SUMMARY A new method of segmentation for Synthetic Aperture Radar (SAR) images using the skewness wavelet energy has been presented. The skewness is the third order cumulant which measures the local texture along the region-based active contour. Nonlinearity in intensity inhomogeneities often occur in SAR images due to the speckle noise. In this paper we propose a region-based active contour model that is able to use the intensity information in local regions and to cope with the speckle noise and nonlinear intensity inhomogeneity of SAR images. We use a wavelet coefficients energy distribution to analyze the SAR image texture in each sub-band. A fitting energy called skewness wavelet energy is defined in terms of a contour and a functional so that, the regions and their interfaces will be modeled by level set functions. A functional relationship has been calculated on these level sets in terms of the third order cumulant, from which an energy minimization is derived. Minimizing the calculated functions derives the optimal segmentation based on the texture definitions. The results of the implemented algorithm on the test images from the Radarsat SAR images of agricultural and urban regions show a desirable performance of the proposed method.

key words: segmentation, synthetic aperture radar; active contours, level set method, third order cumulant

1. Introduction

Many applications of synthetic aperture radar (SAR) imaging are for reconnaissance, surveillance and targeting. SAR can provide sufficiently high resolution to distinguish terrain features and to recognize and identify selected targets [1]. In remote sensing applications, SAR imaging is a very powerful and attractive tool due to its high spatial resolution. Yet, automatic interpretation of SAR images is extremely difficult [2] because of the speckle noise. The speckle noise is a fully developed noise which SAR images are usually modeled as affected by it. Speckle phenomenon affects all coherent imaging systems [3] and can be described as multiplicative noise, with standard deviation equal to pixel reflectivity value [4]. This fact introduces significant difficulties for designing effective despeckling algorithms [5]. The probability density function (PDF) of the pixel intensities in images of SAR also impressed by speckle noise. This phenomenon can be expressed by the nonlinear intensity inhomogeneity in SAR images. This is one of the main points for SAR imaging which is a crucial issue for accurate segmentation.

The main goal of the image segmentation is to cluster pixels into conspicuous image regions that are easier to analyze. Segmentation can be seen as a partition problem. Active contour models or snakes have been one of the most successful methods for image segmentation [6]. The snake methods can be categorized into two major classes: edge-based methods [7]–[9], and region-based methods [10]–[12]. Edge-based methods use image gradient to stop the evolving contours on the object boundaries. Edge-based active contours utilize two terms to control the motion of the contour: an edge-based stopping term and a balloon force term [6]. These methods have weak performance in encounter with weak object boundaries and are also sensitive to the location of initial contours. Region-based active contours often draw upon intensity information in local regions at a controllable scale[13]–[15]. These properties guide the motion of the active contour.

However, recent SAR segmentation models [16]–[19] tend to rely on intensity homogeneity in each region to be segmented as well as no intention for segmenting the special objects in SAR images. These methods are only useful to segment the extended areas such as rivers, urban and agricultural areas. On the other hand, nonlinearity of intensity inhomogeneity often occurs in SAR images from different modalities. Intensity inhomogeneity can be addressed by some active contour models such as Vese and Chan [14], Tsai et al. [20], Mumford-Shah functional [21] and Michailovich et al. [22]. These techniques are widely known as piecewise smooth (PS) models. Recently, Chunming Li et al. [15] proposed a region-based active contour model by defining a region-scalable fitting (RSF) energy functional that locally approximate the image intensities on two sides of a contour. The model relies on the intensity inhomogeneity and overcome the limitations of PS models.

In this paper, we propose a new region-based active contour model for object segmentation of images from synthetic aperture radar. We first define the skewness wavelet energy (SWE) function in terms of a contour and a fitting function where it locally measures the object textures along the contour and image intensities on two sides of the contour. The energy of wavelet coefficients of each region in each sub-band of the decomposition has been used. In fact, we characterize a texture of each region by using the energy of its wavelet coefficients. This energy is then formed into
variational level sets with a third order cumulant regularization term. With these level set functions, we can model the regions and their interfaces. Then a functional formulation can be calculated on these level sets in terms of third order cumulant, from which an energy minimization is derived. By minimizing of this functional, the optimal segmentation based method will be defined on textures.

Note that our method, termed as skewness wavelet energy model, is also related to the local binary fitting energy model which is presented in [15] where the local intensity average values are derived as the minimizers of this energy functional in a variational formulation. Whereas in this paper, a local texture of each region of a SAR object are computed as the minimizers of the proposed skewness wavelet energy functional.

This paper is organized as follows. In Sect. 2, we introduce a region-based skewness level set model. The higher order cumulants are reviewed and a functional formulation based on the skewness level set model is given. In Sect. 3, calculation of the proposed SWE energy formulation is presented. The implementation and test results of our method are given in Sect. 4. Finally, Sect. 5 draws some conclusions.

