along with the server's identity S and the authenticator V_S . After computing $\text{DHK} \equiv Y^x$ from Y , client C verifies the received authenticator V_S . If $V_S \neq H_1(\text{SID} \parallel \text{DHK})$, the client terminates the protocol. Otherwise, client C generates his/her authenticator $V_C = H_2(\text{SID} \parallel \text{DHK})$ and a session key $\text{SK} = H_3(\text{SID} \parallel \text{DHK})$, and then sends the authenticator V_C . Similarly, server S verifies the received authenticator V_C . If $V_C \neq H_2(\text{SID} \parallel \text{DHK})$, the server terminates the protocol. Otherwise, server S generates a session key $\text{SK} = H_3(\text{SID} \parallel \text{DHK})$. In order to avoid so-called partition attacks [22], [34], both of client and server should check the subgroup order of Y and X^* , respectively: $Y^q \equiv (X^*)^q \equiv 1$.^{††}

3.2.2 Recovering Private Key and Refreshing Stored Secrets

Using the session key in an authenticated encryption AE, server S sends the credential (K_{12}, Cert_C) to client C through the secure channel. Finally, the client recovers his/her private key from one partial secret (stored on own devices) and the other secret (received from the server), $K = K_{11} \oplus K_{12}$, so that a pair of private key and certificate can be used as in the usual PKI. If K is not the correct private key, client C terminates the protocol. Otherwise, the client stores the next counter $(i + 1)$ and newly-generated secrets $(s_{(i+1)}, K_{(i+1)1})$ that are refreshed from the hash of the Diffie-Hellman key and other information: $s_{(i+1)} \equiv s_i + H_4(\text{SID} \parallel \text{DHK}) \pmod{q}$ and $K_{(i+1)1} = K_{11} \oplus H_5(\text{SID} \parallel \text{DHK})$. In the same way,

^{††}Alternatively, one can use a secure prime p in [30].

server S stores the next counter $(i + 1)$ and refreshed secrets $(W_{(i+1)}, (K_{(i+1)2}, Cert_C))$ on its database: $W_{(i+1)} \equiv W_i \times h^{H_4(SID||DHK)}$ and $K_{(i+1)2} = K_{i2} \oplus H_5(SID||DHK)$. These stored secrets will be used for the next session between the parties without changing the client's password. Note that the frequent change of passwords might incur the risk of password to be exposed, simply because people tends to write it down on somewhere or needs considerable efforts to remember new passwords. The hash functions H_4 and H_5 used here can be replaced by pseudo random functions to produce appropriate output sizes.

4. Security Model

Here, we introduce the security model [31], which was used to prove security of the LRP-AKE protocol.

We denote by C and S two parties that participate in an authenticated key exchange protocol P . Each of them may have several instances called oracles involved in distinct, possibly concurrent, executions of P . We denote C (resp., S) instances by C^ζ (resp., S^η) where $\zeta, \eta \in \mathbb{N}$, or by U in the case of any instance. For the i -th ($i \geq 1$) session in the LRP-AKE protocol, client C remembers a low-entropy secret pw drawn from a small dictionary of password $\mathbb{D}_{\text{Password}}$, whose cardinality is N , and holds another secret s_i and a partial private key K_{i1} on insecure devices. On the other hand, server S stores a verification data W_i and the client's credential $(K_{i2}, Cert_C)$. During the execution of the protocol, an adversary has the entire control of the network and additionally has access to the parties' stored secrets where the latter simulates *insecure* devices and databases. Let us show the capability of adversary $\mathcal{A}$ each query captures:

- **Execute**(C^ζ, S^η): This query models passive attacks, where the adversary gets access to honest executions of P between the instances C^ζ and S^η by eavesdropping.
- **Send**(U, m): This query models active attacks by having $\mathcal{A}$ send a message to instance U . The adversary $\mathcal{A}$ gets back the response U generates in processing the message m according to the protocol P . A query **Send**(C^ζ, Start) initializes the protocol, and thus the adversary receives the first flow message.
- **Reveal**(U): This query handles the misuse of the session key (e.g., use in a weak symmetric-key encryption) by any instance U . The query is only available to $\mathcal{A}$, if the instance actually holds a session key, and the latter is released to $\mathcal{A}$.
- **Leak**(U, i): This query handles the leakage of the i -th stored secrets by any instance U . The adversary $\mathcal{A}$ gets back the secrets (i, s_i, K_{i1}) or $(i, W_i, (K_{i2}, Cert_C))$ where the former (resp., the latter) is released if the instance corresponds to C^ζ (resp., S^η) and an instance of the $(i - 1)$ -th session[†] was accepted. The query is available to $\mathcal{A}$ since the stored secrets might be leaked out due to lost/stolen devices, viruses or mis-behaviors of system

[†]When $i = 1$, the 0-th session corresponds to the initialization phase of the LRP-AKE protocol.

administrators.

- **Test**(U): This oracle is used to see whether or not the adversary can obtain some information on the session key by giving a hint on the key. The **Test**-query can be asked at most once by the adversary $\mathcal{A}$ and is only available to $\mathcal{A}$ if the instance U is "fresh" (see below). This query is answered as follows: One flips a (private) coin $b \in \{0, 1\}$ and forwards the corresponding session key SK (**Reveal**(U) would output) if $b = 1$, or a random value with the same size except the session key if $b = 0$.

We say that an instance U is fresh only if an adversary $\mathcal{A}$ can not distinguish the session key SK_b in a trivial way (e.g., with the **Reveal**(U)-query or the **Leak**($S^\eta, *$)-query). The latter is due to the fact that in a 2-party password-based AKE protocol one cannot prove the security when an adversary gets the verification data. However, the **Leak**($S^\eta, *$)-query would be considered for security of the client's password and private key (see Sect. 4.2 of [31]).

5. A Problem

Here, we discuss what would be a problem [26] in the security proof of the LRP-AKE protocol.

In [31], Shin et al. showed that the LRP-AKE protocol is provably secure in the random oracle model [3], and discussed security of the client's password and private key. In order to simplify the security proof, they only considered the first two flows of the i -th protocol execution (i.e., unilateral authentication of S to C) where the **Leak**-query is restricted to instance C^ζ as discussed in Sect. 4.

Now, we consider all the flows of the i -th protocol execution of the LRP-AKE protocol. In this case, there exists a problem in the security proof because adversary $\mathcal{A}$ can distinguish the session key SK_b by decrypting the ciphertext of the credential $(K_{i2}, Cert_C)$. If $\mathcal{A}$ correctly decrypts the ciphertext with SK_b , then b will be 1. Otherwise, b will be 0.

Remark 1: The LRP-AKE protocol was renamed as "LKAM1" in the ISO/IEC 11770-4:2017/Amd 2:2021 IS (International Standard) [10]. However, this LKAM1 does not inherit the above problem since the recovering private key and certificate part of the LRP-AKE protocol [31] was excluded.

