Learning of Finite Unions of Tree Patterns with Internal Structured Variables from Queries

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SUMMARY A linear term tree is defined as an edge-labeled rooted tree pattern with ordered children and internal structured variables whose labels are mutually distinct. A variable can be replaced with arbitrary edge-labeled rooted ordered trees. We consider the polynomial time learnability of finite unions of linear term trees in the exact learning model formalized by Angluin. The language $L(t)$ of a linear term tree t is the set of all trees obtained from t by substituting arbitrary edge-labeled rooted ordered trees for all variables in t . Moreover, for a finite set S of linear term trees, we define $L(S) = \bigcup_{t \in S} L(t)$. A target of learning, denoted by T_* , is a finite set of linear term trees, where the number of edge labels is infinite. In this paper, for any set T_* of m linear term trees ($m \geq 0$), we present a query learning algorithm which exactly identifies T_* in polynomial time using at most $2mn^2$ *Restricted Subset* queries and at most $m + 1$ *Equivalence* queries, where n is the maximum size of counterexamples. Finally, we note that finite sets of linear term trees are not learnable in polynomial time using *Restricted Equivalence*, *Membership* and *Subset* queries.

key words: exact learning, computational learning theory, finite union of tree pattern languages

1. Introduction

Large number of documents such as HTML/XML files are accessible on Internet. Since such documents have tree structures but no rigid structure, they are called tree-structured data, which are represented by rooted trees with ordered children and edge labels. As an example of representations of tree-structured data, we give a sample document SAMPLE.Doc in Fig. 1 and the rooted tree T as a representation of SAMPLE.Doc in Fig. 2. In the fields of Web mining and text mining, many researchers have been developing data mining techniques based on machine learning methods for analyzing tree-structured data. If we can construct oracles which answer any query in practical time, we can design efficient and effective data mining tools based on query learning algorithms using such oracles. Therefore, from the viewpoint of computational and algorithmic learning theory, in this paper, we consider a polynomial time learnability of finite unions of ordered tree patterns with internal structured variables in the exact learning model formalized by Angluin [4].

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<Sec1> Introduction </Sec1>
<Sec2> Preliminary </Sec2>
<Note> </Note>
<Sec3>
  <SubSec3.1> Result I </SubSec3.1>
  <SubSec3.2> Result II </SubSec3.2>
  <SubSec3.3> Result III </SubSec3.3>
</Sec3>
<Sec4> Conclusion </Sec4>
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Fig. 1 A sample XML file SAMPLE.Doc.

Fig. 2 A rooted ordered tree T .

As a knowledge representation for tree-structured data, we use a rooted ordered tree pattern with internal structured variables, called a *term tree*. A variable in a term tree is a pair of vertices for which an arbitrary rooted ordered tree can be substituted. As an example of term trees, we give a term tree t in Fig. 3. By replacing the variables labeled with x_1 , x_2 , x_3 and x_4 of t in Fig. 3 by ordered rooted trees T_1 , T_2 , T_3 and T_4 , respectively, the rooted ordered tree T in Fig. 2 can be obtained from t , T_1 , T_2 , T_3 and T_4 . After Fig. 3, we assume that all children are ordered from left to right. Term trees are more powerful than or incomparable to other representations of tree-structured data, which were proposed in computational learning theory, such as ordered tree patterns [1] and ordered gapped tree patterns [6]. For example, in Fig. 3, the tree pattern $f(b, x, g(a, z), y)$ can be represented by the term tree s , but the term tree t cannot be represented by any standard tree pattern because of the existence of internal structured variables represented by x_2 and x_3 in t .

Let Λ be an infinite set of edge labels. A term tree t is said to be *linear* if all variable labels in t are mutually distinct. In this paper, we deal with linear term trees only. By μOTT_Λ we denote the set of all linear term trees all of whose edge labels are in Λ . For a linear term tree

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Fig. 3 Term trees t , s and rooted ordered trees T_1, T_2, T_3, T_4 . A variable is represented by a box with lines to its elements. The label inside a box is the variable label.

$t \in \mu OTT_\Lambda$, the *term tree language* $L_\Lambda(t)$ of t is the set of all rooted ordered trees which are obtained from t by substituting arbitrary rooted ordered trees for all variables in t . We remark that the subtrees which are obtained from t by removing the variables in t represent common subtree structures in the trees in $L_\Lambda(t)$. In order to give a learning algorithm for tree-structured data including multiple or heterogeneous sources, we deal with finite unions of term tree languages of linear term trees in μOTT_Λ . The language represented by a finite set of k linear term trees $H = \{t_1, t_2, \dots, t_k\}$ is the union of k term tree languages of $t_1, t_2, \dots, t_k$, i.e., $L_\Lambda(H) = L_\Lambda(t_1) \cup L_\Lambda(t_2) \cup \dots \cup L_\Lambda(t_k)$. The class μOTF_Λ is defined as the collection of all finite sets of linear term trees in μOTT_Λ . Our object in this paper is to make clear the polynomial time learnability of the class μOTF_Λ in exact learning model.

In exact learning model using queries, a learning algorithm accesses to oracles, which answer specific kinds of queries, and collects information about an unknown target. A learning algorithm is said to *exactly learn a target set of linear term trees* $\mathcal{T}_*$ in polynomial time if it outputs a set S of linear term trees in μOTT_Λ in polynomial time such that $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$, after it uses some queries and additional information. Since an edge label in tree structured data such as HTML/XML files is a word, we assume that the set of edge labels attached to edges of input trees is infinite. When a finite set S of linear term trees in μOTT_Λ is given as an input, an oracle for *Restricted Subset* queries answers “yes” if $L_\Lambda(S) \subseteq L_\Lambda(\mathcal{T}_*)$, and answers “no”, otherwise. An oracle for *Equivalence* queries returns “yes” if $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$, and returns a counterexample in $(L_\Lambda(S) \cup L_\Lambda(\mathcal{T}_*)) - (L_\Lambda(S) \cap L_\Lambda(\mathcal{T}_*))$, otherwise.

