Gaze Estimation Method Involving Corneal Reflection-Based Modeling of the Eye as a General Surface of Revolution about the Optical Axis of the Eye

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SUMMARY We have proposed a novel geometric model of the eye in order to avoid the problems faced while using the conventional spherical model of the cornea for three dimensional (3D) model-based gaze estimation. The proposed model models the eye, including the boundary region of the cornea, as a general surface of revolution about the optical axis of the eye. Furthermore, a method for calculating the point of gaze (POG) on the basis of our model has been proposed. A prototype system for estimating the POG was developed using this method. The average root mean square errors (RMSEs) of the proposed method were experimentally found to be smaller than those of the gaze estimation method that is based on a spherical model of the cornea.

key words: gaze estimation, eye model, eye movement, one-point calibration

1. Introduction

Computer vision-based gaze trackers are widely used for scientific studies and human-computer interactions [1]–[3] because they are nonintrusive. One new trend in such trackers is three dimensional (3D) model-based gaze estimation that uses a geometric model of the eye and calculates a 3D gaze direction vector [4]; this reduces the number of calibration points that a user has to gaze at before the system can be used, and it enables the user to move freely. The geometric model typically used for 3D model-based gaze estimation is a spherical model of the front surface of the cornea [5]–[9]. As the surface of the cornea is very smooth, it behaves like a mirror. The point of gaze (POG) is estimated using the reflections of the point light sources on the corneal surface. However, Shih et al. and Guestrin et al. reported that a spherical model may not be suitable for modeling the boundary region of the cornea [5], [9]. Gaze estimation methods that use a spherical model of the front surface of the cornea can exhibit POG estimation biases because the real cornea is not truly spherical (only the central portion of the cornea is approximately spherical).

This paper presents a novel method for gaze estimation, in which the cornea is modeled as a surface of revolution about the optical axis of the eye. This model is a more accurate depiction of the corneal region near the sclera boundary than is a spherical model of the cornea.

The main contributions of this paper are as follows:

• A novel geometric model of the eye that models the eye, including the boundary region of the cornea, as a general surface of revolution about the optical axis of the eye;
• A corneal reflections-based method for estimating the optical axis of the eye on the basis of our model;
• A POG estimation method including a one-point user-calibration method.

The remainder of the paper is organized as follows. In Sect. 2, we describe a novel geometric model of the eye for 3D model-based gaze estimation. In Sect. 3, we propose a method for estimating the optical axis of the eye based on the proposed model. In Sect. 4, we describe a method for estimating user-dependent parameters (user calibration). In Sect. 5, we describe a method for estimating the visual axis of the eye and POG after user calibration. In Sect. 6, we explain the implementation of the system and the evaluation of the results. We discuss our results in Sect. 7, which is followed by the conclusion of the paper.

A preliminary version of this work has been reported in [10].

2. A Novel Geometric Model of the Eye for 3D Model-Based Gaze Estimation: General Surface of Revolution about the Optical Axis of the Eye

2.1 Structure of an Eyeball

The structure of an eyeball is shown in Fig. 1. The light is converged by the cornea and the lens onto the retina. Part of the light is reflected on the smooth surface of the cornea like a mirror. The shape of the central portion of the outer surface of the cornea can be approximated as a sphere. The edge of the outer surface of the cornea is smoothly connected to the sclera. Because the surface of the sclera is
2.2 Conventional Model of the Eye for 3D Model-Based Gaze Estimation

A typical eye model that is used in a model-based approach [5]–[9] is shown in Fig. 2. The model contains a mixture of large and small spheres. In this model, the cornea is modeled as a small sphere. However, the shape of the real cornea is not spherical near its edge. Because the model-based approach uses corneal reflections, estimated gaze direction using the boundary region of the cornea will have estimation bias.

