PAPER

Improvement of CT Reconstruction Using Scattered X-Rays

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SUMMARY A neural network that outputs reconstructed images based on projection data containing scattered X-rays is presented, and the proposed scheme exhibits better accuracy than conventional computed tomography (CT), in which the scatter information is removed. In medical X-ray CT, it is a common practice to remove scattered X-rays using a collimator placed in front of the detector. In this study, the scattered X-rays were assumed to have useful information, and a method was devised to utilize this information effectively using a neural network. Therefore, we generated 70,000 projection data by Monte Carlo simulations using a cube comprising 216 ($6 \times 6 \times 6$) smaller cubes having random density parameters as the target object. For each projection simulation, the densities of the smaller cubes were reset to different values, and detectors were deployed around the target object to capture the scattered X-rays from all directions. Then, a neural network was trained using these projection data to output the densities of the smaller cubes. We confirmed through numerical evaluations that the neural-network approach that utilized scattered X-rays reconstructed images with higher accuracy than did the conventional method, in which the scattered X-rays were removed. The results of this study suggest that utilizing the scattered X-ray information can help significantly reduce patient dosing during imaging.

key words: CT reconstruction, scattered X-rays, neural network, Monte Carlo simulation

1. Introduction

X-ray computed tomography (CT) is a medical diagnostic technique in which cross-sectional images of the human body are reconstructed using X-ray intensities measured with a rotating X-ray source–detector pair [1]. In the conventional system, the scattered X-rays generated when the incident radiation penetrates an object are removed, because they can degrade image quality, unless an algorithm that considers their influence were to be used [1]–[3]. Various correction methods, ranging from hardware to software solutions, have thus been proposed to remove the influence of scattered X-rays. The correction method that uses hardware includes an anti-scatter grid or post-patient collimator [1], [4], [5] and beam-stop arrays [6]–[8], whereas the correction method using software includes a kernel-based scheme [9], [10] and a technique based on Monte Carlo simulations [10]–[13]. All of these methods can remove scattered X-rays.

However, considering that scattered X-rays are not gen-

erated unless there is an object, it should be assumed that the scatter contains information about the target object. Based on this perspective, the scattered X-ray effect has been investigated by many researchers when developing CT reconstruction approaches [14]–[19]. Furthermore, if the information contained in the scattered X-rays is independent of that in the primary X-rays, it is expected that the reconstruction accuracy could be improved using this additional information. Toda et al. [20] proposed a simplified mathematical model, the T-junction model, and theoretically demonstrated its ability to improve reconstruction accuracy using scattered X-rays. This pioneering work was the first of its kind. From this, Nagae et al. [3] proposed an iterative reconstruction method for reproducing the projection data that contained scattered X-rays using Monte Carlo simulations and numerically presented the possibility of accuracy improvements for a 2×2 object phantom. However, because this method involves performing Monte Carlo simulation at each iteration, it requires a large amount of calculations. Therefore, other effective methods are needed to offset the computational overhead.

X-ray CT reconstruction methods using deep learning and neural networks [21]–[23] have also been proposed in recent years. These methods construct maps from projection data to obtain the reconstructed images by learning numerous input/output data pairs. Although the learning requires a finite amount of time, the reconstructed images can be obtained in an extremely short time if the projection data are input to a learned network. Since the advantages of reconstruction by this method have been recognized, various alternatives have been considered, such as the conventional filtered back-projection (FBP) methods or iterative reconstruction methods for applications involving denoising [21] or reconstruction from partial projections [22]. Some methods that suppress the influence of the scattered X-rays have also been proposed [23], but research on improving the reconstruction accuracy using scattered X-rays is yet to be explored.

This study aims to verify the effects of utilizing scattered X-rays for image reconstruction using a neural network. We generated a large amount of projection data through Monte Carlo simulations via EGS5 [24], a multi-purpose Monte Carlo simulation package, and evaluated the reconstruction performance using a neural network that was trained with the data. Finally, numerical experiments were conducted to verify that the reconstructed images that used both primary and scattered X-rays were more accurate than

Manuscript received November 16, 2020.

Manuscript revised March 21, 2021.