2. Region-Based Skewness Level Set Model

Active contour models were first formulated by Mumford and Shah for the image segmentation problem [21]. Their functional is practically difficult to be minimized because of the unknown contour and the nonconvexity of the functional. Afterwards, Chan and Vese optimized the active contour of the Mumford-Shah problem for a special case where the image is considered as a piecewise constant function. Their model does not contain any local intensity information. Then, Chunming Li et al. [15] proposed the LBF model to optimize the Chan-Vese model. However, their model does not contain any statistical information, which is crucial for segmentation of SAR images with nonlinear intensity inhomogeneity. In this section, we first investigate the LBF model proposed by Chunming Li et al. and then introduce the skewness level set model instead of the LBF model.

For a given image vector of $I : \Omega \rightarrow \mathbb{R}^d$, where $\Omega \subset \mathbb{R}^d$ is the image domain and $d \geq 1$ is dimension of the vector $I(x)$. Chunming Li et al. [15] proposed to minimize the following energy functional to find the object boundary

$$F_x(\Phi, f_1, f_2) = \epsilon_x^{LBF}(\Phi, f_1, f_2) + \mu \rho(\Phi) + \nu \ell(\Phi) \quad (1)$$

where $\epsilon_x^{LBF}$ is the LBF energy for each point $x \in \Omega$, $\Phi$ is the zero level set of a Lipschitz function $\Phi : \Omega \rightarrow R$, $f_1$ and $f_2$ are functions of the center point $x$ that minimize the LBF energy $\epsilon_x^{LBF}(\Phi, f_1, f_2)$. $\rho(\Phi)$ is a distance regularization term to penalize the deviation of the level set function $\Phi$ from a signed distance function, $\ell(\Phi)$ is the length of the zero level curve (surface) of $\Phi$, and finally $\mu$ and $\nu$ are nonnegative constants.

In the above LBF model, local information in the neighborhood of a center point $x$ is utilized to segment images with intensity inhomogeneity, especially for MR images. Obviously, such non statistical local fitting will not be useful for SAR images with nonlinear intensity inhomogeneity. Moreover, the segmentation process in the LBF model depends on the location of initial contour created by user. For example, if user locates the initial contour in the corners of the image, the LBF model will not converge correctly. So, we introduced the Skewness energy instead the LBF energy to use in level set function.

2.1 Higher Order Cumulants and Skewness Feature

Consider a random process $x(m)$ and let $h(m)$ represents the impulse response of a desired linear filter. In this paper, we propose that $h(m)$ is the impulse response of the equivalent wavelet coefficients filter in a given sub-band. Hence, we have

$$y \doteq W_x(m) = \sum_{k=0}^{N-1} h(k)x(m-k) = \sum_{k=0}^{N-1} y_k$$

where $y_k \doteq h(k)x(m-k)$.

The moment generating function is defined by

$$\Phi_X(\omega) = E_X \{ e^{j \omega x} \}$$

The cumulant generating function of a random variable $X$ is given by the logarithm of the moment generating function, as below

$$C_X(\omega) = \ln(\Phi_X(\omega)) = \ln \left[ E_X \{ e^{j \omega x} \} \right]$$

If we make a Taylor series expansion in the $\omega$ variable about the cumulant generating function, we can rewrite the Eq. (4) as follow

$$C_X(\omega) = \ln \left[ E_X \{ e^{j \omega x} \} \right] = \sum_{n=1}^{\infty} k_n (j\omega)^n n!$$

The coefficient of the Taylor series term $\omega^n$, multiplied by $(-j)^n$, is called the $n$-th order cumulant of the random variable $X$ and is defined by

$$c^{(n)}_X = (-j)^n \frac{d^n}{d\omega^n} C_X(\omega) \big|_{\omega=0} \quad \text{for } n = 1, 2, 3, \ldots$$

If we compute the first and the second order cumulants from (6) and rewrite them in terms of the moments, it can be found that the first order cumulant is the first order moment (or mean) and the second order cumulant is the covariance (or center of moment). These relations are defined by

$$\mu = c^{(1)}_x = m^{(1)}_x = E\{X\}$$

and

$$\text{cov.} = c^{(2)}_x = m^{(2)}_x - (m^{(1)}_x)^2 = E\{X^2\} - (E\{X\})^2$$

respectively.

We claim that whatever the order of cumulant as a feature for a SAR image increases, this feature will give the more statistical characteristics of a specific region from a
SAR image. However, implementation of higher order cumulants will impose more calculations and time consuming. So, we should perform a trade of between the higher order and the complexity of calculations. In this paper, we propose to use the 3rd-order cumulant named skewness as a texture feature for segmentation of SAR images.