6. A Modified LRP-AKE Protocol

In this section, we propose a modified LRP-AKE protocol that has a simple and effective measure to the problem discussed in Sect. 5, and then formally prove its security for the entire modified LRP-AKE protocol.

6.1 The Protocol

The problem in Sect. 5 is due to the fact that the session key is directly used for an authenticated encryption in the recovering private key and certificate part (see Sect. 3.2.2)

Fig. 1 The initialization phase of the (modified) LRP-AKE protocol where the enclosed values in the rectangle represent stored secrets of client and server, respectively

Fig. 2 The i -th ($i \geq 1$) protocol execution phase of the modified LRP-AKE protocol where $SID = C || S || i || X^* || Y || W_i$ plays a role of session identifier and the enclosed values in the rectangle represent stored secrets of client and server, respectively

of the LRP-AKE protocol [31].

In the modified LRP-AKE protocol, we slightly change only the recovering private key and certificate part (Sect. 3.2.2) as follows: Instead of using the session key SK , both of client C and server S generate an encryption key $EK = H_6(SID || DHK)$ which will be used for an authenticated encryption AE in order to encrypt/decrypt the credential $(K_{i2}, Cert_C)$. Since we are working in the random oracle model, this modification is the simplest solution anyway. A graphical description of the modified LRP-AKE protocol

can be found in Fig. 1 and Fig. 2.

6.2 Security Definitions and Assumption

The adversary $\mathcal{A}$ is provided with random coin tosses, some oracles and then is allowed to invoke any number of queries as described in Sect. 4, in any order. The aim of the adversary is to break the privacy of the session key (a.k.a., semantic security) or authentication of the parties in the context of executing P .

Definition 1: (AKE Security) The AKE security is defined by the game $\text{Game}^{\text{ake}}(\mathcal{A}, P)$, in which the ultimate goal of the adversary is to guess the bit b involved in the Test-query by outputting this guess b' . We denote the AKE advantage, by $\text{Adv}_P^{\text{ake}}(\mathcal{A}) = 2 \Pr[b = b'] - 1$, as the probability that $\mathcal{A}$ can correctly guess the value of b . The protocol P is said to be (t, ε) -AKE-secure if $\mathcal{A}$'s advantage is smaller than ε for any adversary $\mathcal{A}$ running time t .

Definition 2: (MA) Another goal is to consider mutual authentication of C (C -auth) and S (S -auth) wherein the adversary impersonates either party. We denote by $\text{Succ}_P^{\text{ma}}(\mathcal{A})$ the probability that $\mathcal{A}$ successfully impersonates an C instance (resp., an S instance) in an execution of P , which means that S (resp., C) agrees on a key while the latter is shared with no instance of C (resp., S). A protocol P is said to be (t, ε) -MA-secure if $\mathcal{A}$'s success probability for breaking either C -auth or S -auth is smaller than ε for any adversary $\mathcal{A}$ running time t .

Also, there is a computational assumption on which the modified LRP-AKE protocol is based.

Assumption 1: (CDH) A (t, ε) -CDH $_{g, \mathbb{G}}$ attacker, in a finite cyclic group $\mathbb{G}$ of prime order q with g as a generator, is a probabilistic machine $\mathcal{B}$ running in time t such that its success probability $\text{Succ}_{g, \mathbb{G}}^{\text{cdh}}(\mathcal{B})$, given random elements g^x and g^y to output g^{xy} , is greater than ε . We denote by $\text{Succ}_{g, \mathbb{G}}^{\text{cdh}}(t)$ the maximal success probability over every adversaries running within time t . The CDH assumption states that $\text{Succ}_{g, \mathbb{G}}^{\text{cdh}}(t) \leq \varepsilon$ for any t/ε not too large.

6.3 Security Proof

Here, we show that the modified LRP-AKE protocol of Fig. 2 is provably secure in the random oracle model [3]. In other words, the modified LRP-AKE protocol distributes session keys that are semantically-secure and provides mutual authentication.

Theorem 1: Let P be the modified LRP-AKE protocol of Fig. 2 where passwords are chosen from a dictionary of size N . For any adversary $\mathcal{A}$ within a polynomial time t , with less than q_{se} active interactions with the parties (Send-queries), q_{ex} passive eavesdroppings (Execute-queries) and asking q_{H} hash queries to any H_j , $\text{Adv}_P^{\text{ake}}(\mathcal{A}) \leq 4\varepsilon$ and $\text{Succ}_P^{\text{ma}}(\mathcal{A}) \leq \varepsilon$, with ε upper-bounded by

$$\frac{3q_{\text{se}}}{N} + 7q_{\text{H}}^2 \times \text{Succ}_{g, \mathbb{G}}^{\text{cdh}}(t + 3\tau_e) + \frac{q_{\text{se}}}{2^{k_1}} + \frac{q_{\text{se}}}{2^{k_2}} + \frac{8q_{\text{se}} + 2q_{\text{ex}} + (q_{\text{ex}} + q_{\text{se}})^2}{2q}, \quad (1)$$

where k_1 (resp., k_2) is the output length of H_1 (resp., H_2) and τ_e denotes the computational time for an exponentiation in $\mathbb{G}$.

This theorem indicates that the modified LRP-AKE protocol is secure against offline dictionary attacks since the advantage of the adversary essentially grows with the ratio of interactions to the number of passwords. Note that in order to

perform online dictionary attacks successively the adversary should obtain the client's stored secrets every time.

Proof 1: In this proof, we define a sequence of games starting at the real protocol $\mathbf{G}_0$ and ending up at $\mathbf{G}_5$ where we bound the probability of each event by using Shoup's difference lemma [27]. For clarity, we denote by Event_n an event Event considered in **Game $\mathbf{G}_n$** .

Game $\mathbf{G}_0$: This is the real protocol in the random oracle model. We are interested in the following two events:

- $\mathbf{S}_0$ which occurs if the adversary correctly guesses the bit b involved in the Test-query;
- $\mathbf{A}_0$ which occurs if an instance C^ζ (resp., S^η) accepts with no partner instance S^η (resp., C^ζ) with the same transcript $((C, i, X^*), (S, Y, V_S), V_C)$

$$\text{Adv}_P^{\text{ake}}(\mathcal{A}) = 2 \Pr[\mathbf{S}_0] - 1, \quad \text{Succ}_P^{\text{ma}}(\mathcal{A}) = \Pr[\mathbf{A}_0]. \quad (2)$$

In any game $\mathbf{G}_n$ below, we study the event $\mathbf{A}_n$ and the restricted event $\text{SwA}_n = \mathbf{S}_n \wedge \neg \mathbf{A}_n$.