In this paper, we show that any finite union of languages defined by m linear term trees is exactly identifiable in polynomial time using at most $2mn^2$ *Restricted Subset* queries and at most $m + 1$ *Equivalence* queries, where n is the maximum size of counterexamples and the set of edge labels is infinite. The idea of our learning algorithm is as follows: First, we get a counterexample by an *Equivalence* query. Second, we generalize the counterexample by trying to replace an edge with a variable repeatedly as much as possible, in order to determine the number of edges in one of the linear term trees in the target set. Third, by trying to

replace two adjacent variables with one variable repeatedly as much as possible, we determine the number of variables in the target linear term tree. We repeat these three steps until all linear term trees in the target set are found.

An oracle for *Restricted Equivalence* queries returns “yes” for an input set S if $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$, and answers “no”, otherwise. An oracle for *Membership* queries answers “yes” for a rooted ordered tree T if $T \in L_\Lambda(\mathcal{T}_*)$, and answers “no”, otherwise. An oracle for *Subset* queries answers “yes” for an input set S if $L_\Lambda(S) \subseteq L_\Lambda(\mathcal{T}_*)$, and returns a counterexample in $L_\Lambda(S) - L_\Lambda(\mathcal{T}_*)$, otherwise. Then, we show that finite sets of linear term trees are not learnable in polynomial time using *Restricted Equivalence*, *Membership* and *Subset* queries.

As previous work [10], we showed that the class of languages defined by restricted string patterns are exactly learnable using *Membership* queries and a positive example. In [11], we showed that unions of string subsequences are exactly learnable using *Membership* and *Equivalence* queries. Also our work [8], [15], [17] gave some classes of languages defined by extended term trees which are polynomial time inductively inferable from positive data. As an application [14], we proposed a data mining method from tree-structured data, using a learning algorithm for linear term trees based on another learning model [3]. As other related works, the works [1], [2], [5], [6] show the learnability of tree-structured pattern in exact learning model using queries. Especially, Arimura et al. showed in [5] that the unions of languages defined by ordered tree patterns are exactly learnable in polynomial time using *Equivalence* and *Membership* queries. Moreover, Arimura et al. showed in [6] that the unions of languages defined by m ordered gapped tree patterns are exactly learnable in polynomial time using $O(m)$ *Equivalence* queries and $O(mn^2)$ *Restricted Subset* queries, where n is the maximum size of counterexamples.

This paper is organized as follows. In Sect. 2, we introduce a linear term tree as a tree-structured pattern and a term tree language. Next, we briefly explain an exact learning model using queries. In Sect. 3, we show that any finite union of languages defined by linear term trees is exactly identifiable in polynomial time using *Restricted Subset* queries and *Equivalence* queries. In Sect. 4, we show fi-

nite sets of linear term trees are not learnable in polynomial time using *Restricted Equivalence*, *Membership* and *Subset* queries.

2. Preliminaries

In this section, we define a term tree which is a tree-structured pattern and then define a term tree language of a term tree. Next, we introduce an exact learning model using queries due to Angluin [4].

For a set S , the number of elements in S , called the *size* of S , is denoted by $|S|$.

2.1 Term Trees and Term Tree Languages

Let X be an infinite alphabet whose element is called a *variable label*, and Λ an alphabet where $\Lambda \cap X = \emptyset$. We call an element in Λ an *edge label*, and in this paper, we assume that $|\Lambda|$ is infinite.

Definition 1: Let $T = (V_T, E_T)$ be an edge-labeled rooted tree which has a set V_T of vertices and a set E_T of edges labeled with elements of $\Lambda \cup X$. Let H_t be the set of all edges in E_T whose labels are in X . Let $V_t = V_T$ and $E_t = E_T - H_t$ (i.e., $E_t \cup H_t = E_T$ and $E_t \cap H_t = \emptyset$). A triplet $t = (V_t, E_t, H_t)$ is called a *term tree*, and elements in V_t , E_t and H_t are called a *vertex*, an *edge* and a *variable*, respectively.

For a term tree $t = (V_t, E_t, H_t)$ and its vertices v_1 and v_i , a *path* from v_1 to v_i is a sequence $v_1, v_2, \dots, v_i$ of distinct vertices of t such that for any j with $1 \leq j < i$, there exists an edge or a variable which consists of v_j and v_{j+1} . If there is an edge which consists of v and v' such that v lies on the path from the root to v' , then v is said to be the *parent* of v' and v' is a *child* of v . We denote by (v, v') the edge in E_t . If there is a variable which consists of v and v' such that v lies on the path from the root to v' , Then v is said to be the *parent port* of v' and v' is a *child port* of v . We denote by $[v, v']$ the variable in H_t .

A term tree t is called *ordered* if every internal vertex u in t has a total ordering on all children of u . We define the *size* of t as the number of vertices in t and denote it by $|t|$, that is, $|t| = |V_t|$. For example, the ordered term tree $t = (V_t, E_t, H_t)$ in Fig. 3 is defined as follows. $V_t = \{v_1, \dots, v_{11}\}$, $E_t = \{(v_1, v_2), (v_2, v_3), (v_1, v_4), (v_7, v_8), (v_1, v_{10}), (v_{10}, v_{11})\}$ with the root v_1 and the sibling relation displayed in Fig. 3. $H_t = \{[v_4, v_5], [v_1, v_6], [v_6, v_7], [v_6, v_9]\}$.

