SUMMARY In this paper we aim to group visual correspondences in order to detect objects or parts of objects commonly appearing in a pair of images. We first extract visual keypoints from images and establish initial point correspondences between two images by comparing their descriptors. Our method is based on two types of graphs, named relational graphs and correspondence graphs. A relational graph of a point is constructed by thresholding geometric and topological distances between the point and its neighboring points. A threshold value of a geometric distance is determined according to the scale of each keypoint, and a topological distance is defined as the shortest path on a Delaunay triangulation built from keypoints. We also construct a correspondence graph whose nodes represent two pairs of matched points or correspondences and edges connect consistent correspondences. Two correspondences are consistent with each other if they meet the local consistency induced by their relational graphs. The consistent neighborhoods should represent an object or a part of an object contained in a pair of images. The enumeration of maximal cliques of a correspondence graph results in groups of keypoint pairs which therefore involve common objects or parts of objects. We apply our method to common visual pattern detection, object detection, and object recognition. Quantitative experimental results demonstrate that our method is comparable to or better than other methods. 

key words: correspondence, graph, object recognition, object detection

1. Introduction

Not a few papers have shown that parts-based approaches in object detection and tracking are effective to robustly handle appearance variations, background clutter, or occlusions [1]–[4]. Likewise, spatial configurations of parts can be applied to object category recognition [5]–[8] and action recognition [9], [10]. A method to find corresponding parts of objects among images can be plugged into these algorithms as a fundamental tool. For example, an importance weighting of patches or a biased sampling strategy plays this kind of role. On the other hand, mismatched keypoints can occur due to background clutter, ambiguous points, or severe deformation of objects even by using effective visual descriptors [11]–[14]. A method for validating the consistency of point correspondences will sort out the mismatches. Therefore, it is important to develop a method for finding sets of correspondences consistent with each other which are directly relevant to objects or parts of objects. Because matched keypoints, or correspondences, are already verified through their descriptors as visual appearance, some geometric consistency among correspondences should be ensured [15]–[17]. However, it is not an easy task to determine such consistent correspondences because there may be outliers which disturb spatial distributions of correspondences.

We propose an approach to grouping visual correspondences which are consistent within each group in terms of their spatial configuration. Our method is based on two graphs named relational graphs and correspondence graphs. A relational graph of a keypoint is constructed by thresholding geometric and topological distances between the point and its neighboring points. A threshold value of a geometric distance is determined according to the scale of each keypoint, and a topological distance is defined as the shortest path on a Delaunay triangulation built from keypoints. The geometric and topological consistency between two correspondences is evaluated according to their relational graphs which describe a spatial neighborhood of each point. Relational graphs reflect the fact that two points involving an object part should be mapped to points close to each other. A correspondence graph is composed of nodes which represent a pair of keypoints matched between two images and edges which connect consistent correspondences based on relational graphs. A maximal clique of a correspondence graph can be viewed as a group of correspondences consistent with each other and thus involves a common object or a part of an object.

In the rest of the paper, we first introduce some related work to our method in Sect. 2, and then, propose our approach to visual correspondence grouping in Sect. 3. In Sect. 4, we present experimental results, and qualitatively evaluate our method in comparison with other methods. Finally, we conclude this paper in Sect. 5.

2. Related Work

Liu and Yan [15] have proposed a method to find common visual patterns which correspond to dense subgraphs of a correspondence graph. A weighted adjacency matrix of the correspondence graph is calculated from geometric and visual consistency scores among two correspondences. Their formulation is a weighted version of the maximum clique problem, and the difference between their method and ours is how to construct a correspondence graph and then find its dense subgraphs. Our relational graphs of a pair of matched keypoints determine a set of possible correspondences consistent with each other in geometric and topological point of view, so a correspondence graph constructed from relational
graphs is unweighted. If we suppose that two consistent correspondences are located close each other within two images, the local property induced by our relational graphs is more reasonable, and also can be introduced into the calculation of their correspondence graph and adjacency matrix.

The method proposed by Cho et al. [16] is based on hierarchical agglomerative clustering where the geometric consistency is defined as a projection error among two correspondences. Their method may accidentally consider an ambiguous correspondence which has high consistency with another one as an inlier of a cluster. By contrast, such a correspondence is less likely to be one of vertices of a clique in our correspondence graph because it must be consistent with all other correspondences of a clique.

Zhao and Yuan [17] have proposed a method to find common objects by counting occurrences of spatial point groups among images. A spatial group of a keypoint is defined as its k-spacial nearest neighbors (k-SNN) and the value of k gradually increases to detect a larger part of an object. Although k-SNN may change its shape due to background clutter, our relational graph can stably cover a certain spatial extent whenever the keypoints are extracted from (a part of) an object.

Estrada et al. [18] have proposed an approach to clustering of keypoints according to their similarity of appearance, where spectral clustering is applied to a set of keypoints extracted from an image, and then constrained matching with clusters is performed. The appearance of a keypoint, especially located on an object boundary, can vary dramatically to make matching difficult. Our approach, on the other hand, is to first match keypoints between two images, and then group the corresponding points according to both their geometric and topological consistency. Mismatch removal is achieved as a by-product of our method.