Game $\mathbf{G}_1$: In this game, we simulate the hash oracles (H_j , but as well additional hash functions, for $j = 1, 2, 3, 4, 5, 6$, $H'_j : \{0, 1\}^* \rightarrow \{0, 1\}^{k_j}$ that will appear in the game $\mathbf{G}_3$) by maintaining hash lists Λ_{H} and $\Lambda_{\text{H}'}$ (see below). We also simulate all the instances, as the real parties would do, for the Send-queries and for the Execute, Reveal, Leak and Test-queries (see further below). From this simulation, we can easily see that the game is perfectly indistinguishable from the real attack.

Simulation of the hash functions: H_j oracles

- For a hash-query $H_j(q)$ (resp., $H'_j(q)$), such that a record (j, q, r) appears in Λ_{H} (resp., $\Lambda_{\text{H}'}$), the answer is r . Otherwise one chooses a random element $r \xleftarrow{R} \{0, 1\}^{k_j}$, answers with it, and adds the record (j, q, r) to Λ_{H} (resp., $\Lambda_{\text{H}'}$).

Simulation of the LRP-AKE protocol

Send-queries to C

We answer to the Send-queries to a C -instance as follows:

- A $\text{Send}(C^\zeta, \text{Start})$ -query is processed according to the following rules:

► **Rule C1⁽¹⁾**

Choose random elements $(b, v_i) \xleftarrow{R} (\mathbb{Z}_q^*)^2$, compute $h \equiv g^b$ and $W_i \equiv h^{v_i}$, and set $s_i \equiv s_{(i-1)} + r \pmod q$ where r is the reply of H_4 in the $(i-1)$ -th "accepted" instance. Also, we set $K_{i1} = K_{(i-1)1} \oplus r$ where r is the reply of H_5 in the $(i-1)$ -th "accepted" instance.

► **Rule C2⁽¹⁾**

Choose a random element $\theta \xleftarrow{R} \mathbb{Z}_q^*$, and compute $X \equiv g^\theta$ and $X^* \equiv X \times W_i$. Then the query is

answered with (C, i, X^*) , and the instance goes to an expecting state.

- If the instance C^ζ is in an expecting state, a query $\text{Send}(C^\zeta, (S, Y, V_S))$ is processed by computing the Diffie-Hellman key, authenticators, session key and encryption key. We apply the following rules.

► **Rule C3⁽¹⁾**

Compute $DHK \equiv Y^\theta$.

► **Rule C4⁽¹⁾**

Compute the expected authenticators, session key and encryption key:

$$V'_S = H_1(C\|S\|i\|X^*\|Y\|W_i\|DHK),$$

$$V_C = H_2(C\|S\|i\|X^*\|Y\|W_i\|DHK),$$

$$SK = H_3(C\|S\|i\|X^*\|Y\|W_i\|DHK),$$

$$EK = H_6(C\|S\|i\|X^*\|Y\|W_i\|DHK).$$

If $V'_S \neq V^S$, it terminates. Otherwise the query is answered with V_C , and the instance goes to an expecting state.

- If the instance C^ζ is in an expecting state, a query $\text{Send}(C^\zeta, \text{AE}_{EK}(K_{i2}, \text{Cert}_C))$ is processed by recovering the private key, and updating the stored secrets. We apply the following rules.

► **Rule C5⁽¹⁾**

Recover the private key and update the stored secrets:

$$K = K_{i1} \oplus K_{i2},$$

$$s_{(i+1)} \equiv s_i + H_4(C\|S\|i\|X^*\|Y\|W_i\|DHK) \bmod q,$$

$$K_{(i+1)1} = K_{i1} \oplus H_5(C\|S\|i\|X^*\|Y\|W_i\|DHK).$$

If K is the correct private key, then the instance accepts. In any case, it terminates.

Send-queries to S

We answer to the Send-queries to a S -instance as follows:

- A $\text{Send}(S^\eta, (C, i, X^*))$ -query is processed according to the following rule:

► **Rule S1⁽¹⁾**

If i is not correct, it terminates. Otherwise, choose a random element $\varphi \xleftarrow{R} \mathbb{Z}_q^*$ and compute $Y \equiv g^\varphi$. Then, the instance computes the Diffie-Hellman key and authenticator. We apply the following rules:

► **Rule S2⁽¹⁾**

Compute $X \equiv X^*/W_i$ and $DHK \equiv X^\varphi$.

► **Rule S3⁽¹⁾**

Compute the authenticator: $V^S = H_1(C\|S\|i\|X^*\|Y\|W_i\|DHK)$. Then the query is answered with (S, Y, V_S) , and the instance goes to an expecting state.

- If the instance S^η is in an expecting state, a query $\text{Send}(S^\eta, V_C)$ is processed by computing the alleged authenticator, session key and encryption key, and updating the stored secrets. We apply the following rule.

► **Rule S4⁽¹⁾**

Compute the expected authenticator, session key and encryption key:

$$V'_C = H_2(C\|S\|i\|X^*\|Y\|W_i\|DHK),$$

$$SK = H_3(C\|S\|i\|X^*\|Y\|W_i\|DHK),$$

$$EK = H_6(C\|S\|i\|X^*\|Y\|W_i\|DHK).$$

Update the stored secrets:

$$W_{(i+1)} \equiv W_i \times h^{H_4(C\|S\|i\|X^*\|Y\|W_i\|DHK)},$$

$$K_{(i+1)2} = K_{i2} \oplus H_5(C\|S\|i\|X^*\|Y\|W_i\|DHK).$$

If $V'_C \neq V_C$, it terminates. Otherwise, the query is answered with $\text{AE}_{EK}(K_{i2}, \text{Cert}_C)$ and the instance accepts.

Other queries

- An $\text{Execute}(C^\zeta, S^\eta)$ -query is processed using successively the above simulations of the Send-queries: $(C, i, X^*) \leftarrow \text{Send}(C^\zeta, \text{Start})$, $(S, Y, V_S) \leftarrow \text{Send}(S^\eta, (C, i, X^*))$, $V_C \leftarrow \text{Send}(C^\zeta, (S, Y, V_S))$, $\text{AE}_{EK}(K_{i2}, \text{Cert}_C) \leftarrow \text{Send}(S^\eta, V_C)$, and then outputting the transcript $((C, i, X^*), (S, Y, V_S), V_C, \text{AE}_{EK}(K_{i2}, \text{Cert}_C))$.
- A $\text{Reveal}(U)$ -query returns the session key SK computed by the instance U (if the former has accepted).
- A $\text{Leak}(C^\zeta, i)$ -query returns the i -th secret (i, s_i, K_{i1}) held by the instance C^ζ .
- A $\text{Test}(U)$ -query first gets SK from $\text{Reveal}(U)$, and flip a coin b . If $b = 1$, we return the value of the session key SK , otherwise we return a random value drawn from $\{0, 1\}^{k_3}$.