We call an ordered term tree a *term tree* simply. In particular, a term tree $t = (V_t, E_t, H_t)$ is said to be *linear* if all variables in H_t have mutually distinct variable labels in X . A linear term tree in this paper is a special form of a linear term tree of the previous work [18]. A variable in a linear term tree is also called a read-once variable and a formula with read-once variables is called a μ -formula in computational learning theory. In this paper, we deal with linear term trees only. We denote by $\mu\mathcal{OTT}_\Lambda$ the set of all linear term trees with Λ as a set of edge labels, and by $\mu\mathcal{OTF}_\Lambda$ the set of all finite sets of linear term trees with Λ as a set of edge labels,

that is, $\mu\mathcal{OTF}_\Lambda = \{S \subseteq \mu\mathcal{OTT}_\Lambda \mid |S| \text{ is finite}\}$. A linear term tree with no variable is called a *ground term tree* and considered to be a tree with ordered children. $\mathcal{OT}_\Lambda$ denotes the set of all ground term trees with Λ as a set of edge labels.

For any linear term tree t , a vertex u of t , and two children u' and u'' of u , we write $u' <_u^t u''$ if u' is smaller than u'' in the order of the children of u . Let $f = (V_f, E_f, H_f)$ and $g = (V_g, E_g, H_g)$ be linear term trees. We say that f and g are *isomorphic*, denoted by $f \equiv g$, if there is a bijection φ from V_f to V_g such that (i) the root of f is mapped to the root of g by φ , (ii) $(u, u') \in E_f$ if and only if $(\varphi(u), \varphi(u')) \in E_g$ and the two edges have the same edge label, (iii) $[u, u'] \in H_f$ if and only if $[\varphi(u), \varphi(u')] \in H_g$, and (iv) for any vertex u in f which has more than one child, and for any two children u' and u'' of u , $u' <_u^f u''$ if and only if $\varphi(u') <_{\varphi(u)}^g \varphi(u'')$. Two isomorphic linear term trees are considered to be identical.

Let f and g be linear term trees with at least two vertices. Let $h = [v, v']$ be a variable in f with a variable label x and $\sigma = [u, u']$ a list of two distinct vertices in g , where u is the root of g and u' is a leaf of g . The form $x := [g, \sigma]$ is called a *binding* for x . A new linear term tree $f' = f\{x := [g, \sigma]\}$ is obtained by applying the binding $x := [g, \sigma]$ to f in the following way. Let g' be a copy of g and w, w' the vertices of g' corresponding to u, u' of g , respectively. For the variable $h = [v, v']$, we attach g' to f by removing the variable h from H_f and by identifying the vertices v, v' with the vertices w, w' of g' respectively. All variable in f' are labeled by mutually distinct variable labels in X .

A *substitution* θ is a finite collection of bindings $\{x_1 := [g_1, \sigma_1], \dots, x_n := [g_n, \sigma_n]\}$, where x_i 's are mutually distinct variable labels in X . The linear term tree $f\theta$, called the *instance* of f by θ , is obtained by applying all the bindings $x_i := [g_i, \sigma_i]$ on f simultaneously. We define a new total ordering $<_{f\theta}^v$ on every vertex v in $f\theta$ in the following natural way. Suppose that v has more than one child and let v' and v'' be two children of v in $f\theta$. There are five cases in which the ordering between v' and v'' have to be newly defined. (1) $v \in V_{f\theta} - V_f$: In this case, there is a linear term tree $g \in \{g_1, \dots, g_n\}$ such that all of v, v', v'' are in V_g . Then $v' <_{f\theta}^v v''$ is defined if and only if $v' <_v^g v''$. If $v \in V_f$, we have the following four subcases. (2) $v' \in V_f$ and $v'' \in V_f$: $v' <_{f\theta}^v v''$ is defined if and only if $v' <_v^f v''$. (3) $v' \in V_f$ and there is a linear term tree $g \in \{g_1, \dots, g_n\}$ such that $v'' \in V_g$: Let w be the child port of the variable for which g is substituted. We note that v is the parent port of the variable. Then $v' <_{f\theta}^v v''$ is defined if and only if $v' <_v^f w$. (4) There is a linear term tree $g \in \{g_1, \dots, g_n\}$ such that both of v' and v'' are in V_g : Since v is identified with the root of g (say u), $v' <_{f\theta}^v v''$ is defined if and only if $v' <_u^g v''$. (5) There are two distinct linear term trees $g, g' \in \{g_1, \dots, g_n\}$ such that $v' \in V_g$ and $v'' \in V_{g'}$: Let w (resp. w') be the child port of the variable for which g (resp. g') is substituted. Then $v' <_{f\theta}^v v''$ is defined if and only if $w <_v^f w'$. In Fig. 4, we give an example of the new ordering on vertices in a linear term tree.

Fig. 4 The new ordering on vertices in the linear term tree $f' = f\{x := [g_1, [u_1, u_1']], y := [g_2, [u_2, u_2']]\}$.

We define the root of the resulting linear term tree $f\theta$ as the root of f . Consider the examples in Fig. 2 and 3. An example of a linear term tree t is given. Let $\theta = \{x_1 := [T_1, [u_1, w_1]], x_2 := [T_2, [u_2, w_2]], x_3 := [T_3, [u_3, w_3]], x_4 := [T_4, [u_4, w_4]]\}$ be a substitution, where T_1, T_2, T_3 , and T_4 are ground term trees in Fig. 3. Then the instance $t\theta$ of the linear term tree t by θ is isomorphic to the tree T in Fig. 2. Let f and g be linear term trees. We write $f \leq g$ if there exists a substitution θ such that $f \equiv g\theta$. If $f \leq g$ and $f \not\equiv g$, then write $f < g$.