Due to the relations that exist between moments and cumulants, the desired cumulant (the third) of the interested variables can be derived. The expression yielding the third order cumulant of a random variable \( Y \) from its moments is the following

\[
c^{(3)}_Y = E[(Y - \mu)^3] = m^{(3)}_Y - 3m^{(2)}_Y m^{(1)}_Y + 2(m^{(1)}_Y)^3
\]  

(9)

In the SAR image segmentation approach under development, a desirable feature is that it be invariant to image size. Unfortunately, the concept of cumulant formulation as represented in Eq. (6) is inherently dependent on the size of the random variable. In order to cope with this problem, we use a normalized cumulant. With this remembering, we consider the \( m \)-th order cumulant normalized with \( n \)-th order cumulant as in (10)

\[
k^{(m,n)}_X = \frac{c^{(m)}_X}{c^{(n)}_X^{m/n}}
\]  

(10)

In Eq. (10), \( k^{(m,n)}_X \) is the normalized cumulant of order \( (m, n) \) associated with random variable \( X \). In this equation, the denominator \( c^{(n)}_X \) is not zero, for this reason, we select \( n = 2 \) to provide a logical choice, since \( c^{(2)}_X \) is covariance and it is always nonzero for any random variable. If \( m = 3 \) and \( n = 2 \) and rewrite the Eq. (10) with these values of \( m \) and \( n \), the skewness proposed in this paper is achieved as below

\[
k^{(3,2)}_X = \frac{c^{(3)}_X}{c^{(2)}_X^{3/2}}
\]  

(11)

In Eq. (11), skewness is the third order cumulant normalized with covariance. The skewness cumulant indicates the value of symmetry of the PDF histogram of a SAR image.

The simple equation of the skewness of a random variable \( X \) is as follow

\[
\text{skewness}(X) = \frac{E[(X - \mu)^3]}{\sigma^3} = \frac{c^{(3)}_X}{(\text{var})^{3/2}}
\]  

(12)

where, \( \mu \) is the mean of \( X \), \( \sigma \) var is the variance \( \sigma^2 \) of \( X \) that it is a nonzero parameter, \( \text{var} > 0 \), and \( E[t] \) represents the expected value of the quantity \( t \).

With a view to statistical calculations, skewness is a measure of the asymmetry of the data around the sample mean. If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or any perfectly symmetric distribution) is zero.

2.2 Level Set Definition for Segmentation

Suppose that \( r \) be an open subset of \( R^2 \), \( K \) be the number of segmented regions (number of \( r_k \)), \( k \) be a parameter that shows the region index, \( p \) be a point of region, \( R_k \) be the interface between region \( r_k \) and region \( r_l \) and the image be a function considered as \( I : r \rightarrow R \). We define the region \( R_k = \{ p \in r | p \text{ belongs to the region } k \} \). We denote that for all \( k = 1, \ldots, K \), \( R_k \) is an open set \( r_k \) given by a Lipschitz function \( \Phi_k : r \rightarrow R \) such that

\[
\begin{align*}
\Phi_k(p) & > 0 \quad \text{if} \quad p \in r_k \\
\Phi_k(p) & = 0 \quad \text{if} \quad p \in R_k \\
\Phi_k(p) & < 0 \quad \text{otherwise}
\end{align*}
\]  

(13)

where \( R_k \) is the boundary of \( r_k \) and \( \Phi_k \) is the signed distance function to \( R_k \). We can determine \( r_k \) using the sign of \( \Phi_k \) and the Heaviside distribution function \( H \) is approximated by

\[
H_\alpha(\beta) = \begin{cases} 
\frac{1}{\alpha} & \beta < \alpha \\
1 & \beta = \alpha \\
0 & \beta > \alpha
\end{cases}
\]  

(14)

In the distributional sense, when \( \alpha \to 0 \), we have \( H_\alpha \to H \). \( \Phi_k(p) \) is introduced as level set function. If \( \Phi_k(p) \) is calculated for any point \( p \) and get the sign of \( \Phi_k(p) \), we can determine that if \( p \) is in the region \( r_k \) or not. For example, if \( \Phi_k(p) > 0 \) then \( H(\Phi_k(p)) = 1 \), so \( p \in r_k \).

2.3 Skewness Level Set Functional Formulation

Let \( p \in r \) be an arbitrary point, and \( I(p) : r \rightarrow R^1 \) be a given vector of SAR image, where \( I \) is the dimension of the vector \( I(p) \). Dimension of SAR images corresponds to the dimension of gray level images. We define the following functional

\[
e_{\Phi_k}^{SWE}(\Phi_1, \Phi_2, \ldots, \Phi_K, f_1, f_2) = e_p^{SWE}(\Phi_1, \Phi_2, \ldots, \Phi_K, f_1, f_2) + \mu \rho(\Phi)
\]  

(15)

where \( \mu \) is a positive constant, \( e_p^{SWE} \) is the skewness wavelet energy, \( f_1 \) and \( f_2 \) are the functions that minimize the \( e_p^{SWE} \), and \( \rho(\Phi) \) is the deviation of the level set function \( \Phi \) from a signed distance function as below

\[
\rho(\Phi) = \int r \frac{1}{2} (|\nabla \Phi(p)| - 1)^2 dp
\]  

(16)

We propose the skewness of the wavelet coefficients energy to be used as contour energy \( e_p^{SWE} \) in (15). For this purpose, the parameters of \( e_p^{SWE} \) should be computed. In the next section, we explain the manner of calculations.