Game G₂: For an easier analysis in the following, we cancel games in which some collisions (Coll_2) are unlikely to happen:

- Collisions on the partial transcripts $((C, i, X^*), (S, Y, V_S))$: Any adversary tries to find out one pair (X^*, Y) , coinciding with the challenge transcript, and then obtain the corresponding session key using the Reveal -query. However, at least one party involves with the transcripts, and thus one of X^* and Y is truly uniformly distributed.

The probability is bounded by the birthday paradox:

$$\Pr[\text{Coll}_2] \leq \frac{(q_{\text{ex}} + q_{\text{se}})^2}{2q}. \quad (3)$$

Game G₃: In order to make the authenticators, session key, encryption key and stored secrets unpredictable to any adversary, we compute them using the private oracles H'_j (instead of H_j) so that the values are completely independent from the random oracles. We reach this aim by using the following rules:

► **Rule C4/C5⁽³⁾**

Compute the authenticators, session key and encryption key:

$$V'_S = H'_1(C\|S\|i\|X^*\|Y),$$

$$V_C = H'_2(C\|S\|i\|X^*\|Y),$$

$$SK = H'_3(C\|S\|i\|X^*\|Y),$$

$$EK = H'_6(C\|S\|i\|X^*\|Y).$$

Update the stored secrets:

$$s_{(i+1)} \equiv s_i + H'_4(C\|S\|i\|X^*\|Y) \pmod{q},$$

$$K_{(i+1)1} = K_{i1} \oplus H'_5(C\|S\|i\|X^*\|Y).$$

► **Rule S3/S4**⁽³⁾

Compute the authenticators, session key and encryption key:

$$V^S = H'_1(C\|S\|i\|X^*\|Y).$$

$$V'_C = H'_2(C\|S\|i\|X^*\|Y),$$

$$SK = H'_3(C\|S\|i\|X^*\|Y),$$

$$EK = H'_6(C\|S\|i\|X^*\|Y).$$

Update the stored secrets:

$$W_{(i+1)} \equiv W_i \times h^{H'_4(C\|S\|i\|X^*\|Y)},$$

$$K_{(i+1)2} = K_{i2} \oplus H'_5(C\|S\|i\|X^*\|Y).$$

Since we do no longer need to compute the value DHK , we can simplify the following rules:

► **Rule C3/S2**⁽³⁾

Do nothing.

Finally, the mask W_i is not used anymore either so that we can also simplify the generation of X^* using the group property of $\mathbb{G}$.

► **Rule C2**⁽³⁾

Choose a random element $x \xleftarrow{R} \mathbb{Z}_q^*$ and compute $X^* \equiv g^x$.

The games $\mathbf{G}_3$ and $\mathbf{G}_2$ are indistinguishable unless some specific hash queries are asked, denoted by event $\text{AskH}_3 = \text{AskH1}_3 \vee \text{AskH2w1}_3 \vee \text{AskH3w12}_3 \vee \text{AskH4w123}_3 \vee \text{AskH5w1234}_3 \vee \text{AskH6w12345}_3$:

- **AskH1₃**: $H_1(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_1 for some execution transcripts $((C, i, X^*), (S, Y))$;
- **AskH2w1₃**: $H_2(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_2 for some execution transcripts $((C, i, X^*), (S, Y))$, but event **AskH1₃** did not happen.
- **AskH3w12₃**: $H_3(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_3 for some execution transcripts $((C, i, X^*), (S, Y))$, but event **AskH1₃** and **AskH2w1₃** did not happen.
- **AskH4w123₃**: $H_4(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_4 for some execution transcripts $((C, i, X^*), (S, Y))$, but event **AskH1₃**, **AskH2w1₃** and **AskH3w12₃** did not happen.
- **AskH5w1234₃**: $H_5(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_5 for some execution transcripts $((C, i, X^*), (S, Y))$, but event **AskH1₃**, **AskH2w1₃**, **AskH3w12₃** and **AskH4w123₃** did not happen.
- **AskH6w12345₃**: $H_6(C\|S\|i\|X^*\|Y\|W_i\|DHK)$ has been queried by $\mathcal{A}$ to H_6 for some execution transcripts $((C, i, X^*), (S, Y))$, but event **AskH1₃**, **AskH2w1₃**, **AskH3w12₃**, **AskH4w123₃** and **AskH5w1234₃** did not happen.

The authenticators are computed with random oracles that are private to the simulator, then one can remark that it cannot be guessed by the adversary, better than

at random for each attempt, unless the same partial transcript $((C, i, X^*), (S, Y))$ appeared in another session with a real instance (C^ζ or S^η). But such a case has already been excluded (in game $\mathbf{G}_2$). A similar remark holds on the session key:

$$\Pr[A_3] \leq \frac{q_{se}}{2^{k_1}} + \frac{q_{se}}{2^{k_2}} \quad \Pr[\text{Sw}A_3] = \frac{1}{2}. \quad (4)$$

When collisions of the partial transcripts have been excluded, the event **AskH1** can be split in three disjoint sub-cases:

- **AskH1-Passive₃**: The transcript $((C, i, X^*), (S, Y))$ comes from an execution between instances of C and S (Execute-queries or forward of Send-queries, relay of part of them). This means that both X^* and Y have been simulated;
- **AskH1-WithC₃**: The execution involved an instance of C , but Y has not been sent by any instance of S . This means that X^* has been simulated, but Y has been produced by the adversary;
- **AskH1-WithS₃**: The execution involved an instance of S , but X^* has not been sent by any instance of C . This means that Y has been simulated, but X^* has been produced by the adversary.

Game $\mathbf{G}_4$: In order to evaluate the above events, we introduce a random Diffie-Hellman instance (P, Q) where both P and Q are generators of $\mathbb{G}$. (Otherwise, the Diffie-Hellman problem is easy.) We first modify the simulation of C , involving the element Q .

► **Rule C1**⁽⁴⁾

Choose a random element $w \xleftarrow{R} \mathbb{Z}_q^*$ and compute $W_i \equiv h^{v_i}$ such that $h = Q$ and $v_i \equiv s_i + w \pmod{q}$.

By the isomorphic property from $\mathbb{G}$ to $\mathbb{G}$, the new W_i is perfectly indistinguishable from before since there exists a unique discrete logarithm for W_i . We also introduce the other part P of the Diffie-Hellman instance in the simulation of S .