Manuscript publicized May 6, 2021.

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DOI: 10.1587/transinf.2020EDP7241

those using only the primary X-rays.

The remainder of this paper is organized as follows. In Sect. 2, the model for data generation via Monte Carlo simulation is described. In Sect. 3, data conditioning and structures of reconstruction networks for both proposed and conventional methods are explained. In Sect. 4, through numerical experiments, the accuracy improvement by utilizing the scattered X-rays is shown. Finally, in Sect. 5, concluding remarks and future works are provided.

2. Model for Data Generation by Monte Carlo Simulation

When training a neural network, a method of preparing a large quantity of training data must be considered. In the present study, we require the true density distributions of many objects and measurements of their scattered X-rays. However, it is difficult to prepare several phantoms with accurate density distributions for this purpose. However, because Monte Carlo simulations have recently been shown to be capable of accurately calculating scattered X-rays [24], we use this approach to generate the large amount of data necessary for training the neural network. In the following sections, the different elements of the simulation model are described.

2.1 X-Ray Tube

The radiation source point of the X-ray tube is placed at $(x, y, z) = (0, 0, -40)$ (each axis is coordinated in cm) in 3D space, as shown in Fig. 1. A cone beam having a fan angle of 36.0° and cone angle of 24.0° is emitted. These beams have adequate spreads, such that the entire object phantom is irradiated with X-rays in consideration of its rotation. The material of the X-ray-tube target is assumed to be tungsten, and we use the data quantized in one channel as 1keV for the energy spectrum of the tube voltage (i.e., 140keV). Figure 2 shows the energy spectrum used in this study.

2.2 Object Phantom

The object phantom is placed such that the origin $(0, 0, 0)$ is at its center, as shown in Fig. 1. The object phantom is a $20 \times 20 \times 20$ cm cube that is equally divided into 216 ($6 \times 6 \times 6$) smaller cubes (hereinafter called “voxels”). Each voxel is fully filled with water having a random density value that is varied within the range of 0.3 to 3.0 g/cm³. The parameter, $\rho(i_x, i_y, i_z)$, ($0 \leq i_x, i_y, i_z \leq 5$) denotes the density of each voxel, where i_x, i_y, i_z are the discrete positions of x -axis, y -axis, and z -axis, respectively. In this study, the reconstruction is accomplished by estimating the recovered density parameters, which are proportional to attenuation coefficients from the projection data.

2.3 Arrangement of Detectors and Projection-Data Acquisition Method

We consider the method that uses only the primary X-

Fig. 1 Object phantom and X-ray tube

Fig. 2 Energy spectrum

rays for reconstruction as the conventional method and that which uses the scattered as well as primary X-rays in the reconstruction as the proposed method. The detector arrangements and data acquisition methods in the simulation models differ between the conventional and proposed methods, as described below.

2.3.1 Conventional Method

Figure 3 shows the geometry of the simulation model for the conventional method. As shown in the figure, to measure the primary X-rays, a square flat detector array is located behind the object phantom, such that it is perpendicular to the central X-ray beamline. Hereinafter, this detector is called the “forward detector”. The detector is $80 \times 80 \times 1$ [cm] in size, and the number of square detector cells (hereinafter “cell”) is 64 (8×8). The center of the forward detector is at the coordinate position $(0, 0, 40)$. The detector cell is assumed to count all incident photons. An ideal anti-scatter grid is assumed to be installed on the forward detector, such that no scattered X-rays are detected. Rotational scanning is performed every 24° from 0° to 336° . The number of irradiation photons is 3.0×10^4 for each rotation angle, and the total number of irradiation photons is $15 \times 3.0 \times 10^4$. These data are the inputs to the neural network.

2.3.2 Proposed Method

Figure 4 shows the geometry of the simulated model for the proposed method, which uses both the primary and scattered X-rays from all directions. In the conventional method, an

Fig. 3 Detector arrangement in the conventional method

Fig. 4 Detector arrangement for the proposed method

ideal grid is attached to the forward detector to remove the scattered X-rays, but in the proposed method, no grid is installed on any detector and the scattered X-rays are measured together with the primary X-rays without distinction. Moreover, as in the conventional method, no energy analysis is performed. In addition to the forward detector, five flat detectors are deployed on the surface of a cube having dimensions $80 \times 80 \times 80$ cm to capture the scattered X-rays. However, regarding the backscattered X-ray detector, the number of detector cells is 60, because the four-cell area at the center of the detector array is used as the X-ray irradiation port. Thus, the total number of detector cells in this model is 380. Under this geometry, all X-ray photons arriving at the detector are measured and used to estimate the density of the object phantom.