Definition 2: The term tree language $L_\Lambda(t)$ of a linear term tree $t \in \mu OTT_\Lambda$ is $\{s \in OT_\Lambda \mid s \leq t\}$. For a subset S of μOTT_Λ , we define $L_\Lambda(S) = \bigcup_{t \in S} L_\Lambda(t)$, and $L_\Lambda(S)$ is called the term tree language defined by S . In particular, we define $L_\Lambda(\emptyset) = \emptyset$.

Let t be a linear term tree in μOTT_Λ and S a set in μOTF_Λ . Then we consider the following property: $L_\Lambda(t) \subseteq L_\Lambda(h)$ for some $h \in S$ if and only if $L_\Lambda(t) \subseteq L_\Lambda(S)$. This property is important in the learning of unions of languages, is called *compactness*, which was proposed in [6], [7] and is enjoyed by similar languages, e.g., monotone DNF.

Lemma 1: Let t be a linear term tree in μOTT_Λ , S a set in μOTF_Λ and $|\Lambda|$ infinite. Then, $t \leq h$ for some $h \in S$ if and

Fig. 5 An example which violates the property *compactness*: If $\Lambda = \{a_1, \dots, a_k\}$ ($k \geq 1$), then these linear term trees satisfy $L_\Lambda(f) = L_\Lambda(g_1) \cup \dots \cup L_\Lambda(g_{k+2})$, but $L_\Lambda(f) \not\subseteq L_\Lambda(g_i)$ for all i ($1 \leq i \leq k+2$).

only if $L_\Lambda(t) \subseteq L_\Lambda(S)$.

Proof. The only if part is obvious. Then, we only prove the if part. Since $|\Lambda|$ is infinite, there is an edge label c which does not appear in any linear term tree in S . Let w_t be a ground term tree obtained from t by substituting all the variables in t with edges labeled with c . Since $w_t \in L_\Lambda(t)$ and $L_\Lambda(t) \subseteq L_\Lambda(S)$, there exists a linear term tree $t' \in S$ such that $w_t \in L_\Lambda(t')$. Since the edge label c does not appear in S , we have $t \leq t'$ by inverting the substitutions. $\square$

If $|\Lambda|$ is finite, then μOTF_Λ does not satisfy the property of compactness (see Fig. 5). By Lemma 1, μOTF_Λ satisfies the property of compactness if $|\Lambda|$ is infinite.

2.2 Learning Model

Let $\mathcal{T}_* \in \mu OTF_\Lambda$ to be identified, and we say that the set $\mathcal{T}_*$ is a *target*. First we introduce the exact learning model via queries due to Angluin [4]. In this model, learning algorithms can access to *oracles* that will answer queries about the target set $L_\Lambda(\mathcal{T}_*)$. A learning algorithm collects information about $\mathcal{T}_*$ by using queries. We consider the following queries.

1. **Membership query:** The input is a ground term tree T in OT_Λ . The output is “yes” if $T \in L_\Lambda(\mathcal{T}_*)$, and “no” otherwise. The oracle which answers the Membership query is called a *Membership oracle*.
2. **Subset query and Restricted Subset query:** The input of both queries is a set S in μOTF_Λ . The output of Subset query is “yes” if $L_\Lambda(S) \subseteq L_\Lambda(\mathcal{T}_*)$, otherwise, a ground term tree, called a *counterexample*, in $L_\Lambda(S) - L_\Lambda(\mathcal{T}_*)$. The oracle which answers the Subset query is called a *Subset oracle*. The output of Restricted Subset query is “yes” if $L_\Lambda(S) \subseteq L_\Lambda(\mathcal{T}_*)$, otherwise “no”. The oracle which answers the Restricted Subset query is called a *Restricted Subset oracle*.
3. **Equivalence query and Restricted Equivalence query:** The input of both queries is a set S in μOTF_Λ . The output of Equivalence query is “yes” if $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$, otherwise a ground term tree, called a *counterexample*, in $(L_\Lambda(S) \cup L_\Lambda(\mathcal{T}_*)) - (L_\Lambda(S) \cap L_\Lambda(\mathcal{T}_*))$. The oracle which answers the Equivalence query is called an *Equivalence oracle*. The output of Restricted Equivalence query is “yes” if $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$, otherwise

“no”. The oracle which answers the *Restricted Equivalence query* is called a *Restricted Equivalence oracle*.

A learning algorithm $\mathcal{A}$ is said to *exactly identify a target* $\mathcal{T}_*$ in polynomial time if $\mathcal{A}$ outputs a set $S \in \mu OTF_\Lambda$ in polynomial time such that $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$.

For two linear term trees t and t' , we write $t \approx t'$ if $\tilde{t} \equiv \tilde{t}'$, where $\tilde{t}$ and $\tilde{t}'$ are linear term trees obtained from t and t' by replacing all edges with variables, respectively. For a set of linear term trees H , $N(H)$ denotes the total size of all linear term trees in H . The above oracles are implemented practically. We have the following propositions 1, 2 and 3 with respect to the time complexities of *Membership query*, *Restricted Subset query* and *Restricted Equivalence query*, respectively.

Proposition 1 ([16]): Given a set of linear term trees H and a ground term tree T , the problem of deciding whether or not $T \in L_\Lambda(H)$ is solvable in $O(N(H)|T|)$ time.

In order to show Proposition 2, we give the following lemma.