► **Rule S1**⁽⁴⁾

If i is not correct, it terminates. Otherwise, choose a random element $y \xleftarrow{R} \mathbb{Z}_q^*$ and compute $Y \equiv P^y$. Note that we excluded the cases $W_i \equiv 1$ and $Y \equiv 1$. By Shoup's difference lemma, the probability is upper-bounded by a random guess for Y :

$$|\Pr[\text{AskH}_4] - \Pr[\text{AskH}_3]| \leq \frac{q_{se} + q_{ex}}{q}. \quad (5)$$

Game $\mathbf{G}_5$: It is now possible to evaluate the probability of the event **AskH** (or more precisely, the sub-cases). Indeed, one can see that the password is never used during the simulation. It does not need to be chosen in advance, but at the very end only. Then, an information-theoretic analysis can be done which simply uses cardinalities of some sets.

To this aim, we first cancel a few more games, wherein for some pairs $(X^*, Y) \in \mathbb{G}^2$, involved in a communication between an instance S^η and either the adversary

or an instance C^ζ , there are two distinct elements W_i such that the tuple $(i, X^*, Y, W_i, \text{CDH}_{g,\mathbb{G}}(X^*/W_i, Y))$ is in Λ_H (which event is denoted CollH_5). By Shoup's difference lemma, the probability is upper-bounded by $\Pr[\text{CollH}_5]$:

$$|\Pr[\text{AskH}_5] - \Pr[\text{AskH}_4]| \leq \Pr[\text{CollH}_5]. \quad (6)$$

The event CollH_5 can be upper-bounded, granted the following lemma:

Lemma 1: If for any pair $(X^*, Y) \in \mathbb{G}^2$, involved in a communication with an instance S^η , there are two elements W_{i0} and W_{i1} such that $(i, X^*, Y, W_{ij}, Z_{ij} = \text{CDH}_{g,\mathbb{G}}(X^*/W_{ij}, Y))$ is in Λ_H , one can solve the computational Diffie-Hellman problem:

$$\Pr[\text{CollH}_5] \leq q_H^2 \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t + \tau_e). \quad (7)$$

Proof. We prove this lemma by showing the reduction to the CDH problem when event CollH_5 happens. We assume that there exist $(X^*, Y \equiv P^y) \in \mathbb{G}^2$ involved in a communication with an instance S^η , and two elements $W_{i0} \equiv Q^{v_{i0}}$ and $W_{i1} \equiv Q^{v_{i1}}$ such that the tuple $(i, X^*, Y, W_{ij}, Z_{ij} \stackrel{\text{def}}{=} \text{CDH}_{g,\mathbb{G}}(X^*/W_{ij}, Y))$ is in Λ_H , for $j = 0, 1$. Then,

$$\begin{aligned} Z_{ij} &= \text{CDH}_{g,\mathbb{G}}(X^*/W_{ij}, Y) \\ &= \text{CDH}_{g,\mathbb{G}}(X^* \times Q^{-v_{ij}}, Y) \\ &= \text{CDH}_{g,\mathbb{G}}(X^*, Y) \times \text{CDH}_{g,\mathbb{G}}(Q, Y)^{-v_{ij}} \\ &= \text{CDH}_{g,\mathbb{G}}(X^*, Y) \times \text{CDH}_{g,\mathbb{G}}(P, Q)^{y(-v_{ij})}. \end{aligned} \quad (8)$$

As a consequence, $Z_{i1}/Z_{i0} = \text{CDH}_{g,\mathbb{G}}(P, Q)^{y(v_{i0}-v_{i1})}$, and thus $\text{CDH}_{g,\mathbb{G}}(P, Q) = (Z_{i1}/Z_{i0})^\psi$ where ψ is the inverse of $y(v_{i0} - v_{i1})$ in $\mathbb{Z}_q^*$. The latter exists since $W_{i0} \neq W_{i1}$ and $y \neq 0$. By guessing the two queries asked to the H_j , one concludes the proof. $\square$

In order to conclude the proof, let us study separately the three sub-cases of AskH_1 , and then AskH2w1 , AskH3w12 , AskH4w123 , AskH5w1234 and AskH6w12345 (keeping in mind the absence of several kinds of collisions for partial transcripts, and for W_i in H-queries):

- **AskH1-Passive:** About the passive transcripts (in which both X^* and Y have been simulated), one can state the following lemma:

Lemma 2: If for any pair $(X^*, Y) \in \mathbb{G}^2$, involved in a passive transcript, there is an element W_i such that $(i, X^*, Y, W_i, Z_i = \text{CDH}_{g,\mathbb{G}}(X^*/W_i, Y))$ is in Λ_H , one can solve the computational Diffie-Hellman problem:

$$\Pr[\text{AskH1-Passive}_5] \leq q_H \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t + 2\tau_e). \quad (9)$$

Proof. We prove this lemma by showing

the reduction to the CDH problem when event AskH1-Passive_5 happens. We assume that there exist $(X^* \equiv g^x, Y \equiv P^y) \in \mathbb{G}^2$ involved in a passive transcript and $W_i \equiv Q^{v_i}$ such that the tuple (i, X^*, Y, W_i, Z_i) is in Λ_H . As above,

$$\begin{aligned} Z_i &= \text{CDH}_{g,\mathbb{G}}(X^*, Y) \times \text{CDH}_{g,\mathbb{G}}(Q, Y)^{-v_i} \\ &= P^{xy} \times \text{CDH}_{g,\mathbb{G}}(P, Q)^{-yv_i}. \end{aligned} \quad (10)$$

As a consequence, $\text{CDH}_{g,\mathbb{G}}(P, Q) = (Z/P^{xy})^\psi$ where ψ is the inverse of $-yv_i$ in $\mathbb{Z}_q^*$. The latter exists since we have excluded the cases where $y = 0$ or $v_i = 0$. By guessing the query asked to the H_j , one concludes the proof. $\square$

- **AskH1-WithC:** This corresponds to an attack where the adversary tries to impersonate S to C . But each authenticator sent by the adversary (who may have obtained the secret s_i in the Leak -query, denoted by event Leak_5) has been computed with at most one W_i value:

$$\begin{aligned} \Pr[\text{AskH1-WithC}_5] &= \Pr[\text{AskH1-WithC}_5 | \neg \text{Leak}_5] \\ &\quad + \Pr[\text{AskH1-WithC}_5 | \text{Leak}_5] \\ &\leq \frac{q_{se}}{q} + \frac{q_{se}}{N}. \end{aligned} \quad (11)$$

- **AskH1-WithS:** The Lemma 1, applied to games where the event CollH_5 did not happen, states that for each pair (X^*, Y) involved in a transcript with an instance S^η , there is at most one element W_i such that for $W_i \equiv Q^{s_i} \times Q^{pw}$, the corresponding tuple is in Λ_H : The probability for the adversary (who may have obtained the secret s_i in the Leak -query) over a random password is as above:

$$\Pr[\text{AskH1-WithS}_5] \leq \frac{q_{se}}{q} + \frac{q_{se}}{N}. \quad (12)$$