3. Neural Networks for Reconstruction

3.1 Data Conditioning and Reconstruction Network

The initial value of the random number is changed to generate N_{all} ($= 70,000$) object phantoms. As mentioned at the beginning of Sect. 2, the object phantom is irradiated with X-rays with the energy shown in Fig. 2 by Monte Carlo simulation, and the primary X-rays and scattered X-rays are measured with the detector set shown in Fig. 3 and Fig. 4. The network is trained using N_{train} ($= 56,000$) projection data as the input and the true density $\rho_k^*(i_x, i_y, i_z)$ of the object phantom as the target, where k ($1 \leq k \leq N_{all}$) denotes the index of the object phantom. As explained later

Table 1 Detector positions

Detector number	Parallel plane	Center position
$n = 1$	xy	$z = -40$
$n = 2$	yz	$x = -40$
$n = 3$	yz	$x = 40$
$n = 4$	xz	$y = 40$
$n = 5$	xz	$y = -40$
$n = 6$	xy	$z = 40$

in Sect. 3.3, N_{t1} ($= 7,000$) input / output data pairs (Test1 data) were used for termination of learning, and the remaining N_{t2} ($= 7,000$) pairs (Test2 data) were used for evaluation. Because the learned networks output $\hat{\rho}_k(i_x, i_y, i_z)$ as the estimate for the density, the number of output units is the same as the number of voxels, i.e., $216 (= 6 \times 6 \times 6)$.

Reconstruction is carried out using neural networks for both the conventional and proposed methods. However, the types of networks and data conditioning methods differ for each method, as described below.

3.1.1 Proposed Method

The measurement position, $\mathbf{d}_p$, in the proposed method is defined as

$$\mathbf{d}_p = \begin{pmatrix} \theta \\ n \\ l \\ m \end{pmatrix}, \quad (1)$$

where $\theta = 24h[^\circ]$ (h is an integer, $0 \leq h \leq 14$) denotes the rotational scanning angle, and integer n denotes the number of detector arrays. Table 1 summarizes the correspondence between the integers and detector positions. Integer l denotes the cell number in the horizontal direction (first letter of the parallel plane in Table 1) of the detector array, and integer m denotes the cell number in the vertical direction (second letter of the parallel plane in Table 1). $I_{pk}(\mathbf{d}_p)$ is the k -th ($1 \leq k \leq N_{all}$) measured datum for the proposed method. Figure 5 shows the schematic of the projection data. $I'p_k(\mathbf{d}_p)$, obtained by normalizing $I_{pk}(\mathbf{d}_p)$, can be expressed as

$$I'p_k(\mathbf{d}_p) = \frac{1}{I_{max}} I_{pk}(\mathbf{d}_p), \quad (2)$$

where $I_{max} = \max\{I_{pk}(\mathbf{d}_p) : 1 \leq k \leq N_{all}, \mathbf{d}_p \in \mathbf{D}\}$, $\mathbf{D} = \{\mathbf{d}_p : 1 \leq n \leq 6, 1 \leq l \leq 8, 1 \leq m \leq 8\}$, and $I'p_k(\mathbf{d}_p)$ are the inputs to the neural network. Because the output of the network is an estimate of the density of the object phantom, it can be expressed as

$$\hat{\rho}_k(i_x, i_y, i_z) = \text{NR}_p(I'p_k(\mathbf{d}_p)), \quad (3)$$

where NR_p is a function type, F_1 , from the input to the output of the neural network, as described in 3.2.1.