3. Skewness Wavelet Coefficients Energy Formulation

The high order cumulants are able to characterize texture information of an image. Thus, a texture of each region in
SAR images can be explained with the skewness parameters. On the other hand, it can be shown that a texture of each region in a SAR image is characterized by its wavelet coefficients energy. We are now in position to characterize regions of a SAR image through their wavelet decomposition.

3.1 Wavelet Representation for Texture Discrimination of Regions

Let \( R_0(x, y) \) be the function which illustrates the texture of each region. We denote

\[
R_0(x, y) = \frac{1}{\sqrt{MN}} \sum_{m} \sum_{n} W_{\psi}(j_0, m, n)\varphi_{j_0, m, n}(x, y) + \frac{1}{\sqrt{MN}} \sum_{i=H, V, D} \sum_{j=j_0}^{-1} \sum_{m} \sum_{n} W_{\psi}(j, m, n)\varphi_{j, m, n}(x, y)
\]

(17)

where \( \psi \) is the wavelet function, \( \varphi \) the scaling function and \( j_0 \) the order of the decomposition. So, we propose that the texture of each region can be presented by the sequence as below

\[
\left( \left[ |W_{\psi}(j_0, m, n)|^2, m, n \in \mathbb{Z} \right], \\
\left[ |W_{\psi}(j, m, n)|^2, m, n \in \mathbb{Z}, -j_0 \leq j \leq -1 \right] \right)
\]

(18)

It can be shown that the distribution of the modulus of the wavelet coefficients in a sub-band of any image can be modeled with the following family of histograms [23]

\[
h(W_{\psi}) = K \exp \left( -\left( \frac{|W_{\psi}(j_0, m, n)|}{\alpha} \right)^{\beta} \right)
\]

(19)

where \( \beta \) is the parameter that modifies the decreasing rate of the peak and \( \alpha \) models the variance. Note that \( h(W_{\psi}) \) is the probability density function of a sub-band of each region, so we have

\[
\int_{-\infty}^{\infty} h(W_{\psi})dW_{\psi} = \sum_{m} \sum_{n} W_{\psi}(j_0, m, n) = N
\]

(20)

where \( N \) is the total number of pixels of the given detail image.

It can be experimentally shown [24] that the distribution of the energy of the wavelet packet coefficients in a sub-band of any image follows a picky generalized Gaussian law of the form

\[
p_{\chi^2}(\chi) = \frac{K}{2\sqrt{\pi}} \exp \left( -\left( \frac{\sqrt{\chi}}{\alpha} \right)^{\beta} \right) I_{\beta-1} \left( \frac{2\sqrt{\chi}}{\beta} \right)
\]

(21)

where, \( \alpha \) and \( \beta \) is the segment parameters. One idea is to compute \( \alpha \) and \( \beta \) from the first and second order moments of the energy distribution in each sub-band. Such definition is not good for our SAR image segmentation problem due to the instability that exists in the curve of moments.

Let us compute the parameters \( K, \alpha \) and \( \beta \) using the first and second order moments and considering the total number of pixels of the given detail image \( N \).

We can compute the parameter \( K \) by define the Gamma function \( \Gamma(t) \) on \( R^+ \) in (22)

\[
\Gamma(t) = \int_{0}^{\infty} e^{-u} u^{t-1} du
\]

(22)

By using the Eqs. (19), (20) and (22), and making the change of variable \( \left( \frac{W_{\psi}}{\alpha} \right)^{\beta} = u \), we get

\[
\int_{0}^{+\infty} h(W_{\psi})dW_{\psi} = \int_{0}^{+\infty} K e^{-\left( \frac{W_{\psi}}{\alpha} \right)^{\beta}} dW_{\psi}
\]

\[
= \int_{0}^{+\infty} K e^{-\left( \frac{W_{\psi}}{\alpha} \right)^{\beta}} \frac{1}{\beta} du
\]

\[
= \frac{K \alpha}{\beta} \int_{0}^{+\infty} e^{-\left( \frac{W_{\psi}}{\alpha} \right)^{\beta}} \frac{1}{\beta} du
\]

\[
= \frac{K \alpha}{\beta} \Gamma \left( \frac{1}{\beta} \right) = N
\]

(23)

So we have

\[
K = N \beta \left( \frac{1}{\beta} \right)
\]

(24)

