LETTER

Hellinger Distance-Based Parameter Tuning for ε -Filter

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SUMMARY In this letter, we propose a new tuning method of ε value, which is a parameter in the ε -filter, using a metric between signal distributions, i.e., Hellinger distance. The difference between the input and output signals is evaluated using Hellinger distance and used for the parameter tuning in the proposed method.

key words: ε -filter, parameter tuning, Hellinger distance

1. Introduction

The ε -filter is a nonlinear smoothing filter proposed by Harashima et al. [1] to effectively remove the superimposed small amplitude noise. ε -filter has been widely applied to not only speech signals but also image signals in many practical scenes [2]–[5]. In the use of ε -filter, to obtain a fine result, a smoothing parameter ε should be appropriately adjusted for each input signal. However, the adjustment has been done empirically so far. In order to determine the ε value efficiently, a tuning method must be developed for it.

To realize efficient parameter tuning, a method based on the noncorrelation property between a true signal and noise has been proposed by Matsumoto et al. [6], [7]. In the method, the parameter tuning is achieved using the noncorrelation property between a restored image signal and a residual signal—a difference between an input and the restored image signals. In the method, however, there are some cases where the noncorrelation property is not effective to determine ε . The performance of the method heavily depends on the type of images handled.

To cope with the problem, we propose a new parameter tuning method, which is based on Hellinger distance (HD) [8] between distributions of the residual signal estimated as noise and the noise assumed beforehand such as Gaussian noise. Concretely, the tuning is carried out by finding the ε value giving the minimum HD between the distributions.

The rest of this letter is organized as follows: Section 2 gives a brief overview of the ε -filter. Details of the proposed method are explained in Sect. 3. Finally, experimental re-

sults for various images and conclusions are described in Sects. 4 and 5, respectively.

2. ε -Filter

The ε -filter [1] in image filtering is explained. An ε -filter output I_ε at a pixel (x, y) is obtained as follows:

$$I_\varepsilon(x, y) = I(x, y) - \sum_{i=-r}^r \sum_{j=-r}^r [a(i, j) \times F(I(x, y) - I(x+i, y+j))], \quad (1)$$

where I is an input image corrupted by noise. r is a positive integer standing for a window size. The following function and a filter coefficient are typical examples of F and a , and used in this letter:

$$F(z) = \begin{cases} z & -\varepsilon \leq z \leq \varepsilon \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

and

$$a(i, j) = \frac{1}{(2r+1)^2}. \quad (3)$$

The filter coefficient shown in Eq. (3) gives the effect of an average filter to ε -filter. ε -filter becomes an average filter when ε is set to an enough large value (e.g., 255).

3. Proposed Method

In the proposed method, the smoothing parameter ε of ε -filter is adjusted based on similarity of distributions between a noise model beforehand assumed and the residual signal which is obtained as a difference between the input signal and restored one. HD is employed in the similarity calculation. When the similarity is large, a fine noise removal is carried out. Figure 1 shows an overview of the proposed method and details of it are described in Sect. 3.1.

In the proposed method, an assumption concerning the distribution of superimposed noise is needed and its parameter should be estimated from a noisy input signal. In this letter, the additive Gaussian noise is assumed as the superimposed noise and its parameter, i.e., the standard deviation is estimated. The estimation algorithm employed in the proposed method is explained in Sect. 3.2.

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Fig. 1 Overview of the proposed method.

3.1 Hellinger Distance-Based Parameter Tuning

In the proposed method, the residual image I_{res} —a difference between an input image I and a restored image I_{ϵ} obtained by ϵ -filter—is obtained as follows:

$$I_{\text{res}}(x, y) = I(x, y) - I_{\epsilon}(x, y). \quad (4)$$

Then, HD between the assumed noise distribution H_{ass} and the distribution H_{res} , which is a normalized histogram of I_{res} , is calculated. Concretely, HD is calculated as follows [8]:

$$\text{HD} = \left\{ 1 - \sum_{i=-255}^{255} [H_{\text{ass}}(i)H_{\text{res}}(i)]^{1/2} \right\}^{1/2}. \quad (5)$$