3.1.2 Conventional Method-1

The FBP method, which is widely used for CT reconstruction

Fig. 5 Schematic of the projection data

tion, can be regarded as a linear operation when the logarithm of the projection data of the primary X-rays is considered. Therefore, if the number of network parameters and datasets are sufficient, the function for the FBP method can be executed using a linear network that does not require a nonlinear activation function, such as the sigmoid function or rectified linear unit. In this work, we refer to this method as conventional method-1, and the corresponding network is trained with the logarithm of the measured primary photon-number data to estimate the densities of the target object. To obtain the logarithmic equivalent value, an X-ray dose is required such that one or more photons are incident on each cell. In conventional methods, the scattered X-rays are not used for reconstruction. Thus, the measurement position, $\mathbf{d}_c$, is defined as

$$\mathbf{d}_c = \begin{pmatrix} \theta \\ l \\ m \end{pmatrix}, \quad (4)$$

where θ is the same as that in Eq. (1), and l, m denote the cell numbers in the horizontal and vertical directions of the forward detector, which is the same (detector number $n = 6$ in Table 1) as in the proposed method. Let $Ic_k(\mathbf{d}_c)$ be the number of detected photons at position $\mathbf{d}_c$ of the k -th measurement in the conventional method. Then, $I'c1_k(\mathbf{d}_c)$, obtained by taking the logarithm of $Ic_k(\mathbf{d}_c)$, is expressed as

$$I'c1_k(\mathbf{d}_c) = \log Ic_k(\mathbf{d}_c). \quad (5)$$

Here, $I'c1_k(\mathbf{d}_c)$ is the input to the linear network, and the output is expressed as

$$\widehat{\rho}_k(i_x, i_y, i_z) = \text{NR}_{c1}(I'c1_k(\mathbf{d}_c)), \quad (6)$$

where NR_{c1} is a function type, F_2 , from the input to the output of the linear network, as described in 3.2.2.

3.1.3 Conventional Method-2

The conventional method-2 is a scheme in which data having only the primary X-rays are input to the neural network with sigmoid-type hidden units, and learning is performed to output the density values. $I'c2_k(\mathbf{d}_c)$ is obtained by normalizing $Ic2_k(\mathbf{d}_c)$ and is expressed as

Table 2 Common specifications of the neural networks

Number of data: N_{all}	70,000
Number of training data: N_{train}	56,000
Number of test data	
Test1 data: N_{t1}	7,000
Test2 data: N_{t2}	7,000
Batch size	2,000
Optimizer	Adam
Activation function	Sigmoid
Number of outputs (number of voxels)	216
Initial learning rate	0.01
GPU	Nvidia GeForce RTX 2080Ti (TensorFlow)
Evaluation function	mean-squared error (MSE)
Weight initial value	Xavier
Bias initial value	0.1

$$I'c2_k(\mathbf{d}_c) = \frac{1}{I_{max}} Ic_k(\mathbf{d}_c), \quad (7)$$

where I_{max} is as defined in Eq. (2), and $I'c2_k(\mathbf{d}_c)$ is the input to the neural network. The output is then expressed as

$$\widehat{\rho}_k(i_x, i_y, i_z) = \text{NR}_{c2}(I'c2_k(\mathbf{d}_c)), \quad (8)$$

where NR_{c2} is a function type, F_3 , from the input to the output of the neural network, as described in 3.2.3.

3.2 Structures of the Neural Networks

The structures of the different neural networks used in this study are described herein. Although the networks for the proposed and conventional methods have different structures, the number of output units is fixed at the number of voxels in the object phantom (i.e., 216).

A suitable regularization strategy or constraint on the structure of the network for reconstruction may probably exist, but because such an efficient approach is not known, full-connection networks were employed in this paper. The common specifications of these neural networks are summarized in Table 2.

3.2.1 Neural Network for the Proposed Method

The proposed method employs one hidden layer, for which the number of units in the input layer is 5,700, corresponding to the number of measurement positions, $\mathbf{d}_p$. The number of hidden layer units is u_{hidden} . The mapping from the input to the output of the neural network is defined as function type F_1 . The sigmoid function is used as the activation function of the hidden layer.

3.2.2 Linear Network for Conventional Method-1

The linear network in this method has no hidden layers, and the number of units in the input layer is 960, which corresponds to the number of measurement positions, $\mathbf{d}_c$. The mapping from the input to the output of this linear network is defined as function type F_2 .