Lemma 2: Given two linear term trees t and t' , if $|\Lambda|$ is infinite then the problem of deciding whether or not $t \leq t'$ is solvable in $O(|t||t'|)$ time.

Proof. Let c be an edge label which does not appear in both t and t' . Here, we call an edge labeled with c a c -edge simply. Let T be the ground term tree obtained from t by replacing all variables in t with c -edges. We show the next claim: $T \in L_\Lambda(t')$ if and only if $t \leq t'$. The if direction is obvious. Then, we assume $T \in L_\Lambda(t')$. There exists a substitution θ satisfying $t'\theta \equiv T$. Since no c -edge appear in t' , all c -edges in T must be c -edges of substituted trees in θ . Let θ' be the substitution obtained from θ by replacing all c -edges with variables. Then, since $t'\theta' \approx t'\theta$ and $T \approx t$, it is easy to see that $t'\theta' \equiv t$. Therefore, we have $t \leq t'$. Then, from this claim and Proposition 1, this lemma holds. $\square$

Proposition 2: Given two sets of linear term trees H and H' , if $|\Lambda|$ is infinite then the problem of deciding whether or not $L_\Lambda(H) \subseteq L_\Lambda(H')$ is solvable in $O(N(H)N(H'))$ time.

Proof. From Lemma 1, $L_\Lambda(t) \subseteq L_\Lambda(H')$ if and only if there exists a linear term tree $t' \in H'$ with $t \leq t'$. Then, from Lemma 2, the problem of deciding whether or not $L_\Lambda(t) \subseteq L_\Lambda(H')$ is solvable in $O(|t|N(H'))$ time. Therefore, the problem is solvable in $O(N(H)N(H'))$ time. $\square$

From Proposition 2, it is easy to see that the following proposition holds.

Proposition 3: Given two sets of linear term trees H and H' , if $|\Lambda|$ is infinite then the problem of deciding whether or not $L_\Lambda(H) = L_\Lambda(H')$ is solvable in $O(N(H)N(H'))$ time.

If $|\Lambda|$ is infinite, then, for two sets of linear term trees H and H' such that $L_\Lambda(H) \not\subseteq L_\Lambda(H')$, there exists a linear term tree $t \in H$ such that $t \not\leq t'$ for any $t' \in H'$. Let c be an edge label which does not appear in any linear term tree in H' . Let T be the ground term tree which is obtained from t

by replacing all variables with edges of label c . It is easy to see that $T \in L_\Lambda(H) - L_\Lambda(H')$. From these observations, we can see that *Subset query* and *Equivalence query* have also square time complexities with respect to the input size.

3. Learning Finite Unions of Term Tree Languages

By presenting a learning algorithm *LEARN_UNION* in Fig. 6, we give the following theorem as one of the main results of this paper. The formal definitions of notations used in the algorithm *LEARN_UNION* are stated after Theorem 1 and in Definition 3.

Theorem 1: The algorithm *LEARN_UNION* of Fig. 6 exactly identifies any set $\mathcal{T}_* \in \mu OTF_\Lambda$ in polynomial time using at most $m + 1$ *Equivalence queries* and at most $2mn^2$ *Restricted Subset queries*, where $m = |\mathcal{T}_*|$, n is the maximum size of counterexamples and $|\Lambda|$ is infinite.

First, we consider the internal repeat-loop between lines 5 and 12 in the algorithm *LEARN_UNION*. For a linear term tree t , let t' be a linear term tree obtained from t by replacing an edge e of t with a new variable. Then, t' is a generalized linear term tree of t such that $t \leq t'$. Let $t = (V_t, E_t, H_t) \in \mu OTF_\Lambda$ and $e = (u, v) \in E_t$. We denote by $t/\{e\}$ the linear term tree obtained from t by replacing the edge e with a new variable $[u, v]$, that is, $t/\{e\} = (V_t, E_t - \{e\}, H_t \cup \{[u, v]\})$. For example, for a linear term

Algorithm LEARN_UNION

Given: Oracles *Equiv_{T_{*}}* and *rSub_{T_{*}}*
for the target $\mathcal{T}_* \in \mu OTF_\Lambda$;
Output: A set $S \in \mu OTF_\Lambda$ with $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$.

```

begin
1   $S := \emptyset$ ;
2  while EquivT*( $S$ )  $\neq$  yes do
3    begin
4      Let  $t$  be a counterexample;
5      repeat
6        foreach edge  $e$  in  $t$  do
7          begin
8            Let  $t' := t/\{e\}$ ;
9            if rSubT*( $\{t'\}$ ) = yes then
10             begin  $t := t'$ ; break; end;
11          end
12        until  $t$  does not change;
13      repeat
14        foreach  $\theta \in C(t)$  do
15          begin
16            Let  $t'$  be a linear term tree with  $t \vdash_\theta t'$ ;
17            if rSubT*( $\{t'\}$ ) = yes then
18              begin  $t := t'$ ; break; end;
19          end
20        until  $t$  does not change;
21       $S := S \cup \{t\}$ ;
22    end
23  output  $S$ ;
end

```

Fig. 6 Learning algorithm *LEARN_UNION*. We denote by *Equiv_{T_{*}}*, *Equivalence query* and by *rSub_{T_{*}}*, *Restricted Equivalence query*.

Fig. 7 Linear term trees t_1 and t_2 such that $t_2 \equiv t_1 / \{(v_2, v_4)\}$.

Fig. 8 Linear term trees g_1 , g_2 , g_3 .

tree t_1 in Fig. 7, we give a linear term tree $t_2 = t_1 / \{(v_2, v_4)\}$ in Fig. 7 obtained from t by replacing the edge $e = (v_2, v_4)$ with a new variable $[v_2, v_4]$. We can see that $t_1 \leq t_2$. Therefore, we have the following lemma.