2.3 A Novel Geometric Model of the Eye for 3D Model-Based Gaze Estimation

We propose a novel geometric model of the eye for gaze estimation that models the eye as a general surface of revolution about the optical axis of the eye, as shown in Fig. 3. This model involves the entire eyeball, including the central region of the cornea, the boundary region of the cornea, and the scleral surface. Because our gaze estimation method needs only the fact that the model is a general surface of revolution about the optical axis of the eye, it is not necessary to express the shape of the model as an equation.

In addition, we assumed that the optical axis and visual axes of the eye intersect at the center of corneal curvature around the central portion of the outer surface of the cornea (Fig. 4).

3. Estimation of the Optical Axis of the Eye, Based on the Proposed Model

To calculate the optical axis of the eye by using our model, we use a special arrangement of cameras and light sources (Fig. 5). We use two calibrated cameras with a light source attached to each camera. The position of each light source is supposed to be the same as the nodal point of the camera.

3.1 Reflection and Refraction at the Corneal Surface

A ray tracing diagram of the system in the case of two cameras (C_0, C_1) and two light sources (L_0, L_1) is shown in Fig. 6. A ray from light source 0 L_0 is reflected at P_{00} on the corneal surface. The ray is reflected back to camera 0 in the plane that includes the optical axis of the eye (Π_0) because the cornea is modeled as a general surface of the revolution about the optical axis of the eye. The ray goes through the nodal point C_0 (= L_0) of camera 0 and reaches P_{00}' on the camera image plane. The center of the pupil B is observed in the same plane, i.e., Π_0. A ray originating from B gets refracted at a point B_0'', passes through the nodal point C_0 of camera 0, and intersects the camera image plane at a point B_0'.

P_{11}' and B_1' are determined by the same way using the camera 1 and the light source 1.

3.2 Image Processing of Eye Images

Figure 7 shows the camera image of camera j in the camera coordinate system j in two dimensions, which is the case where we use two cameras with a light source. In the images captured by camera j, the cross /B = (/B_x, /B_y) is the center of the pupil of the observed image; this is detected by ellipse fitting. The small circles /P_j = (/P_{1j}, /P_{2j}), which are
Fig. 5 Arrangement of cameras and light sources for estimating POG on the basis of our model.

reflections of the light source \(i\) \((i = 0, 1)\), are found seeking near the center of the pupil. The reflection of a light source from the outer surface of the cornea is called the first Purkinje image.

\(j\) and \(jP_i\) are located on the image sensor of camera \(j\) in the camera image coordinate system. They are converted to the position vectors \(B'_j = (B'_{jx}, B'_{jy}, B'_{jz})^T\) and \(P'_{ji} = (P'_{jix}, P'_{jiy}, P'_{jiz})^T\) in the world coordinate system in three dimensions by using the intrinsic and extrinsic camera parameters.

3.3 Estimation of the Optical Axis of the Eye

A cross section of the model of the eyeball is shown in Fig. 8. \(A\) is the center of corneal curvature around the central portion of the outer surface of the cornea. The optical axis of the eye is defined by the line connecting \(A\) and \(B\). \(L_j\) and \(C_j\) denote the position of the light source \(j\) and the nodal point of camera \(j\), respectively; \(C_j\) is assumed to be located at the same position as \(L_j\). The value of \(C_j\) \((= L_j)\) is determined
by calibrating the camera beforehand.

As described in 3.1, a ray from \( \mathbf{L}_j \) is reflected at a point \( \mathbf{P}_{jj} \) on the corneal surface back along its incident path. It passes through \( \mathbf{C}_j \) and intersects the camera image plane at a point \( \mathbf{P}_{jj}' \). If the cornea is perfectly spherical, the line connecting \( \mathbf{L}_j \) and \( \mathbf{P}_{jj} \) would pass through \( \mathbf{A} \), and the position of \( \mathbf{A} \) can be easily determined using two cameras as described in Chen’s method [11]. However, the position of \( \mathbf{A} \) cannot be accurately estimated when light is reflected from around an edge of the surface of the cornea. For example, in Fig. 6 the line connecting \( \mathbf{P}_{00} \) and \( \mathbf{C}_0 \) and the line connecting \( \mathbf{P}_{11} \) and \( \mathbf{C}_1 \) do not intersect on the optical axis of the eye.