When HD is close to 0, the two distributions are similar each other. The optimal value ϵ^* in the proposed adjustment is obtained as follows:

$$\epsilon^* = \arg \min_{\epsilon} \text{HD}^2(\epsilon). \quad (6)$$

3.2 Estimation of Noise Standard Deviation

In this letter, the Gaussian noise is assumed as the noise superimposed on an input image, i.e., H_{ass} is the normal distribution $N(0, \sigma^2)$. σ is estimated by the standardized median of absolute deviation from the median (MAD) [9]. MAD is calculated in each block cropped as a square with $k \times k$ pixels on an input image. MAD in the i th block $\hat{\sigma}_i$ is calculated as follows:

$$\hat{\sigma}_i = 1.483 \text{ med} \left\{ \left| I_{i,j} - \text{med} \{ I_{i,j} : 1 \leq j \leq k^2 \} \right| : 1 \leq j \leq k^2 \right\}, \quad (7)$$

where “med” is a function to obtain a median and $I_{i,j}$ is j th pixel value in the i th block. In this letter, the estimated standard deviation of the Gaussian noise $\hat{\sigma}_{\text{est}}$ is obtained as follows:

$$\hat{\sigma}_{\text{est}} = \min_i \hat{\sigma}_i. \quad (8)$$

Fig. 2 Images employed in experiments: (a) Lenna, (b) Barbara, (c) Airplane, (d) Boat, (e) Bridge, (f) Cameraman, (g) Girl, (h) Lax, (i) Woman.

4. Experimental Results

The effectiveness and validity of the proposed method are illustrated through experiments in which various digital images are employed. Images employed here are shown in Fig. 2. Each image constitutes of 256×256 pixels. In experiments, the Gaussian noises whose standard deviations σ s are 10, 20, and 30, are employed as the superimposed noises on images. The method proposed by Matsumoto and Hashimoto [6] is used as a method for comparison.

In the setup of ϵ -filter used both in the proposed and Matsumoto's method, Eqs. (2) and (3) are used as F and a filter coefficient, respectively. Further, r and k , which are the ϵ -filter window size and the block size in σ estimation, are empirically set as 2 and 30, respectively.

A mean square error (MSE) is used as an index for the quantitative evaluation. MSE is calculated from the original noise-free image I_{ori} and the restored image I_{ϵ} . MSE becomes small when the restored image is similar to the original one.

4.1 Relationships between ϵ and Metrics

The relationships between ϵ and MSE are shown in Fig. 3 when Lenna (Fig. 2 (a)) and Barbara (Fig. 2 (b)) are employed. In Fig. 3, the ϵ giving the minimum MSE is the optimal value ϵ_{opt} . Concerning Lenna and Barbara, from Fig. 3, ϵ_{opt} s are around 40, 70, and 100 in the cases σ s are 10, 20, and 30, respectively.

The relationships between ϵ and a correlation coefficient R calculated between I_{ϵ} and I_{res} are shown in Fig. 4. In

Fig. 3 ε vs. MSE: (a) Lenna, (b) Barbara.

Fig. 4 ε vs. R : (a) Lenna, (b) Barbara.

Fig. 5 ε vs. HD: (a) Lenna, (b) Barbara.

Matsumoto's method, it is assumed that I_{ori} is uncorrelated with the superimposed noise, and the ε giving the minimum R is regarded as the optimal value. However, R does not always reflect MSE well as seen in the results for Lenna when σ s are 20 and 30 and Barbara with all σ s. It suggests that there are some cases Matsumoto's method does not work well in the parameter adjustment.

The relationships between ε and HD are shown in Fig. 5. As shown in Figs. 3 and 5, ε^* s obtained based on HD are sufficiently similar to ε_{opt} s although ε^* s are little bit different from ε_{opt} s. The HD-based parameter tuning is highly promising in comparison with Matsumoto's method.

Additionally, we attempted to use the Kullback-Leibler divergence (KLD) [10], [11] instead of HD measuring distance between signal distributions. ε^* s obtained by using KLD are equivalent with the case of HD and calculation cost of KLD is also equivalent with that of HD. There was no significant difference between using HD and KLD. In the following experiments, we employ the HD as a metric.

4.2 Performance of Each Method

Figure 6 shows resulting images for Lenna. Figures 6(a) and 6(b) show the input image corrupted by the Gaussian noise with $\sigma = 10$ and the optimal result, i.e., the re-

Fig. 6 Experimental results for Lenna (partial image): (a) noisy input image ($\sigma = 10$), (b) optimal result ($\varepsilon_{opt} = 38$), (c) Matsumoto's method ($\varepsilon = 38$), (d) proposed method ($\varepsilon^* = 48$).