Lemma 3: Let $t = (V_t, E_t, H_t) \in \mu OTT_\Lambda$ and $e \in E_t$. Then $t \leq t/\{e\}$.

For a linear term tree t given after executing the internal repeat-loop between lines 5 and 12 in the algorithm *LEARN_UNION*, we can see that $L_\Lambda(t) \subseteq L_\Lambda(\mathcal{T}_*)$ and $L_\Lambda(t/\{e\}) \not\subseteq L_\Lambda(\mathcal{T}_*)$ for any $e \in E_t$. Then, we have the following lemma.

Lemma 4: Let $t = (V_t, E_t, H_t) \in \mu OTT_\Lambda$ and $S \in \mu OTF_\Lambda$. If $L_\Lambda(t) \subseteq L_\Lambda(S)$ and $L_\Lambda(t/\{e\}) \not\subseteq L_\Lambda(S)$ for any $e \in E_t$, then there exists a linear term tree $t' = (V_{t'}, E_{t'}, H_{t'}) \in S$ such that $t \leq t'$ and $|E_t| = |E_{t'}|$.

Proof. Since $L_\Lambda(t) \subseteq L_\Lambda(S)$, there exists a linear term tree $t' = (V_{t'}, E_{t'}, H_{t'}) \in S$ such that $t \leq t'$ by Lemma 1. Since $t \leq t'$, we have $|E_{t'}| \leq |E_t|$. We assume $|E_{t'}| < |E_t|$. There exists an edge $e \in E_t$ such that $t/\{e\} \leq t'$. This implies $L_\Lambda(t/\{e\}) \subseteq L_\Lambda(S)$. This is a contradiction. Therefore, we have $|E_t| = |E_{t'}|$. $\square$

Second, we consider the internal repeat-loop between lines 13 and 20 in *LEARN_UNION*.

Definition 3: For linear term trees t , t' and a substitution $\theta = \{x := [g, [u, v]]\}$, we write $t \vdash_\theta t'$ if $t' \equiv t$ and g is one of the following linear term trees g_1 , g_2 and g_3 (see Fig. 8).

1. $g_1 = (\{u, v, w\}, \emptyset, \{[u, w], [w, v]\})$.
2. $g_2 = (\{u, v, w\}, \emptyset, \{[u, w], [u, v]\})$ where $w <_u^{g_2} v$.
3. $g_3 = (\{u, v, w\}, \emptyset, \{[u, v], [u, w]\})$ where $v <_u^{g_3} w$.

If θ is clear from the context, we may write $\vdash$ instead of $\vdash_\theta$.

For linear term trees $t = (V_t, E_t, H_t)$ and $t' = (V_{t'}, E_{t'}, H_{t'})$ in

μOTT_Λ , if $t \vdash_\theta t'$, then there are three vertices u_1, u_2, u_3 such that t' can be transformed from t by applying the following operations to t .

1. Contracting a subgraph consisting of two variables $[u_1, u_2]$, $[u_2, u_3]$ in t such that u_2 has exactly one child port in t into the variable $[u_1, u_3]$, that is $V_{t'} := V_t - \{u_2\}$, $E_{t'} := E_t$ and $H_{t'} := H_t \cup \{[u_1, u_3]\} - \{[u_1, u_2], [u_2, u_3]\}$.
2. Removing a variable $[u_1, u_2]$ from t if there exists a variable $[u_1, u_2]$ in t such that u_2 is a leaf of t and u_3 is the next sibling of u_2 , that is $V_{t'} := V_t - \{u_2\}$, $E_{t'} := E_t$ and $H_{t'} := H_t - \{[u_1, u_2]\}$.
3. Removing a variable $[u_1, u_3]$ from t if there exists a variable $[u_1, u_3]$ in t such that u_3 is a leaf of t and u_3 is the next sibling of u_2 , that is $V_{t'} := V_t - \{u_3\}$, $E_{t'} := E_t$ and $H_{t'} := H_t - \{[u_1, u_3]\}$.

For example, in Fig. 9, $f_1 \vdash_{\theta_1} f_2$, $f_3 \vdash_{\theta_2} f_4$ and $f_5 \vdash_{\theta_3} f_6$, where $\theta_1 = \{x_1 := [g_1, [u, v]]\}$, $\theta_2 = \{x_2 := [g_2, [u, v]]\}$, $\theta_3 = \{x_1 := [g_3, [u, v]]\}$ and g_1, g_2, g_3 are described in Fig. 8. We can see that $f_1 \leq f_2$, $f_3 \leq f_4$ and $f_5 \leq f_6$.

For a linear term tree $t = (V_t, E_t, H_t)$, we denote by $C(t)$ the set of all substitutions θ with $t \vdash_\theta t'$. Note that $|C(t)| < |t|$.

For linear term trees t , t' , if $t \vdash_\theta t'$ then t' is a generalized linear term tree of t such that $t \leq t'$. Therefore we have the following lemmas.

Lemma 5: Let t and t' be linear term trees in μOTT_Λ . If $t \vdash t'$, then $t \leq t'$.

For a linear term tree t given after executing the internal repeat-loop between lines 13 and 20 in the algorithm *LEARN_UNION*, we can see that $L_\Lambda(t'') \not\subseteq L_\Lambda(\mathcal{T}_*)$ for any $t'' \in \mu OTT_\Lambda$ with $t \vdash t''$.