A ray originating from the center of the pupil \( \mathbf{B} \) gets refracted at a point \( \mathbf{B}_j' \), passes through the nodal point \( \mathbf{C}_j \) of the camera \( j \), and intersects the camera image plane at a point \( \mathbf{B}_j' \).

Because we are using the model of a general surface of revolution about the optical axis of the eye, the ray from \( \mathbf{L}_j \) that is reflected back from \( \mathbf{P}_{jj} \) and the ray from \( \mathbf{B} \) that is refracted at \( \mathbf{B}_j' \) are in the plane that includes the optical axis of the eye, i.e., \( \Pi_j \). Therefore, \( \mathbf{A} \), \( \mathbf{B} \), \( \mathbf{B}_j' \), \( \mathbf{C}_j \), \( \mathbf{L}_j \), \( \mathbf{P}_{jj} \), \( \mathbf{P}_{jj}' \), and the optical axis of the eye are all coplanar. The normal vector of the plane including these points is \(( \mathbf{C}_j - \mathbf{B}_j') \times (\mathbf{P}_{jj} - \mathbf{C}_j) \); therefore the plane \( \Pi_j \) is expressed as

\[ (\mathbf{C}_j - \mathbf{B}_j') \times (\mathbf{P}_{jj} - \mathbf{C}_j) \cdot (\mathbf{X} - \mathbf{C}_j) = 0, \quad (1) \]

where \( \mathbf{X} = (x, y, z)^T \) is a point on the plane. We obtain two planes when we use two cameras (\( j = 0, 1 \)). The optical axis of the eye can be determined from the intersection of the two planes (the two planes must not be coplanar) as shown in Fig. 6.

Thus, our method can calculate the optical axis of the eye from the reflections from all the outer surface of the eye, while the conventional model uses only the spherical portion of the cornea.

3.4 Toward Accurate Estimation of POG

Only the optical axis of the eye can be determined using the above-mentioned method. It is necessary to determine \( \mathbf{A} \) in order to determine the visual axis of the eye because we assume that the visual axis and the optical axis of the eye intersect at \( \mathbf{A} \) (Fig. 4). The visual axis of the eye is described as \( \mathbf{X} = \mathbf{A} + t \mathbf{e} \) in a parametric form, where \( \mathbf{e} \) is the unit direction vector of the visual axis of the eye and can be estimated by the method described in Sect. 5.2. Errors in estimating \( \mathbf{A} \) lead to a parallel shift of the visual axis of the eye. Therefore, as the distance between a user and the object gazed at increases, the error in estimating the POG on the object in terms of the view angle decreases. A point on the optical axis of the eye nearest from the intersection of two lines (\( \mathbf{X} = \mathbf{C}_j + t_j (\mathbf{C}_j - \mathbf{B}_j') \), \( j = 0, 1 \)) can give an approximation of \( \mathbf{A} \); the approximation error is less than approximately 7.8 mm (the average value of the radius of the cornea). However, when the distance between the user and the object is small, the error in estimating POG, which is caused by the error in estimating \( \mathbf{A} \), is relatively large in terms of the view angle. To solve this problem, we have proposed a method for estimating the user-dependent parameters and determining the visual axis of the eye, in Sects. 4 and 5.

4. Estimation of User-Dependent Parameters (User Calibration)

The user-dependent parameters that have to be estimated are as follows: \( R \), the radius of corneal curvature around the central portion of the outer surface of the cornea; \( K \), the distance between the centers of the corneal curvature and the pupil; the offset between the optical and visual axes of the eye. This estimation is considered as a user calibration.

In the user-calibration process, the user is instructed to gaze at a single point (calibration point), the position of which is known. The position of the calibration point is selected such that the light from the camera is reflected from the central portion of the corneal surface that is approximated as a sphere.

Furthermore, it is assumed that the pupil can be observed through the portion of the corneal surface that is approximated as a sphere because the pupil is observed sufficiently inside the corneal edge even when the pupil is enlarged. Therefore, the refraction at the corneal surface can be determined on the basis of the spherical model of the cornea.