We denote the energy of each wavelet coefficient in a sub-band with the square of that wavelet coefficient \( |W_{\psi}|^2 \). Since \( h(W_{\psi}) \) in (19) is the probability density function of a sub-band of each region thus the cumulative distribution function of \( |W_{\psi}|^2 \) can be computed as follow

\[
F_{|W_{\psi}|^2}(x) = P \left( |W_{\psi}|^2 \leq x \right)
\]

\[
= P \left( W_{\psi} \leq \sqrt{x} \right) = \int_{0}^{\sqrt{x}} p_{|W_{\psi}|} \left( |W_{\psi}| \right) d \left( |W_{\psi}| \right)
\]

\[
= \sqrt{x} \int_{0}^{\sqrt{x}} h \left( |W_{\psi}| \right) d \left( |W_{\psi}| \right)
\]

\[
= K \sqrt{x} \exp \left( -\left( \frac{|W_{\psi}(j_0, m, n)|}{\alpha} \right)^{\beta} \right) d \left( |W_{\psi}| \right)
\]

(25)

where \( x \) is a random variable that represents the square of the wavelet coefficient in each sub-band.

We make the change of variable \( |W_{\psi}| = \sqrt{x} \), so we have (for \( x \geq 0 \))

\[
F_{|W_{\psi}|^2}(x) = \frac{K}{2} \int_{0}^{\sqrt{x}} \exp \left( -\left( \frac{\sqrt{\chi}}{\alpha} \right)^{\beta} \right) d x
\]

(26)
$F_{|W_{j}|^2}(x)$ is the distribution of the energy of the wavelet coefficients in a sub-band. Hence, we have the probability density function as below

$$f_{|W_{j}|^2}(x) = \frac{dF_{|W_{j}|^2}(x)}{dx} = \frac{K}{2x} \exp \left( - \left( \frac{x}{\alpha} \right)^{\beta} \right)$$

(27)

where $x$ is the square of the wavelet coefficient in each subband ($x = |W_{j}|^2$).

From (7), and making the change of variable $(\frac{W_{j}}{\alpha})^\beta = u$, we have

$$m_{|W_{j}|^2}^{(1)} = E[|W_{j}|^2] = \int_0^{+\infty} (W_{j})^2 h(W_{j}) d(W_{j})$$

$$= \int_0^{+\infty} (W_{j})^2 \cdot K \cdot \exp(- \left( \frac{W_{j}}{\alpha} \right)^{\beta}) d(W_{j})$$

$$= \int_0^{+\infty} \alpha^2 \cdot u^{1/2} \cdot Ke^{-u} \cdot \frac{\alpha}{\beta} \cdot u^{(1-1)/2} du$$

$$= \frac{K \alpha^3}{\beta} \int_0^{+\infty} e^{-u} \cdot u^{1/2} du$$

$$= \frac{K \alpha^3}{\beta} \cdot \Gamma \left( \frac{3}{\beta} \right)$$

(28)

In the same way, the second order moment of the energy distribution $m_{|W_{j}|^2}^{(2)}$ can be written as

$$m_{|W_{j}|^2}^{(2)} = E[|W_{j}|^4] = \int_0^{+\infty} (W_{j})^4 h(W_{j}) d(W_{j})$$

$$= \frac{K \alpha^5}{\beta} \cdot \Gamma \left( \frac{5}{\beta} \right)$$

(29)

Dividing (29) by (28), we obtain

$$\alpha = \sqrt{\frac{\Gamma \left( \frac{3}{\beta} \right) m_{|W_{j}|^2}^{(2)}}{\Gamma \left( \frac{5}{\beta} \right) m_{|W_{j}|^2}^{(1)}}}$$

(30)

As well as by dividing squared (28) by (29), we get

$$\frac{(m_{|W_{j}|^2}^{(1)})^2}{m_{|W_{j}|^2}^{(2)}} = \frac{N \cdot \Gamma^2 \left( \frac{3}{\beta} \right)}{\Gamma \left( \frac{1}{\beta} \right) \Gamma \left( \frac{5}{\beta} \right)}$$

(31)

We assume that $F(x)$ is a function defined by

$$F(x) = \frac{\Gamma \left( \frac{3}{\beta} \right)}{\Gamma \left( \frac{1}{\beta} \right) \Gamma \left( \frac{5}{\beta} \right)}$$

(32)

Thus

$$\beta = F^{-1} \left( \frac{(m_{|W_{j}|^2}^{(1)})^2}{N \cdot m_{|W_{j}|^2}^{(2)}} \right)$$

(33)

The function $F^{-1}(x)$ is depicted in Fig. 1. We can obtain the value of the parameter $\beta$ from this plot.