Lemma 6: Let $t = (V_t, E_t, H_t)$ be a linear term tree in μOTT_Λ and S a set in μOTF_Λ such that there exists a linear term tree $t' = (V_{t'}, E_{t'}, H_{t'}) \in S$ with $t \leq t'$ and $|E_t| = |E_{t'}|$. If $L_\Lambda(t'') \not\subseteq L_\Lambda(S)$ for any $t'' \in \mu OTT_\Lambda$ with $t \vdash t''$, then $t \equiv t'$.

Proof. Since $t \leq t'$, we have $|t| \geq |t'|$. We assume $|t| > |t'|$. Since $t \leq t'$, there exists a substitution $\theta = \{x_1 := [g_1, \sigma_1], \dots, x_n := [g_n, \sigma_n]\}$ such that $t \equiv t'\theta$. Since $|E_t| = |E_{t'}|$, we have $|H_t| > |H_{t'}|$ by the assumption. We assume $|g_i| = 2$ for any binding $x_i := [g_i, \sigma_i]$ in θ ($1 \leq i \leq n$). Then we have $|t| \neq |t'\theta|$. This contradicts with $t \equiv t'\theta$. Next, we assume that there exists a binding $x_i := [g_i, \sigma_i]$ in θ such that $E_{g_i} \neq \emptyset$, where $g_i = (V_{g_i}, E_{g_i}, H_{g_i})$ ($1 \leq i \leq n$). Then, we have $|E_t| = |E_{t'}| \neq |E_{t'\theta}|$, where $t'\theta = (V_{t'\theta}, E_{t'\theta}, H_{t'\theta})$. This contradicts with $t \equiv t'\theta$. Therefore, there exists a binding $x_\ell := [g_\ell, \sigma_\ell]$ in θ such that $|g_\ell| \geq 3$ and $E_{g_\ell} = \emptyset$, where $g_\ell = (V_{g_\ell}, E_{g_\ell}, H_{g_\ell})$ ($1 \leq \ell \leq n$). We can construct a new substitution θ' such that $t \equiv t'\theta' \vdash t'\theta'$ and $t'\theta' \in \mu OTT_\Lambda$. By Lemma 5, we have $L_\Lambda(t) \subseteq L_\Lambda(t'\theta') \subseteq L_\Lambda(t') \subseteq L_\Lambda(S)$. This is a contradiction. Thus, $|t| = |t'|$. Since $|E_t| = |E_{t'}|$ and $|t| = |t'|$, we have $|H_t| = |H_{t'}|$. Therefore, $t \equiv t'$. $\square$

Let $S_0, S_1, \dots, S_i, \dots$ and $t_1, t_2, \dots, t_i, \dots$ ($i \geq 0$) be the sequence of hypotheses asked in the *Equivalence* queries by *LEARN_UNION* and the sequence of counterexamples

Fig. 9 Operations 1, 2 and 3 constructing linear term trees f_2, f_4, f_6 from linear term trees f_1, f_3, f_5 , respectively.

returned by the queries respectively. Let S_0 be the initial hypothesis $\emptyset$. And we suppose that at each stage $i \geq 1$, *LEARN_UNION* makes an *Equivalence* query $\text{Equiv}_{\mathcal{T}_*}(S_{i-1})$, and receives a counterexample t_i to the query.

Let $P, Q \in \mu\text{OTF}_\Lambda$. If there exists a linear term tree $q \in Q$ such that $p \leq q$ for any $p \in P$, we write $P \sqsubseteq Q$. If $P \sqsubseteq Q$ and $P \not\sqsubseteq Q$, then we write $P \sqsubset Q$.

Lemma 7: For each $i \geq 0$, $t_i \in L_\Lambda(\mathcal{T}_*)$ and $S_i \sqsubseteq \mathcal{T}_*$.

Proof. The proof is by the induction on the number of iterations $i \geq 0$ of the main loop. If $i = 0$, then it is clear.

We assume inductively that the result holds for any number of iterations of the main loop less than i . By the inductive hypothesis, we have $S_{i-1} \sqsubseteq \mathcal{T}_*$. If $L_\Lambda(S_{i-1}) \neq L_\Lambda(\mathcal{T}_*)$, t_i is obtained. Since $S_{i-1} \sqsubseteq \mathcal{T}_*$, we have $L_\Lambda(S_{i-1}) \sqsubseteq L_\Lambda(\mathcal{T}_*)$. Thus, $t_i \in L_\Lambda(\mathcal{T}_*)$.

Let $t'_i = (V_{t'_i}, E_{t'_i}, S_{t'_i})$ be a linear term tree obtained by executing the first repeat-loop from t_i . By Lemma 1 and Lemma 4, there exists a linear term tree $t'_* \in \mathcal{T}_*$ such that $|E_{t'_i}| = |E_{t'_*}|$ and $t'_i \leq t'_*$. Let t''_i be a linear term tree obtained by executing the second repeat-loop from t'_i . By Lemma 1 and Lemma 6, there exists a linear term tree $t''_* \in \mathcal{T}_*$ such that $t''_i \equiv t''_*$. By the inductive hypothesis, we know $S_{i-1} \sqsubseteq \mathcal{T}_*$. Therefore, we have $S_i = S_{i-1} \cup \{t''_i\} \sqsubseteq \mathcal{T}_*$. $\square$

Lemma 8: For each $i \geq 1$, $S_{i-1} \sqsubset S_i$.