4.1 Estimation of the Radius of Corneal Curvature

When a user gazes at an object near the camera in the user-calibration process, light is reflected from the spherical corneal surface. Hence, we can use the spherical model of the cornea in the calibration process.

We now estimate the position of the center of corneal curvature \( \mathbf{A} \). Figure 9 shows a cross section of the cornea including \( \mathbf{A} \), the center of corneal curvature; \( \mathbf{L}_i \), the position of the light source \( i \); \( \mathbf{L}_j \), the position of the light source \( j \); \( \mathbf{C}_i \), the nodal point of camera \( i \); and \( \mathbf{C}_j \), the nodal point of the camera \( j \). The positions of \( \mathbf{C}_i (= \mathbf{L}_i) \) and \( \mathbf{C}_j (= \mathbf{L}_j) \)
are known from the camera calibration. A ray from \(L_i\) reflected from the corneal surface returns to \(C_j\) and reaches \(P_{ji}'\). The extension of the path of the ray includes \(A\) because the corneal surface is supposed to be a sphere. Similarly, the line connecting \(C_j\) and \(P_{jj}'\) includes \(A\). Therefore, \(A\) can be estimated from the intersection of two lines as follows:

\[
X = C_i + t_{ii} \left( C_j - P_{ji}' \right),
\]

\[
X = C_j + t_{jj} \left( C_j - P_{jj}' \right),
\]

where \(t_{ii}\) and \(t_{jj}\) are parameters.

A ray from \(L_i\) is reflected at a point \(P_{ji}\) on the corneal surface such that the reflected ray passes through \(C_j\) and intersects the camera image plane at a point \(P_{ji}'\). Similarly, a ray from \(L_j\) is reflected at a point \(P_{jj}\) on the corneal surface such that the reflected ray passes through \(C_j\) and intersects the camera image plane at a point \(P_{jj}'\). To estimate the center of the cornea, we estimate the reflection point \(P_{ji}' (= P_{ij})\), that is, the intersection of the lines as follows:

\[
X = C_i + t_{ij} \left( C_j - P_{ij}' \right),
\]

\[
X = C_j + t_{ji} \left( C_j - P_{ji}' \right),
\]

where \(t_{ij}\) and \(t_{ji}\) are parameters. Therefore, the radius of corneal curvature \(R\) is determined as \(R = ||P_{ji} - A||\).

### 4.2 Estimation of the Distance between the Center of Corneal Curvature and the Center of the Pupil

As shown in Fig. 10, a ray originating from the center of the pupil \(B\) gets refracted at a point \(B''\), passes through the nodal point \(C_j\) of camera \(j\), and intersects the camera image plane at point \(B_j''\). \(B_j''\) can be determined by solving the equations given below:

\[
X = C_j + t_j \left( C_j - B_j' \right),
\]

\[
R = ||X - A||.
\]

These equations may have two solutions; we select the one closer to \(C_j\).

The vector \(t_j\) (the refracted vector at \(B_j''\), shown in Fig. 10) can be obtained by using Snell’s law as follows:

\[
t_j = \left(-\rho \mathbf{n}_j \cdot \mathbf{v}_j - \sqrt{1 - \rho^2 \left(1 - (\mathbf{n}_j \cdot \mathbf{v}_j)^2\right)}\right) \mathbf{n}_j + \rho \mathbf{v}_j,
\]

where the incident vector \(\mathbf{v}_j = (C_j - B'_j)/||C_j - B'_j||\); the normal vector at the point of refraction, \(\mathbf{n}_j = (B''_j - A)/||B''_j - A||\); and \(\rho = n_1/n_2\) (\(n_1\), refractive index of air \(\approx 1\); \(n_2\), effective refractive index \(\approx 1.3375\) [6]).