About AskH2w1 (when the above three events did not happen), it means that only executions with an instance of S (and either C or the adversary) may lead to acceptance. Exactly the same analysis as for AskH1-Passive and AskH1-WithS leads to

$$\Pr[\text{AskH2w1}_5] \leq \frac{q_{se}}{q} + \frac{q_{se}}{N} + q_H \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t + 2\tau_e). \quad (13)$$

Also, we can have the same analysis about AskH3w12 , AskH4w123 , AskH5w1234 and AskH6w12345 since only executions involving instances of S and C may lead to acceptance. Thus,

$$\begin{aligned} &\Pr[\text{AskH3w12}_5] + \Pr[\text{AskH4w123}_5] \\ &\quad + \Pr[\text{AskH5w1234}_5] + \Pr[\text{AskH6w12345}_5] \\ &\leq 4q_H \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t + 2\tau_e). \end{aligned} \quad (14)$$

As a conclusion, we get an upper-bound for the probability of AskH_5 by combining all the cases:

$$\Pr[\text{AskH}_5] \leq \frac{3q_{se}}{q} + \frac{3q_{se}}{N} + 6q_H \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t+2\tau_e). \quad (15)$$

Combining inequalities (3), (4), (5), (7) and (15), one gets either

$$\Pr[A_0] \leq \frac{q_{se}}{2^{k_1}} + \frac{q_{se}}{2^{k_2}} + \Delta \quad \Pr[\text{SwA}_0] = \frac{1}{2} + \Delta, \quad (16)$$

where

$$\begin{aligned} \Delta &\leq \frac{3q_{se}}{N} + 6q_H \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t+2\tau_e) + \frac{4q_{se} + q_{ex}}{q} \\ &\quad + q_H^2 \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t+\tau_e) + \frac{(q_{ex} + q_{se})^2}{2q} \\ &\leq \frac{3q_{se}}{N} + 7q_H^2 \times \text{Succ}_{g,\mathbb{G}}^{\text{cdh}}(t+3\tau_e) \\ &\quad + \frac{8q_{se} + 2q_{ex} + (q_{ex} + q_{se})^2}{2q} \end{aligned} \quad (17)$$

and Δ is the common probability for events A_0 and SwA_0 from **Game G₁** to **Game G₅**. One can get the result as desired by noting that $\Pr[S_0] \leq \Pr[\text{SwA}_0] + \Pr[A_0]$.

7. Several Extensions

In this section, we discuss several extensions of the LRP-AKE protocol (c.f. Sect. 6 of [32]). These extensions, of course, can be applied to the modified LRP-AKE protocol.

7.1 Synchronization

An important issue in the LRP-AKE protocol may be how to synchronize the respective stored secrets (s_i, K_{i1}) and (W_i, K_{i2}) of client C and server S . If this synchronization is broken, the LRP-AKE protocol terminates with an error. This indicates that the update-timing of the LRP-AKE protocol should be considered more carefully.

Let us suppose that the last message (i.e., the encrypted credential) of the LRP-AKE protocol did not reach to client C . It is inevitable in the AKE protocols that one party, who sent the last message, cannot confirm whether the other party really received the message. In such a situation, client C also did not know whether the third message V_C correctly reached to server S . If that is the case, the server stores either (W_i, K_{i2}) (if it did not receive the third message) or $(W_{(i+1)}, K_{(i+1)2})$ (if it received the third message). On the other hand, the client just keeps (s_i, K_{i1}) because he/she did not receive the last message. A simple way for synchronization is to change the updating procedure on the client side in Sect. 3.2.2 as follows: 1) Client C computes $(s_{(i+1)}, K_{(i+1)1})$ as usual, and 2) If C received the last message and checked its correctness of the private key, he/she stores $(s_{(i+1)}, K_{(i+1)1})$ and outputs **accept**; Otherwise, he/she stores

two pairs (s_i, K_{i1}) and $(s_{(i+1)}, K_{(i+1)1})$, and terminates the protocol. In any case, client C and server S can run the LRP-AKE protocol with a pair of either (s_i, K_{i1}) and (W_i, K_{i2}) or $(s_{(i+1)}, K_{(i+1)1})$ and $(W_{(i+1)}, K_{(i+1)2})$.

If an attacker obtains the client's stored secrets (i.e., two pairs (s_i, K_{i1}) and $(s_{(i+1)}, K_{(i+1)1})$), the only information the attacker can get are hashed values $H_4(\text{SID}||DHK)$ and $H_5(\text{SID}||DHK)$. As it is clear, these values are useless in attacking the LRP-AKE protocol. Also, online dictionary attacks for these two pairs are not possible because server S holds either (W_i, K_{i2}) or $(W_{(i+1)}, K_{(i+1)2})$.

7.2 Randomized ID

Since the AKE protocols are usually used in a variety of applications, it is especially desirable to provide privacy of client. In the LRP-AKE protocol, an attacker can easily get an information about which client has accessed to which server since client's identity C and its counter i are transmitted in clear to server S .

For the provision of client's privacy, one can use an "randomized ID" in the LRP-AKE protocol. Specifically, the initialization phase is the same as Sect. 3.1, except that client C additionally registers a randomized ID rID_1 , chosen from $\{0, 1\}^k$. The rID_1 is stored, instead of the counter, on the client and server sides, respectively. In the i -th protocol execution phase of the LRP-AKE protocol, client C sends the first message (rID_i, X^*) to server S and, at the end of this phase, rID_i is updated to $\text{rID}_{(i+1)}$ as in the same way for updating (s_i, K_{i1}) and (W_i, K_{i2}) (the other parts remain unchanged).

This is also effective against Denial-of-Service (DoS) or Distributed DoS (DDoS) attacks since an attacker should actively intercept the randomized ID, coming from the legitimate client. Note that these DoS/DDoS attacks have become one of the traditional tools for the purpose of exhausting a server's memory and computational capability (e.g., by making the server to consume its computing power for modular exponentiations).

7.3 Preventing Server Compromise-Impersonation Attacks

In the LRP-AKE protocol, if the server's stored secrets (i.e., $W_i, (K_{i2}, \text{Cert}_C)$) are compromised, an attacker can freely impersonate the client because the W_i can be seen as somewhat 'symmetric type' information between client C and server S though the attacker cannot recover the client's private key K . This is called Server Compromise-Impersonation (SCI) attacks in the client-server setting where an attacker obtains the sever's secret information and then tries to impersonate the client, not the server.