Proof. For each $i \geq 1$, it is clear that $S_{i-1} \sqsubseteq S_i$. We assume that $S_{i-1} \supseteq S_i$ for some $i \geq 1$. Let t_* be a linear term tree which is added to S_{i-1} at the i -th stage, that is, $S_i = S_{i-1} \cup \{t_*\}$. Since $S_{i-1} \supseteq S_i$, there exists a linear term tree $t'_* \in S_{i-1}$ such that $t_* \equiv t'_*$. By Lemma 3 and Lemma 5, we have $t_i \leq t_* \equiv t'_*$. This implies $t_i \in L_\Lambda(t_*) = L_\Lambda(t'_*) \subseteq L_\Lambda(S_{i-1})$. Since t_i is an output counterexample by $\text{Equiv}_{\mathcal{T}_*}(S_{i-1})$, we have $t_i \notin L_\Lambda(S_{i-1})$. This is a contradiction. $\square$

By the above lemmas, we can prove Theorem 1 as follows.

Proof of Theorem 1. By the construction of the algorithm *LEARN_UNION*, Lemma 7 and Lemma 8, we know that the algorithm terminates and outputs S such that $L_\Lambda(S) = L_\Lambda(\mathcal{T}_*)$.

At the i -th stage of the while-loop, we receive a counterexample t_i . The loop of lines 6–11 uses at most $|t_i|$ *Restricted Subset* queries. Since the loop of lines 5–12 is repeated at most $|t_i|$ times, the loop uses at most $|t_i|^2$ *Restricted Subset* queries. We can show that the loop of lines 13–20 uses at most $|t_i|^2$ *Restricted Subset* queries in a similar way. Note that $|\mathcal{T}_*| = m$ and n is the maximum size of counterexamples. By Lemma 7 and Lemma 8, since the while-loop is repeated at most m times. Therefore, *LEARN_UNION* uses

at most $2mn^2$ *Restricted Subset* queries and at most $m + 1$ *Equivalence* queries. $\square$

4. Hardness Result on Learnability

In this section, we show the insufficiency of learning of μOTF_Λ in exact learning model. We use the following lemma to show the insufficiency of learning of μOTF_Λ .

Lemma 9: (László Lovász [9]) Let UT_n be the number of all rooted unordered trees of size n with no edge label. Then, $2^n < UT_n < 4^n$, where $n \geq 6$.

We denote by OT_n the set of all rooted ordered trees of size n with no edge label. From the above lemma, we have $OT_n > 2^n$, where $n \geq 6$. The following lemma is known to show the insufficiency of learning in exact learning model.

Lemma 10: (Angluin [4]) Suppose that a hypothesis space contains a class of distinct sets $L_1, \dots, L_N$. If there exists a set $L_\cap$ in the hypothesis space such that for any pair of distinct indices i and j ($1 \leq i, j \leq N$), $L_\cap = L_i \cap L_j$, then any algorithm that exactly identifies each of the hypotheses L_i using *Restricted Equivalence*, *Membership* and *Subset* queries must make at least $N - 1$ queries in the worst case.

By Lemma 9 and Lemma 10, we have Theorem 2.

Theorem 2: Any learning algorithm that exactly identifies all finite sets of linear term trees of size n using *Restricted Equivalence*, *Membership* and *Subset* queries must make $\Omega(2^n)$ queries in the worst case, where $n \geq 6$ and $|\Lambda| \geq 1$.

Proof. We denote by S_n the class of all singleton sets of ground term trees of size n . The class S_n is a subclass of μOTF_Λ and for any L and L' in S_n , $L \cap L' = \emptyset$. Since the empty set $L_\cap(\emptyset) = \emptyset$ is a hypothesis in μOTF_Λ , by Lemmas 9 and 10, any learning algorithm that exactly identifies all finite sets of linear term trees of size n using *Restricted Equivalence*, *Membership* and *Subset* queries must make $\Omega(2^n)$ queries in the worst case, even when $|\Lambda| = 1$. $\square$

5. Conclusion

We have discussed the learnability of μOTF_Λ in the exact learning model. In Sect. 3, we have shown that any finite set $\mathcal{T}_*$ of linear term trees is exactly identifiable using at most $2mn^2$ *Restricted Subset* queries and at most $m + 1$ *Equivalence* queries, where $m = |\mathcal{T}_*|$, n is the maximum size of counterexamples and $|\Lambda|$ is infinite.

Moreover, as a negative result we have shown that it is hard to exactly identify any set in μOTF_Λ efficiently using *Restricted Equivalence*, *Membership* and *Subset* queries. In the exact learning model using queries, other positive or negative learnability of μOTF_Λ is not known. As future work, we will investigate it.

We denote by OTF_Λ the set of all term trees with repeated variables. In [13], we showed the polynomial time learnability of OTF_Λ in the exact learning model by using *Superset* queries and *Restricted Equivalence* queries.

Table 1 Our results and future works.

	Exact learning	Inductive inference from positive data
μOTT_Λ	Yes [12] $ \Lambda \geq 2$	Yes [15] polynomial time $ \Lambda \geq 1$
$(\mu OTT_\Lambda)^2$	Yes [this work]	Yes [8] polynomial time $ \Lambda $ is infinite
μOTF_Λ	<i>Equivalence & Restricted Subset</i> $ \Lambda $ is infinite	<i>Open</i>
OTF_Λ	Yes [13] $ \Lambda $ is infinite	<i>Open</i>

Suzuki et al. [15] showed the learnability of μOTT_Λ in the framework of polynomial time inductive inference from positive data [3]. Hirashima showed the learnability of the class of unions of two linear term tree languages, denoted by $(\mu OTT_\Lambda)^2$, in the same framework [8]. For learning models other than the exact learning model and inductive inference from positive data, no results on the learnability of μOTF_Λ are known. Thus, we will study the learnabilities of OTF_Λ in the same framework. We summarize our results and future works in Table 1.

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