3.2.3 Neural Network for Conventional Method-2

In conventional method-2, a network having one hidden layer is used, and the number of units in the input layer is 960, which corresponds to the number of measurement positions, d_c . The mapping from the input to the output of the network is defined as function type F_3 , and the sigmoid function is used as the activation function of the hidden layer.

3.3 Termination Condition for Learning

We use the following termination algorithm for conventional method-2 and the proposed method:

1. Divide the test data (14,000) into Test1 (7,000) and Test2 (7,000).
2. Perform one epoch of learning.
3. Calculate the evaluation criterion (i.e., root mean-squared error (RMSE)) for Test1. When the minimum of evaluation criterion (RMSE) is updated, the network parameters at that time are saved, and the count is set to zero. If the criterion is not updated, then the count is incremented by one.
4. If count is 1,000 or more, the network weights are overwritten and saved; go to Step 5. If the count is less than 1,000, then return to Step 2.
5. Calculate the evaluation criteria (i.e., RMSE and correlation coefficient (CC)) for Test2 data and Training data.

The evaluation criteria (RMSE and CC) for Test2 are used as the final network evaluation criteria. In conventional method-1, the number of epochs is set to 70,000, and learning is performed until the evaluation criterion (RMSE) for the training data converges.

3.4 Evaluation Criteria

The reconstruction performance was evaluated through five-fold cross-validation using the RMSE and CC for Test2. The formulas for the RMSE and CC evaluation criteria are given by Eq. (9) and Eq. (10), respectively.

$$RMSE = \sqrt{\frac{1}{N_{t2}} \sum_{k \in Test2} \frac{1}{216} D(\rho_k^*, \hat{\rho}_k)}, \quad (9)$$

$$CC = \frac{1}{N_{t2}} \sum_{k \in Test2} \frac{\langle (\hat{\rho}_k - \bar{\rho}), (\rho_k^* - \bar{\rho}^*) \rangle}{\sqrt{D(\hat{\rho}_k, \bar{\rho})} \cdot \sqrt{D(\rho_k^*, \bar{\rho}^*)}}, \quad (10)$$

where

$$D(\rho^a, \rho^b) = \sum_{i_x, i_y, i_z} (\rho^a(i_x, i_y, i_z) - \rho^b(i_x, i_y, i_z))^2,$$

$$\langle \rho^a, \rho^b \rangle = \sum_{i_x, i_y, i_z} \rho^a(i_x, i_y, i_z) \cdot \rho^b(i_x, i_y, i_z),$$

and $\bar{\rho}$, $\bar{\rho}^*$ are the averaged densities of estimation and true over the dataset Test2, respectively. $\sum_{k \in Test2}$ is the summation over the dataset Test2.

4. Numerical Experiments

In this section, we present comparisons of the reconstruction performances of proposed method with two conventional methods through numerical experiments. To this end, the number of hidden units was optimized beforehand so that the best performance of each method could be compared.

4.1 Optimization of the Number of Hidden Units

The evaluation criteria for the five-fold cross-validation require varying the number of hidden units, u_{hidden} , from 260 to 2,020 in 160 steps. Figure 6 shows the RMSE of the results of the proposed method, and Fig. 7 shows the results of conventional method-2. It is seen that the RMSE is minimum when the number of units is 580 for both the conventional method-2 and proposed method.

4.2 Results

Table 3 summarizes the results of the RMSE for a five-fold cross-validation, and Table 4 summarizes the corresponding CC results. Figure 8 compares these results for Test2. Tables 3 and 4 show that the proposed method had lower RMSE and higher CC than did the conventional methods.

Figures 9, 10, and 11 show the RMSE learning curves for each of these methods. The horizontal axis represents a common logarithm of the epochs of these methods.