A solution to preventing SCI attacks is to use 'asymmetric type' information between the client and the server as proposed in [9], [15]. Although a direct application of their method [9] leads to security against SCI attacks, we describe a different way to integrate any secure signature

schemes (in the sense of [8]) to the LRP-AKE protocol. Specifically, the initialization phase is the same as Sect. 3.1 except that client C generates a signature/verification key pair $(SigK, VerK) \leftarrow \text{SigKG}(1^l)$, where SigKG is a key generation algorithm and l is the security parameter, and registers the $VerK$ to server S . The $SigK$ and $VerK$ are stored on the client and server sides, respectively. The parts to be changed in Sect. 3.2.1 are as follows: 1) Client C generates a signature $Sig_C = \text{Sign}(SigK; V_C)$, where Sign is a signature generation algorithm and V_C corresponds to a message to be signed, and sends Sig_C instead of V_C ; and 2) Correspondingly, server S verifies the Sig_C by checking $\text{Verify}(VerK; Sig_C)$, where Verify is a verification algorithm and outputs 0/1. If the Sig_C is a valid signature for V_C (i.e., $\text{Verify}(VerK; Sig_C) = 1$), server S proceeds to the next step. Otherwise, it terminates the protocol. Note that compromise of the sever's stored secrets does not lead to offline dictionary attacks on the password in the LRP-AKE protocol.

8. Performance Evaluation

Here, we evaluate the performance overhead of the LRP-AKE protocol, and show its test vectors.

To this end, we implemented the LRP-AKE protocol with the elliptic curve domain parameters [25]. In order to compare with the Diffie-Hellman (key exchange) protocol [4], the recovering private key and certificate part in Sect. 3.2.2 is omitted in our implementation. For a fair comparison, we also implemented the 3-pass Diffie-Hellman protocol accompanied by key confirmation. As well known, the (3-pass) Diffie-Hellman protocol itself is not secure against active attacks.

As an efficiency-measurement environment, the LRP-AKE server is running on Ubuntu 64-bit virtual machine (Intel® 64bit Core™ i7-6500U 2.50 GHz CPU with 8 GB RAM) and the LRP-AKE client running on Raspberry Pi 2 Model B [24]. The LRP-AKE server and client are connected through 100 Mbps Ethernet.

Specifically, we measured processing times (unit: ms) of the LRP-AKE client and the 3-pass Diffie-Hellman client 15 times by the real time of 'time' command for each elliptic curve domain parameter [25]. These results are summarized in Table 1, and its comparative graph can be found in Fig. 3. For example, the average processing time of the LRP-AKE client with the elliptic curve domain parameter SECp256r1 [25] is 142.2 ms while that of the 3-pass Diffie-

Table 1 Processing times (unit: ms) of the LRP-AKE client on Raspberry Pi 2 Model B [24] where figures in the second line of each row represent processing times of the 3-pass Diffie-Hellman key exchange client

EC domain parameters [25]	Average	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th
SECp160r1	126.3 (LRP-AKE)	114	121	118	127	124	133	125	131	126	132	121	129	124	121	148
	112.5 (DHKE)	110	112	112	112	112	112	112	112	112	113	122	112	112	112	111
SECp192r1	127.5	115	133	118	126	131	132	137	125	127	130	124	129	138	128	120
	117.9	111	113	112	112	112	122	122	112	123	121	119	122	122	123	122
SECp224r1	169.5	150	191	164	177	168	183	167	163	163	162	168	166	174	166	180
	158.9	169	157	146	158	158	163	159	167	155	172	156	148	159	161	155
SECp256r1	142.2	141	168	141	138	156	132	143	142	145	132	135	142	139	141	138
	129.4	134	132	132	130	124	125	128	128	127	137	128	128	128	128	132
SECp384r1	147.8	130	150	150	178	145	139	148	147	156	137	149	136	155	155	142
	135.7	130	137	130	144	131	142	133	137	137	136	140	139	141	132	127
SECp521r1	211.0	213	219	215	194	203	202	201	223	213	215	209	206	219	224	209
	195.0	204	198	199	193	207	193	188	189	192	198	192	190	194	195	193
SECT163r2	136.3	130	139	137	132	146	141	134	129	136	136	134	149	135	137	130
	130.1	121	132	132	132	135	129	132	122	127	133	132	132	132	129	132
SECT233r1	123.5	118	120	123	131	131	120	124	121	127	123	127	122	120	130	115
	116.9	115	112	117	113	119	117	112	120	112	112	118	120	122	120	124
SECT283r1	167.2	158	181	169	171	165	176	159	170	162	170	174	152	159	170	172
	159.1	155	157	154	183	157	156	155	163	154	163	161	152	160	160	156
SECT409r1	261.2	257	265	275	268	297	254	268	236	261	240	252	264	244	267	270
	256.2	240	258	270	246	259	269	246	262	242	259	269	246	259	273	245
SECT571r1	488.2	507	522	470	471	486	607	483	482	489	484	466	471	471	463	451
	469.5	470	470	483	492	483	456	480	472	444	460	471	460	461	467	473

Fig. 3 Comparison of processing times of the LRP-AKE client and the 3-pass Diffie-Hellman key exchange ((3P-)DHKE) client on Raspberry Pi 2 Model B [24]

Table 2 Average operation times (unit: ns) on Raspberry Pi 2 Model B [24]

EC domain parameters [25]	Average time of							
	EC key pair generation	EC point multiplication	EC point addition	SHA1	SHA224	SHA256	SHA384	SHA512
SECp160r1	11133738	4397089	41583.2	95402.4	-	-	-	-
SECp192r1	12985669	6211648	47190.9	-	103086.5	-	-	-
SECp224r1	41108497.7	8550007.4	48743.1	-	97242.9	-	-	-
SECp256r1	7341093	1717354.2	66704.7	-	-	84850.5	-	-
SECp384r1	36787427	29695497	77569.3	-	-	-	63381.7	-
SECp521r1	79498928.2	68261936.8	125746.2	-	-	-	-	61729.3
SECT163r2	11515782.5	4883670.9	161270.5	-	106912.8	-	-	-
SECT233r1	15085942.2	8226115.6	206412.8	-	-	86867.8	-	-
SECT283r1	22670097.1	15243674.3	262252.8	-	-	-	55249.9	-
SECT409r1	37686282.5	29891232.1	387558.3	-	-	-	-	68419.9
SECT571r1	78301691	70874813.3	627668.8	-	-	-	-	59201.2

Hellman client is 129.4 ms. The communication overhead of the LRP-AKE protocol is 1024 bits (same as the 3-pass Diffie-Hellman protocol) where the client/server identities are not counted. In terms of the number of message exchanges, the LRP-AKE protocol requires 3 passes. From Table 1 and Fig. 3, it is clear that the LRP-AKE protocol has almost the same efficiency as the (plain) Diffie-Hellman protocol that does not provide authentication at all.