The center of the pupil \(B\) can be determined from the intersection of two rays from the two cameras, as follows:

\[
X = B_j'' + s_j t_j \quad (j = 0, 1),
\]

where \(s_j\) is a parameter. Therefore, the distance \(K\) between the centers of the corneal curvature and the pupil is calculated as \(K = ||B - A||\).

### 4.3 Estimation of the Offset between the Optical and Visual Axes of the Eye

In the case of a user gazing at a known position, the offset between the optical and visual axes of the eye is calculated using the method reported by Nagamatsu et al. [12]. The offset is determined as a relative position of the unit direction vectors of the optical and visual axes of the eye when the eye is at the primary position.

### 5. Estimation of the Visual Axis of the Eye and POG after User Calibration

After the user calibration, the user moves his/her eyes freely. The optical axis of the eye can be calculated using the method described in Sect. 3. \(R, K,\) and the offset between the optical and visual axes of the eye are known from the user calibration. Then, the positions of \(A\) (the center of corneal curvature) and \(e\) (the unit direction vector along the visual axis of the eye) are required for calculating the visual axis of the eye \((X = A + te)\).

#### 5.1 Estimation of the Center of Corneal Curvature around the Central Portion of the Outer Surface of the Cornea

Because the pupil is observed sufficiently inside the edge of the cornea, we assume that the corneal surface where the pupil is observed can be approximated as a spherical surface. Therefore, the refraction at the surface of the cornea can be calculated using a spherical model.

A flow chart (based on Fig. 11) that describes the procedure involved in searching for the position of \(A\) is shown in Fig. 12. The algorithm for searching for the position of \(A\) is as follows:

1. Set the initial position of \(A\) on the optical axis of the eye, and select the position that is nearest to the intersection of the two lines, i.e., \(X = C_j + t_j(C_j - B_j')\) \((j = 0, 1)\).
2. Calculate \(B_j''\) and \(t_j\) by using Eqs. (6), (7), and (8), where \(R\) is known from the user calibration.
3. Calculate \( B \) from the intersection of the two lines described by using Eq. (9).
4. Calculate the distance between \( B \) and \( A \), and compare it to \( K \) that was estimated during the user calibration.
5. Shift the position of \( A \) toward the rotation center of the eye along the optical axis of the eye, and repeat steps 2–5 to determine the accurate position of \( A \). The search is finished when \( \| B - A \| = K \). It is sufficient to search for the position of \( A \) for a length of 10 mm because the average radius of the cornea is approximately 7.8 mm.

5.2 Estimation of the Visual Axis of the Eye and POG

Using the method reported by Nagamatsu et al. [12], the unit direction vector of the visual axis of the eye, i.e., \( c \), is determined from the unit direction vector of the optical axis of the eye, i.e., \( d \), the unit direction vector of the visual axis of the eye at the primary position, i.e., \( a \) that is approximated by the face direction (or the normal vector of the display), and the offset between the optical and visual axes of the eye.

The intersection point between the visual axis of the eye \((X = A + tc)\) and the object (e.g., a display) is the POG.

6. Implementation and Experiment

6.1 Producing Dark Pupil Images

Our arrangement of cameras and light sources suggest that our system produces bright pupils. A bright pupil is created by the light reflected off the retina, when the light source is set to be coaxial with the optical path. However, the intensity of the bright pupil varies between individuals and is affected by changes in head position or poses [4]. Therefore, we decided to implement our system such that it produced dark pupils.

We set the light-emitting diodes (LEDs) far enough from the nodal point of the camera such that they produced dark pupils; the two infrared LEDs were attached to each camera such that the midpoint of the two LEDs coincided with the nodal point of the camera, as shown in Fig. 13. By calculating the midpoint of the two first Purkinje images, we virtually realized the proposed arrangement.