Fig. 6 Transition of RMSE for Test2 (proposed method)

Fig. 7 Transition of RMSE for Test2 (conventional method-2)

Table 3 Results of RMSE for each method

	Training	Test1	Test2
Conventional 1	0.5587	0.5645	0.5638
Conventional 2	0.5247	0.5494	0.5488
Proposed	0.4585	0.4925	0.4922

Table 4 Results of CC for each method

	Training	Test1	Test2
Conventional 1	0.7632	0.7577	0.7584
Conventional 2	0.7945	0.7724	0.7729
Proposed	0.8481	0.8223	0.8226

Fig. 8 Results of RMSE and CC for Test2 **Fig. 9** Learning curve of conventional method-1

4.3 Frequency Analysis of Improvements

In the previous section, it is numerically shown that the proposed method is more accurate than the conventional method-2 (hereinafter referred to as the conventional method) for a given X-ray irradiation dose. In this section, the improvement in accuracy is decomposed into frequency components and the tendency is observed. A cross section of the object phantom is an image of 6×6 . Although this image is small, it is possible to decompose the improvement into a constant component, and low frequency, medium frequency, and high frequency components below the Nyquist frequency by using the 6-point discrete Fourier transform (DFT). The average of the absolute values of the DFT is obtained in the horizontal and vertical directions for each cross section of the object phantom, along with the average for all the object phantoms used in the test. Each frequency component is obtained by performing this operation on the true and estimated data. By comparing the frequency components of the reconstruction error with respect to the true density data between the conventional method and the pro-

Fig. 10 Learning curve of conventional method-2 **Fig. 11** Learning curve of proposed method

posed method, it is possible to identify the frequency area where the effect of the scattered X-ray is large. The analysis method and results are shown below.

4.3.1 Frequency Analysis Method

In the cross section (x, z) at the level i_y of the k -th ($1 \leq k \leq N_{t2}$) test object, the average $F_{i_y}^{(k)}(u)$ of the absolute value of the DFT in the x -axis direction and the absolute value of the DFT in the z -axis direction are calculated as follows:

$$F_{i_y}^{(k)}(u) = \frac{1}{2} \left(\frac{1}{6} \sum_{i_z=0}^5 |\mathcal{F}_u[\rho(i_x, i_y, i_z)]_{i_x}| + \frac{1}{6} \sum_{i_x=0}^5 |\mathcal{F}_u[\rho(i_x, i_y, i_z)]_{i_z}| \right), \quad (11)$$

where $u = 0, 1, \dots, 5$ is the index representing the spatial frequency, and $\mathcal{F}_u[f(i)]_i$ denotes the 6-point DFT

$$\mathcal{F}_u[f(i)]_i = \sum_{i=0}^5 f(i) \exp\left(-j \frac{i}{6} u\right), \quad (12)$$

where j is an imaginary unit. By taking the average of the above in all cross sections along the y -axis, the frequency component of the k -th object is given as

$$F^{(k)}(u) = \frac{1}{6} \sum_{i_y=0}^5 F_{i_y}^{(k)}(u). \quad (13)$$

By averaging the above quantities for all test objects, the average frequency component is given as

$$F(u) = \frac{1}{N_{t2}} \sum_{k=1}^{N_{t2}} F^{(k)}(u). \quad (14)$$

Table 5 Frequency components of error

spatial frequency: u	$F^*(u)$	$er_{conv}(u)$	$eRatio_{conv}(u)$	$er_{prop}(u)$	$eRatio_{prop}(u)$	$Er_{reduct} [\%]$
0	9.09	0.885	0.0974	0.803	0.0883	0.909
1	1.90	1.04	0.548	0.952	0.500	4.78
2	1.91	1.18	0.622	1.048	0.550	7.19
3	1.71	1.14	0.669	0.987	0.578	9.06

Let the true frequency component $F^*(u)$ be defined by replacing $\rho(i_x, i_y, i_z)$ on the right side of Eq. (11) with the true density $\rho^*(i_x, i_y, i_z)$. The error in the conventional method is defined as

$$er_{conv}(i_x, i_y, i_z) = \rho^*(i_x, i_y, i_z) - \widehat{\rho}_{conv}(i_x, i_y, i_z). \quad (15)$$

The frequency component of the error in the conventional method, $Er_{conv}(u)$, is obtained by replacing $\rho(i_x, i_y, i_z)$ in Eq. (11) with $er_{conv}(i_x, i_y, i_z)$ and then evaluating the right side of Eq. (14).