In addition, we measured average times (unit: ns) of the EC key pair generation, EC point multiplication, EC point addition, SHA1, SHA224, SHA256, SHA384, and SHA512 15 times by the real time of ‘time’ command for each elliptic curve domain parameter [25]. The library used for each operation is OpenSSL 1.1.1a. These average operation times on Raspberry Pi 2 Model B [24], whose CPU and OS are 32-bit ARM and Raspbian, respectively, can be found in Table 2. For example, the average time of SHA256

with the elliptic curve domain parameter SECp256r1 [25] is 84850.5 ns (i.e., 0.08485 ms), which is negligible in the corresponding processing time 142.2 ms of the LRP-AKE client.

In Table 3 (resp., Table 4), a test vector of the LRP-AKE protocol is listed where the elliptic curve domain parameter SECp256r1 (resp., SECT283r1) [25] and SHA256 (resp., SHA384) are used. Note that points on the elliptic curve are expressed with the compressed form having prefix 02 or 03.

9. Conclusions

In this paper, we pointed out a problem in the security proof of the LRP-AKE protocol [31] and then proposed a modified LRP-AKE protocol that has a simple and effective measure to the problem. Also, we formally proved its AKE security and mutual authentication for the entire modified LRP-AKE

Table 3 Test vector of the LRP-AKE protocol where the elliptic curve domain parameter SECp256r1 [25] and SHA256 are used

h	03836362FFB02357EFF24F4881D96618B2128F55791A445D67E301A5A67B57146B
C	(=lrpakeuser1@aist.go.jp) 6C7270616B65757365723140616973742E676F2E6A70
S	(=lrpakeserver@aist.go.jp) 6C7270616B6573657276657240616973742E676F2E6A70
i	00000001
pw	(=SHA512(0 C S password)) 64C0F6239CCA16866612D8E1115B5ADA038D88B2A1376CC24E3B54F308DCBF1825 6A008ED70342EB5F32BAB141717BEB59B46EAE36D8E50EBA514287CFA66DDB
s_i	08B637BA75234719211718326BE28CB3C45EF4EA366DEEBAD2A1738D6A77E327
W_i	03EDA13583FB00976FA641B29F1DA37D95EC4E38328A3E645A522637C635F7AFB7
x	36B80344B46ACC4185A40A740D8008C47BCF6F5C183787DD3E8CFF1CA38A26EC
X	025460E3BA90D478232E64786C016CDC1CA31B67F9C876CBFF5E59CAD80F417896
X^*	02CA2646F82997E01610F25B2BA8858557A193FEF31C4360DBE81537C96D4ECA03
y	78574F0E3861B458291DF3D3935624E873FF9AB8FECDF7B731E8E4E28312C53A
Y	02A509BBD2DCA18C790960A7D6A616A29E9361C5B3E8EDCD5474EDDC03F4FF680A
DHK	02903FB52457300DC360EE8E29E35C8E2AF029EE2E552F9AC79C7D7414A09BFD4F
V_S	EA9C1114DA052C9946016F3E13E5071F687C0F397FCE48395EB417854F13FA43
V_C	7E44AC7A41F74B739B4119288C00117C4EF697E38E4B4CF9CCA75AF48A220633
SK	642B8E2BAF69CE15311D89111403F530239D324B597998C2E6468F153545BF68
$s_{(i+1)}$	8674A52EC629B2DB1CA283E9F3C1BAB79A060CA25F8FCB3EFC940EC3A5A6DEF9
$W_{(i+1)}$	02ECCBC141C0BD5663378F7F71DD846ADDD11DFEE0160DB8863583680A014A21E7

Table 4 Test vector of the LRP-AKE protocol where the elliptic curve domain parameter SEC283r1 [25] and SHA384 are used

h	0300A28B50B8139FE286B2D2E2C0472F226C08A73E5B46410DC3A855A95E51FC5936EE4CBA
C	(=lrpakeuser1@aist.go.jp) 6C7270616B65757365723140616973742E676F2E6A70
S	(=lrpakeserver@aist.go.jp) 6C7270616B6573657276657240616973742E676F2E6A70
i	00000001
pw	(=SHA512(0 C S password)) 64C0F6239CCA16866612D8E1115B5ADA038D88B2A1376CC24E3B54F308DCBF18256A008ED70342EB5F32BAB141717BEB 59B46EAE36D8E50EBA514287CFA66DDB
s_i	0189B967F2E0578BFF8FF7A91E86AB06C5BFEFE98755F650C785858977F4CF1EACB3566
W_i	020134C6F0D6432C45E17344F56AC1BD32E4773EE5E7FC154B615F70E9232884A6ED89AFCC
x	6D6EA2F06155FB6EF0A501F700E2D5564168A42A04D0982313155CA99F0DE873858272
X	020797C2BD11D4B68D95C9339B844137C7271121124DE28EFB74591E8E960DF00CBD0DDCD8
X^*	0304A03C7473B46D6BD7F317BFDE936D12840118C33CF87CFD42EA5D6C05929C6E4C58846F
y	031C8259C15815B83B5821D6769DE7D1A5749B8F336914EDBB073EB67F26CA36B4F6BED4
Y	0200D87675C56855A645233E8732BCBD2D0AEFEFB7F432887517A10186A69F6D750B0D3A13
DHK	02077C8D10FB2EC7E26F5A25DB4875D9C31B89C90D5609BD0EEDDE9562059AA5DA42371E4E
V_S	8A410DE35B583BD661518CF98D953684E97CC0FAB6149452417277F842D25606C5485C20A27662EBC06F297C37329BC2
V_C	BD7CD009CEE8185B7C6D22513F2BE610A69C9A9FEADA6BE24936EEE61073CFE6B5D6E5FB22A865F1F6280886745AD24AC
SK	B54951F161350F5DB64CC5C9BA5D8B504C8D11F53622779AA75BFE705582C647D288FADEE074199A038EE88976EA8093
$s_{(i+1)}$	02805D7E24ACBC7677DA4489A68EABDD7649FEF32C9369BF2CAAA17A7619BBE65135DFD9
$W_{(i+1)}$	0200B77B97EE94A9D75530C47059A6B3BC88D871F86ECC2B7DD4611E7C0B02619CB6501279

protocol.

In addition, we discussed several extensions of the (modified) LRP-AKE protocol including 1) synchronization

issue between the client's and server's stored secrets; 2) randomized ID for the provision of client's privacy; and 3) a solution to preventing server compromise-impersonation at-

tacks. Finally, we evaluated the performance overhead of the LRP-AKE protocol and showed its test vectors. From the performance evaluation, we confirmed that the LRP-AKE protocol has almost the same efficiency as the (plain) Diffie-Hellman protocol [4] that does not provide authentication at all.

A possible alternative solution to the problem discussed in Sect. 5 is to adopt the authenticated and confidential channel establishment (ACCE) protocols model [11], [14], [19] where a key exchange protocol and an authenticated encryption scheme can be used overlappingly. However, in this case, one should prove the security of the LRP-AKE protocol from scratch.

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