6.2 Implementation

A prototype system has been developed for the estimation of the POG on a display (Fig. 14). This system consists of two synchronized monochrome IEEE-1394 digital cameras (Firefly MV, Point Grey Research Inc.), a 17” (for the user) and a 22” (for checking the system) liquid crystal display (LCD), and a Windows-based personal computer (PC) (Windows XP). The software was developed using OpenCV 1.0 [13]. Each camera is equipped with a 1/3" complementary metal oxide semiconductor (CMOS) image sensor that...
has a resolution of 752 × 480 pixels. A 35-mm lens and an infrared (IR) filter are attached to each camera. Two infrared LEDs are attached to each camera such that the midpoint of the two LEDs coincides with the nodal point of the camera. These cameras are positioned under the display. The intrinsic parameters of the cameras are determined before setting up the system.

6.3 Experiment

The laboratory evaluation of the prototype system involved three adult participants (men, 2; woman, 1) who did not use glasses or contact lenses. The distance between the display and the participant was adjusted such that the participant’s right eye was approximately 450 mm from the display. The diameter of the iris on the camera image was about 140–170 pixels.

The participants were asked to stare at 43 fiducial points on the display. The fiducial points that the participants were asked to gaze at are shown in Fig. 15. The points around the top left and right of the display were plotted at short intervals because the results of a preliminary experiment revealed that corneal asphericity seems to greatly affect the region around these points.

To compare our method to the method that uses a spherical model of the cornea, we also calculated the POG by using Chen’s method [11] at the same time. Chen’s method uses a spherical model of the cornea and the same arrangement of cameras and light sources as our system.

We conducted three trials for each participant and changed the distance between the cameras for each trial (near, middle, and far), because we knew from the preliminary experiments that the distance between the cameras influences the range of the area that the system can measure. The first Purkinje images that reflect on the cornea when the distance between the cameras changes are shown in Fig. 16.

More than 30 data were recorded for each fiducial point. The recorded data were time, the position of the fiducial point, the position of the POG calculated by two methods, the position of the center of the corneal curvature, etc.

The results of the comparison are shown in the display coordinate system in Figs. 17–19. The crosses and triangles indicate the average POGs for each fiducial point obtained using our method and Chen’s method, respectively. The distances of the cameras were calculated by camera calibration; the distances of the cameras were 76.7 mm (near), 168.8 mm (middle), and 216.3 mm (far).

The results have been summarized in Table 1. The third column lists the average root mean square error (RMSE) values (converted from millimeters to degrees) of the POGs of each participant obtained using our method. The fourth column lists the RMSE values of the POGs obtained using Chen’s method. The fifth column lists the average RMSE values of the POGs for the top left and right points (11, 12, 14–17, 19–22, 24–43) of each participant obtained using our method. The sixth column lists the RMSE values of the POGs for the top left and right points obtained using Chen’s method. The seventh column lists the number of points (from a total of 43) that could not be measured. The values within parentheses indicate the number of fiducial points that could not be measured. For some cases, we could not get one of Purkinje images no matter how we waited; the system cannot calculate POGs when both of Purkinje images are not detected.

The RMSE is calculated by the following equation:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} ((X - x_i)^2 + (Y - y_i)^2)}$$

where $X$, $x_i$, $Y$, and $y_i$ are the true and measured values of horizontal axis, and the true and measured values of vertical axis, respectively. We used $\theta = \arctan(d/L)$ as the conversion equation, from $d$ mm to $\theta^\circ$, where $L$ is the distance of the eye from the display.

For reference, the estimated $R$ and $K$ are shown in Table 2.
Fig. 17 Comparison of our method and Chen’s method in the display coordinate system (near).

Fig. 18 Comparison of our method and Chen’s method in the display coordinate system (middle).
7. Discussion

Our results indicate that for all the cases, our method reduces the POG estimation bias than does Chen’s method that uses a spherical model of the cornea (Table 1). In Figs. 17–19, the same number of points estimated by Chen’s and our method are shown. The points calculated by Chen’s method are farther than the points calculated by our method from the corresponding fiducial points around the top-left and right of the display, where one of the lights reflects at the boundary region of the cornea. Therefore, in Chen’s method, the reflection at the boundary region of the cornea influences the POG estimation. The POG estimation bias seems to be caused by the estimation error of the position of the center of corneal curvature from the description in Sect. 3.