3.2 Computation of the Segmentation Parameters of the Skewness Wavelet Energy Distribution

We intend to segment the regions of the SAR images by using the property of nonlinear intensity inhomogeneity. The values of the parameters $\alpha$ and $\beta$ derived in previous section, are not useful for solving our segmentation problem. Hence, we propose to use the higher order cumulant to calculate the values of the segment parameters. We use the skewness defined in (12) to get the formulation for the parameters $\alpha$ and $\beta$. Let us compute the third order moment of the wavelet coefficients energy distribution $m_{|W_{j}|^2}^{(3)}$ as follow

$$m_{|W_{j}|^2}^{(3)} = E[|W_{j}|^6] = \int_0^{+\infty} (W_{j})^6 h(W_{j}) d(W_{j})$$

$$= \frac{K \alpha^7}{\beta} \cdot \Gamma \left( \frac{7}{\beta} \right)$$

(34)

Dividing (34) by (29), we get

$$\alpha = \sqrt{\frac{\Gamma \left( \frac{3}{\beta} \right) m_{|W_{j}|^2}^{(3)}}{\Gamma \left( \frac{5}{\beta} \right) m_{|W_{j}|^2}^{(2)}}}$$

(35)

From (9) and (12), we obtain

$$skewness = \frac{C_{|W_{j}|^2}^{(3)}}{(m_{|W_{j}|^2}^{(2)})^{3/2}}$$

Fig. 1 Plot of $F^{-1}(x)$. 

\[
\epsilon_p^{\text{SWE}}(C, k_1(p), k_2(p)) = \gamma_1 \int_{\text{inside contour(C)}} K(p-q) |I(q) - k_1(p)|^2 dq + \gamma_2 \int_{\text{outside contour(C)}} K(p-q) |I(q) - k_2(p)|^2 dq \tag{39}
\]

where \(C\) is a contour in the image region \(r\), \(\gamma_1\) and \(\gamma_2\) are two persistent numbers, \(K\) is a kernel function with a skewness property so that \(K(w)\) decreases when \(|w|\) increases, and \(k_1(p)\) and \(k_2(p)\) are two functions that fit image intensities near the point \(p\). We call the point \(p\) the center point of the above equation, and the above energy the skewness wavelet energy (SWE) around the center point \(p\).

We propose to select the kernel function \(K(w)\) as the probability density function \(f_{W_i,j}(p)\) defined by (27). So, we have

\[
K_{W_i,j}(p) = \frac{K}{2\sqrt{\pi}} \exp\left(-\frac{\sqrt{\pi}}{\alpha}\right) \tag{40}
\]

where the constant parameter \(K\) is obtained by (24) and the segment parameters \(\alpha\) and \(\beta\) are given by (35) and (38) respectively.

The values of the functions \(k_1(p)\) and \(k_2(p)\) that minimize the SWE energy \(\epsilon_p^{\text{SWE}}(C, k_1, k_2)\) are functions of the center point \(p\) of the kernel function \(K(p-q)\). Thus, we claim that SWE energy \(\epsilon_p^{\text{SWE}}\) has the localization property similar to the local binary fitting model. To find the boundary of regions in SAR images, we should minimize the integral of \(\epsilon_p^{\text{SWE}}\) over all center points \(p\) in the all regions of a SAR image. So, we define the energy functional \(\epsilon(C, k_1, k_2)\) as bellow:

\[
\epsilon(C, k_1(p), k_2(p)) = \int_r \epsilon_p^{\text{SWE}}(C, k_1(p), k_2(p)) dp \tag{41}
\]

If we represent the contour \(C \subset r\) with the level set of a Lipschitz function \(\Phi : r \rightarrow R\), we get

\[
\epsilon^{\text{SWE}}(\Phi, k_1, k_2) = \int_r \epsilon_p^{\text{SWE}}(\Phi, k_1(p), k_2(p)) dp \tag{42}
\]

where \(H\) is the Heaviside function defined by (14).

Now, we define the new entire skewness wavelet energy functional \(F^{\text{SWE}}\) as

\[
F^{\text{SWE}}(\Phi, k_1, k_2) = \epsilon^{\text{SWE}}(\Phi, k_1, k_2) + \mu \Phi(\Phi) + \nu \ell(\Phi) \tag{43}
\]

where \(\mu\) and \(\nu\) are nonnegative constants, \(\Phi(\Phi)\) is given by (16) and \(\ell(\Phi)\) is given by

\[
\ell(x) = \frac{\alpha}{\beta} \sqrt{\frac{\beta}{\alpha}} - x
\]

For each point \(p \in r\), the energy is written in Eq. (39),

\[
\epsilon_p^{\text{SWE}}(C, k_1(p), k_2(p)) = \gamma_1 \int_{\text{inside contour(C)}} K(p-q)|I(q) - k_1(p)|^2 dq + \gamma_2 \int_{\text{outside contour(C)}} K(p-q)|I(q) - k_2(p)|^2 dq
\]

where \(C\) is a contour in the image region \(r\), \(\gamma_1\) and \(\gamma_2\) are two persistent numbers, \(K\) is a kernel function with a skewness property so that \(K(w)\) decreases when \(|w|\) increases, and \(k_1(p)\) and \(k_2(p)\) are two functions that fit image intensities near the point \(p\). We call the point \(p\) the center point of the above equation, and the above energy the skewness wavelet energy (SWE) around the center point \(p\).