Fig. 7 Experimental results for Barbara (partial image): (a) noisy input image ($\sigma = 10$), (b) optimal result ($\varepsilon_{opt} = 33$), (c) Matsumoto's method ($\varepsilon = 199$), (d) proposed method ($\varepsilon^* = 63$).

stored image obtained by ε -filter whose parameter ε is tuned based on MSE, respectively. Figures 6(c) and 6(d) are filtering results in the cases the ε value tuned by the Matsumoto's method and the proposed method, respectively. Figure 7 shows resulting images for Barbara in same manner to Fig. 6. Moreover, experimental results for various images and σ s are shown in Tables 1–3.

The ε -filter can yield fine restored images when opti-

Table 1 Experimental results with $\sigma = 10$. “Opt.,” “Com.,” and “Pro.” denote optimal result, comparison (Matsumoto’s method), and the proposed method, respectively.

	ε			MSE		
	Opt.	Com.	Pro.	Opt.	Com.	Pro.
Airplane	36	36	43	38	38	40
Barbara	33	199	63	61	468	122
Boat	36	38	42	39	39	41
Bridge	30	217	71	77	495	194
Cameraman	34	47	42	36	43	39
Girl	37	53	46	35	42	38
Lax	32	232	50	64	479	87
Lenna	38	38	48	37	37	41
Woman	37	220	42	39	162	41

Table 2 Experimental results with $\sigma = 20$. “Opt.,” “Com.,” and “Pro.” denote optimal result, comparison (Matsumoto’s method), and the proposed method, respectively.

	ε			MSE		
	Opt.	Com.	Pro.	Opt.	Com.	Pro.
Airplane	74	250	82	107	282	111
Barbara	66	237	96	181	480	249
Boat	74	234	81	99	171	101
Bridge	63	251	102	214	510	314
Cameraman	73	255	80	99	343	100
Girl	82	134	96	72	89	76
Lax	65	253	85	177	492	206
Lenna	78	255	82	92	180	93
Woman	76	250	88	97	176	103

Table 3 Experimental results with $\sigma = 30$. “Opt.,” “Com.,” and “Pro.” denote optimal result, comparison (Matsumoto’s method), and the proposed method, respectively.

	ε			MSE		
	Opt.	Com.	Pro.	Opt.	Com.	Pro.
Airplane	109	255	129	184	303	199
Barbara	101	255	156	307	501	426
Boat	120	254	130	159	193	162
Bridge	98	255	140	347	534	426
Cameraman	110	255	71	181	372	294
Girl	125	255	80	110	126	190
Lax	99	255	130	293	513	342
Lenna	117	255	145	148	202	162
Woman	117	255	146	157	199	171

mal ε is set as shown in Figs. 6 (a), 6 (b), 7 (a), and 7 (b). In the results for Lenna shown in Figs. 6 (c) and 6 (d), the parameter tuning by Matsumoto’s method is good, on the other hand, the proposed method is inferior to the method because the stripes on a hat are blurred. In the case of Barbara, as shown in Figs. 7 (c) and 7 (d), the result obtained by Matsumoto’s method is very poor whereas a good result is obtained by the proposed method.

From Table 1, the proposed parameter tuning is well done for many images although ε^* is not same perfectly to ε_{opt} . In the cases where Barbara and Bridge are used as input images of concern, however, differences between the

optimal MSE and the MSE result obtained by the proposed method are especially large comparing with cases other images are used. The reason why the large differences are caused is supposed that the estimation accuracy of σ for Barbara and Bridge is lower than for other images. This matter is also caused in the cases σ is 20 and 30 as shown in Tables 2 and 3. However, in most cases shown in Tables 1–3, the proposed ε adjustment is superior to the Matsumoto’s method and it shows the effectiveness and validity of the proposed method.

5. Conclusions

In this letter, a new parameter tuning method for the ε -filter using HD between signal distributions was proposed. Through experiments comparing it with Matsumoto’s method [6], the effectiveness of the proposed method was demonstrated.

Future works are the sophistication of the estimation method for the standard deviation of the Gaussian noise $\hat{\sigma}_{\text{est}}$, to attempt using metrics except HD and KLD, and the development of a search algorithm to efficiently find the optimal value ε^* .

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