Furthermore, it was difficult to accurately measure the POG for the points at the extremely top left and right of the display. An eye image where the POG was not accurately estimated is shown in Fig. 20; the left glint produced by light source 0 is blurred, and therefore, the position of the glint cannot be accurately determined. Although our model models the shape of the sclera, the surface of the sclera is not smooth; therefore, it is difficult to determine the position of the glint for accurate calculation of the optical axis. When the user rotated his/her eye more to the outside, the left glint produced by light source 0 did not appear on the cornea, as shown in Fig. 21.

The RMSE of our system listed in Table 1 is expected to increase as the distance of the cameras increases because of the increase in the number of points at which the POG was not accurately estimated. As shown in Fig. 21, some points could not be measured. The calculated RMSE values listed in Table 1 do not include the points at which the POG could not be measured. Therefore, for example, for participant B, the RMSE when the distance of the cameras is “far” is less than that when the distance of the cameras is “middle.”

It seems that the limit to which the user can rotate the eye is determined depending on whether the light reflects on the cornea. Figure 22 shows the limit ($\phi \approx 40^\circ$) to which the eye can be rotated, while ensuring that the method can accurately estimate the direction of the optical axis of the eye. Therefore, the candidate area where the POG can be accurately estimated is considered as the inside of the cone whose vertex is the position of the eye and whose axis is the line connecting the eye and the camera. Besides, because our method uses two cameras, the area where the POG can be estimated is the overlapping portion of the two cones that are produced by two cameras. The area in which the system can measure POG is shown in Fig. 23 (dark gray area). Therefore, the area where POG can be measured depends on the distance of the cameras.

If the radii of the areas are large, i.e., if the user sits farther away from the display, the POG can be measured all over the display, and even Chen’s method could work. The relation between the distance of the eye from the display...
and the eye models is shown in Fig. 24. In Fig. 24, the “display” is a cross section along the arrow shown in Fig. 23. $d_s$ and $d_a$ are the minimum distances of the eye from the display, while the optical axis of the eye can be estimated for the entire display using our method (aspherical model) and Chen’s method (spherical model), respectively. $\phi_a$ and $\phi_s$ are the limit rotation angles of the eye from the camera using the aspherical model and spherical model, respectively. $E_a$ and $E_s$ are the positions of the eye for each case. If the distance of user is less than $d_s$, the spherical model does not work well. Using our aspherical model, we can reach $d_a$. $d_s$ and $d_a$ change not only with the size of the display and the positions of the display, camera, and eye, but also with the user, i.e., the person sitting in front of the system, because $\phi_a$ and $\phi_s$ are user dependent. From the experimental results, we calculate the relation between the angle of the optical axis of the eye from camera 0 and the RMSE, for each participant, as shown in Figs. 25–27, where the mea-
sured data for the left half (including the center points) of the fiducial points are used. If we determine whether the system is accurate by the RMSE limit of $2.0^\circ$, the limits using our method are approximately $37.5$, $37.5$, and $42.0^\circ$ for participants A, B, and C, respectively. In contrast, the limits using Chen’s method are approximately $35.0$, $35.0$, and $37.0^\circ$ for participants A, B, and C, respectively. If the eye is on the perpendicular bisector of the display line shown in Fig. 24, $d_x$ and $d_y$ of participants A, B, and C are calculated as approximately $607$, $607$, and $537$ mm, and approximately $653$, $653$, and $616$ mm, respectively.

8. Conclusion

In this paper, we have proposed a novel geometric model of the eye in order to avoid the problems faced while using the conventional spherical model of the cornea for three dimensional (3D) model-based gaze estimation. The proposed model models the eye, including the boundary region of the cornea, as a general surface of revolution about the optical axis of the eye. Furthermore, we have proposed a corneal reflections-based method for estimating the optical axis of the eye on the basis of our model. A POG estimation method including a one-point user-calibration method has also been proposed. We confirmed the effectiveness of our method by experimentally comparing it with the spherical-model method.

Acknowledgments

This work was partially supported by KAKENHI (23300047).

References


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