We propose to select the kernel function \(K(w)\) as the probability density function \(f_{W_i,j}(p)\) defined by (27). So, we have

\[
K_{W_i,j}(p) = \frac{K}{2\sqrt{\pi}} \exp\left(-\frac{\sqrt{\pi}}{\alpha}\right) \tag{40}
\]

where the constant parameter \(K\) is obtained by (24) and the segment parameters \(\alpha\) and \(\beta\) are given by (35) and (38) respectively.

The values of the functions \(k_1(p)\) and \(k_2(p)\) that minimize the SWE energy \(\epsilon_p^{\text{SWE}}(C, k_1, k_2)\) are functions of the center point \(p\) of the kernel function \(K(p-q)\). Thus, we claim that SWE energy \(\epsilon_p^{\text{SWE}}\) has the localization property similar to the local binary fitting model. To find the boundary of regions in SAR images, we should minimize the integral of \(\epsilon_p^{\text{SWE}}\) over all center points \(p\) in the all regions of a SAR image. So, we define the energy functional \(\epsilon(C, k_1, k_2)\) as bellow:

\[
\epsilon(C, k_1(p), k_2(p)) = \int_r \epsilon_p^{\text{SWE}}(C, k_1(p), k_2(p)) dp \tag{41}
\]

If we represent the contour \(C \subset r\) with the level set of a Lipschitz function \(\Phi : r \rightarrow R\), we get

\[
\epsilon^{\text{SWE}}(\Phi, k_1, k_2) = \int_r \epsilon_p^{\text{SWE}}(\Phi, k_1(p), k_2(p)) dp \tag{42}
\]

where \(H\) is the Heaviside function defined by (14).

Now, we define the new entire skewness wavelet energy functional \(F^{\text{SWE}}\) as

\[
F^{\text{SWE}}(\Phi, k_1, k_2) = \epsilon^{\text{SWE}}(\Phi, k_1, k_2) + \mu \Phi(\Phi) + \nu \ell(\Phi) \tag{43}
\]

where \(\mu\) and \(\nu\) are nonnegative constants, \(\Phi(\Phi)\) is given by (16) and \(\ell(\Phi)\) is given by
\[ \ell(\Phi) = \int_{\Omega} \delta(\Phi(p))(|\nabla \Phi(p)| - 1)^2 dp \] (44)

Keeping \( k_1(p) \) and \( k_2(p) \) fixed, and minimizing the \( F_{SWE}(\Phi, k_1, k_2) \), the active contour model proposed in this paper, is derived by

\[ \frac{\partial \Phi}{\partial t} = -\delta_0(\Phi)(\gamma_1 e_1 - \gamma_2 e_2) + \nu \delta_p(\Phi) \text{div} \left( \frac{\nabla \Phi}{|\nabla \Phi|} \right) + \mu \left( \nabla^2 \Phi - \text{div} \left( \frac{\nabla \Phi}{|\nabla \Phi|} \right) \right) \] (45)

where, the functions \( e_1 \) and \( e_2 \) are computed by

\[ e_1(p) = \int r K_{W_\gamma}(q-p) |I(p) - k_1(q)|^2 dq \] (46)

and

\[ e_2(p) = \int r K_{W_\gamma}(q-p) |I(p) - k_2(q)|^2 dq \] (47)

also, \( k_1(q) \) and \( k_2(q) \) are given by

\[ k_1(q) = \frac{K_{W_\gamma}(q) * H_q(\Phi(q))}{K_{W_\gamma}(q) * H_q(\Phi(q))} \] (48)

\[ k_2(q) = \frac{K_{W_\gamma}(q) * (1 - H_q(\Phi(q)))I(q)}{K_{W_\gamma}(q) * 1 - H_q(\Phi(q))} \] (49)

4. Implementation and Results

In this section, the implemented algorithms for both agricultural and urban SAR images have been evaluated. In our experiments, the same parameters have been chosen for all images, which they are listed here: \( \gamma_1 = 1.0, \gamma_2 = 1.0, \nu = 0.004 \times 255^2, c_0 = 2 \) (constant value of step function used as initial contour), time step \( \tau = 0.1, \mu = 1.0, \) and center point \( p = 1.0. \)

To calculate the functions \( k_1(q) \) and \( k_2(q) \) given by Eq. (48) and (49) respectively, we should compute the kernel function \( K_{W_\gamma}(p) \) in (40). For this reason, we first computed the value of \( m_{W_\gamma}^{(1)} \), \( m_{W_\gamma}^{(2)} \), and \( m_{W_\gamma}^{(3)} \) accordingly.

The wavelet decompositions of the maximum level \( L \) have been used to compute the first, second and third moments of the energy distribution of the wavelet coefficients, then we calculate the skewness of these coefficients. We have tested several kinds of mother wavelets and several wavelet coefficients \( W^A, W^E, W^V \) and \( W^D \). We have selected to use the Haar mother wavelet as well.