In the same manner, the error obtained by subtracting the density estimated by the proposed method, $\widehat{\rho}_{prop}(i_x, i_y, i_z)$, from the true density is

$$er_{prop}(i_x, i_y, i_z) = \rho^*(i_x, i_y, i_z) - \widehat{\rho}_{prop}(i_x, i_y, i_z). \quad (16)$$

The frequency component of the error in the conventional method $Er_{prop}(u)$ is obtained by replacing $\rho(i_x, i_y, i_z)$ in Eq. (11) with $er_{prop}(i_x, i_y, i_z)$ and then evaluating the right side of Eq. (14).

$Er_{conv}(u)$ and $Er_{prop}(u)$ are the frequency components of the error in the conventional method and the proposed method, respectively. The ratio of the frequency component of the error of each method to the frequency component $F^*(u)$ of the true density is defined as the error rate $eRatio_{conv}(u)$, $eRatio_{prop}(u)$.

4.3.2 Results of Frequency Analysis

In the 6-point DFT, the spatial frequency u corresponds to a constant component, 1 for low frequency, 2 for medium frequency, and 3 for high frequency (4 and 5 are negative frequencies). Table 5 shows the error rate calculated by the method described in the previous section for each frequency. Because the density of each voxel of the object phantom is given by a uniform random number in the range of 0.3 to 3.0, $F^*(u)$ is almost a constant value except for the frequency $u = 0$.

The error rate of the proposed method is lower than that of the conventional method at all frequencies. The error reduction rate of the proposed method with respect to the conventional method is defined as

$$Er_{reduct} = (eRatio_{conv}(u) - eRatio_{prop}(u)) \times 100\%, \quad (17)$$

and is also shown in Table 5.

The error rate of the conventional method increases with increase in the frequency, whereas the increase in the error rate of the proposed method is not very large. Therefore, the error reduction rate increases slightly with increase in the frequency. This means that by using scattered X-ray

under the conditions of this experiment, the effect of improvement in the accuracy is greater than that in the conventional method, which uses only primary X-ray at higher spatial frequencies.

5. Conclusion

In this study, we verified the reconstruction accuracy of CT images containing scattered X-ray data using a neural network to estimate several density parameters under multi-energy conditions. From the results of these experiments, it was numerically shown that the accuracy of reconstruction in terms of the RMSE and CC could be improved using both the primary and scattered X-rays, rather than the primary X-rays alone, as in conventional CT.

To achieve the same reconstruction accuracy in the conventional method as that of the proposed method, it was necessary to increase the number of X-ray irradiation photons when acquiring the projection data, which implies that the subject must be exposed to a higher dose of radiation. Hence, it is suggested that using the proposed method to reconstruct CT images can reduce the exposure dose of the subject, because the scattered X-ray information can improve accuracy.

In the conventional method, the accuracy of reconstruction improves as the amount of X-ray irradiation is increased. The same tendency can be expected to occur in the proposed method, which uses scattered X-rays. Because there exists an upper limit to the reconstruction accuracy, both methods will approach this limit when the X-ray irradiation dose is increased, and the superiority of the proposed method will decrease. Based on this consideration, we emphasize that the accuracy of reconstruction can be improved by using scattered X-rays in situations where the amount of X-ray irradiation is small and reconstruction is difficult with the conventional method. In this paper, the experiment was performed using an object phantom with a small number of voxels due to the performance limitation of the computer, but we believe that the abovementioned aspects have been verified.

Based on the frequency analysis of the error, the proposed method using scattered X-ray at all frequencies had less error than the conventional method using primary X-ray alone, and this tendency increased with the frequency. It is necessary to clarify in the future whether this property is maintained even when the voxels of the object increase.

As a future work, it will also be necessary to develop reconstruction networks that respond to changes in tube voltage and tube current and to clarify the limits of scattered X-ray utilization through numerical experiments using larger-scale objects. In addition, because it is difficult to

cover the subject with detectors in an actual situation, it is necessary to evaluate the performance when the number of scattered X-ray detectors is small.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 19K08101. We would like to thank Editage (www.editage.com) for English-language editing.

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