The Wavelet coefficients are measured along the different directions as written here: \( W^H \) along columns (horizontal direction), \( W^V \) along rows (vertical direction), and \( W^D \) along diagonals. \( W^H, W^V \) and \( W^D \) are detail coefficients and \( W^E \) contains approximation coefficients.

The value of \( \beta \) can be calculated from the curve of \( F^{-1} \) represented in Fig. 2, and then the value of \( \alpha \) can be obtained from Eq. (35).

Figure 3(a) shows a NASA/JPL AIRSAR 4-look image of an airport in Ontario whose size is 512 \times 512 (http://airsar.jpl.nasa.gov/index_detail.html). In Fig. 3(b), we show the approximation wavelet coefficients energy \( |W_\gamma|^2 \) from level 1 to 9 of the “airport” 4-look SAR image.

The value of the skewness of the approximation wavelet coefficients energy represented in Fig. 3(b) is 202.98. It is obtained by Eq. (36). Now, we can get \( \beta = 0.3 \) from Fig. 4. Figure 4 is the extended curve of Fig. 2 in the range \([0,700]\) of the skewness axis in detail.

Also, we calculated \( \alpha = 1.22 \) and \( K = 2.31 \times 10^4 \) from Eq. (35) and (24), respectively. Note that the skewness parameters, \( \beta, \alpha, \) and \( K \) are different for each SAR image.

The segmentation results of the proposed method with the LBF model for two SAR images have been compared in Fig. 5. The SAR image, as shown in Fig. 5(a), is a NASA Goddard Space Flight Center image with 15 m resolution of Washington, D.C., acquired by LANDSAT 7 on May 9, 2005 (DC falls on LANDSAT WRS-2 Path 15 Row 33). This image consists of three types of land cover: water, building, and vegetation. Figure 5(c) shows the initial contour of Fig. 5(a) which is selected for both methods. The segmentation obtained by the LBF is shown in Fig. 5(d). One can see that the water area (lower left) is incorrectly segmented. Furthermore, the boundary between the vege-

![Fig. 3](image1.png)

(a) Original “airport” SAR image (4-look) (b) Representation of the approximation wavelet coefficients energy \( |W_\gamma|^2 \) from level 1 to 9 of the “airport” SAR image.

![Fig. 4](image2.png)

The curve of \( \beta \) in terms of skewness of Fig. 3(b).
Fig. 5  Segmentation of two SAR images and comparison between the segmentation results of LBF model and our method (a) C/X- SAR image of Washington, DC, obtained from NASA/GSFC (481 × 483). (b) C-SAR image of a rice growing area near Okayama, Japan, obtained by JPL AirSAR. (c) [(f)] Initial active contour for (a) [(b)]. (d) [(g)] Segmentation of (a) [(b)] obtained by LBF. (e) [(h)] Segmentation of (a) [(b)] by our method. (i) [(respectively, (j), (k) and (l)] zoom of an area extracted from (d) [(respectively, (e), (g) and (h))].

tation [region B in Fig. 5 (i)] and the building [region C in Fig. 5 (i)] is not correctly defined. The segmentation obtained by SWE, as shown in Fig. 5 (e), improves the uniformity in the water region, and a local region of the vegetation [part of region B located in region C in Fig. 5 (j)] is consistently identified as region B. Another experiment is carried out on a multilook C-band SAR image of a rice growing area near Okayama, Japan obtained by JPL AirSAR, as shown in Fig. 5 (b). This image is already multilooked 9 times in azimuth to give a pixel spacing of approximately 4.6 m in azimuth and 3.3 m in range and a 1000 × 1400 pixel area of approximately 4.5 km square was used to examine
Table 1  The processing time (in seconds) for our proposed model and LBF model for 5 SAR images. The sizes of images 1 to 5 are 154 × 154, 481 × 483, 268 × 250, 122 × 159, and 225 × 225 pixels, respectively. The iteration number is 300 for all images.

<table>
<thead>
<tr>
<th>Model</th>
<th>Image1</th>
<th>Image2</th>
<th>Image3</th>
<th>Image4</th>
<th>Image5</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBF model</td>
<td>36.14</td>
<td>249.39</td>
<td>76.16</td>
<td>34.11</td>
<td>42.43</td>
</tr>
<tr>
<td>Our model</td>
<td>0.75</td>
<td>4.43</td>
<td>1.79</td>
<td>1.01</td>
<td>1.09</td>
</tr>
</tbody>
</table>

5. Conclusion

We have proposed a new region-based active contour model that uses the wavelet coefficients energy of each region and the skewness characteristics of these coefficients. A new energy named skewness wavelet energy is proposed to use in active contour and level set formulation. This energy is able to draw upon local statistical intensity information of each region, and therefore, it can address the nonlinear intensity inhomogeneity which exists in SAR images. Experimental results show that the proposed method is more efficient and accurate. Our algorithm is about 40 times faster than the LBF method. Experimental results demonstrate the advantages of our model for accurate segmentation of several SAR images.

References

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