Horaud and Skordas [19] have used relational graphs and correspondence graphs to find correspondences between a pair of stereo images. A relational graph is constructed from each image by considering spatial relations among line segments, and a correspondence graph is constructed by a set of rules derived from the compatibility and incompatibility between two correspondences of line segments which must be satisfied in stereo images. Our method can be viewed as an extension to a pair of general images.

Parts-based approaches to object recognition, detection and tracking, as listed above, do not explicitly find corresponding object parts among given images. For articulated objects like humans, explicit models such as pictorial structures [20] can be used to detect each object part. In our method, we do not specify object classes, and do not need any model in advance.

The relational graphs improve the computational efficiency by considering only consistent correspondences. This makes our method applicable to higher level processes and is opposite to the methods of densely matching a pair of images which have more computational complexity [21], [22].

### 3. Algorithm

At first, we extract two sets of keypoints from a pair of images, and then find initial correspondences among them by comparing their descriptors. Let us denote two sets of keypoints extracted from images $I$ and $J$ as $S$ and $T$, respectively. A set of initial correspondences is denoted as $M = \{(s,t)|s \in S, t \in T\}$, where $m : S \rightarrow T$ is a mapping among keypoints. Notice that one-to-many correspondences are allowable in this definition of $M$. Actually, we define $m(s) = \{t \in T | \|f_s - f_t\| < \tau\}$ where $f_s$ is a descriptor calculated locally around a point $s$ and $\tau$ is a threshold.

Below we introduce an algorithm to construct both relational and correspondence graphs from keypoints and initial correspondences.

#### 3.1 Relational Graphs

Two correspondences should be geometrically consistent with each other if they are involved in an object or a part of an object among a pair of images. Geometric consistency can be evaluated in terms of geometric distances and distortions among correspondences [15], [16], [23]. We adopt relational graphs to measure geometric consistency, which tolerate local deformations and transformations such as rotation and scaling.

A relational graph is built as follows: First, Delaunay triangulation is constructed from a set of keypoints over an image. As shown in Fig. 1 (a), two points are connected by the edge of a triangle whose circumcircle contains no other point in its interior. Let us denote a set of these edges as $E_T = \{e = (s,s')|s,s' \in S\}$. An edge weight between two points is set to $w(e) = 1, \forall e \in E_T$.

Next the neighborhood of each point $s$ is determined according to both geometric and topological distances to other points $s'$. Specifically we calculate the Euclidean distance $d(s,s')$ and find the shortest path length $l(s,s')$. Then we define the neighborhood of $s$ with two thresholds $D$ and $L$.

$$N_s = \{s'|s' \in S, d(s,s') < D, l(s,s') < L\}. \tag{1}$$

Notice that $s \in N_{s'} \iff s' \in N_s$ because $d(s,s') = d(s',s)$.
and \( l(s, s') = l(s', s) \).

A relational graph \( G_s^R = (\mathcal{V}_s^R, \mathcal{E}_s^R) \) of a point \( s \in S \) is defined as

\[
\mathcal{V}_s^R = \{s\} \cup \mathcal{N}_s, \quad \mathcal{E}_s^R = \{(s, s') | s', s \in \mathcal{V}_s^R\}
\]

(2)

where \( \mathcal{V}_s^R \) is a set of nodes and \( \mathcal{E}_s^R \) a set of edges of a graph. Figure 1 depicts a relational graph.

We can adequately adjust a threshold of the Euclidean distance by considering the scale of each keypoint, which makes this representation invariant to scaling (and also rotation) [24]. On the other hand, a topological distance reflects the local shapes of regions or a part of an object because keypoints are extracted as corners and blobs [25]. Relational graphs can be calculated efficiently due to their locality. This is an advantage of our definition over conventional methods [26], [27], in which the geometric compatibility of all combinations between \(|M|\) correspondences \( \mathcal{V} \) is the average number of nodes and is usually less than \(|M|\). Note that each relational graph will have a different number of nodes.

3.2 Correspondence Graphs

Given a pair of images \( I \) and \( J \), a set of nodes of a correspondence graph is equivalent to a set of point correspondences \( M \) among two point sets \( S, T \) by the following definition:

\[
\mathcal{V}^C = \{v = (s, t) | s \in S, t \in T, (s, t) \in M\}.
\]

(3)

The presence or absence of an edge between two nodes \( v, v' \in \mathcal{V}^C \) depends on the spatial relationship among four points \( \{s, s', t, t'\} \). The spatial relationship of one point to another on an image is induced by its relational graph. As shown in Fig. 2, two pairs of matched keypoints or correspondences \( (s, t), (s', t') \in M \) are geometrically and topologically consistent with each other if they fulfill the following condition:

\[
s' \in N_s \land t' \in N_t.
\]

(4)

The evaluation by relational graphs can be invariant under scaling. We can also consider the shape of a part of an object because of topological information as opposed to the measure of consistency according to the geometric distance and distortion between keypoints [16], [23]. A set of edges of a correspondence graph is constructed by connecting each pair of consistent nodes as follows:

\[
\mathcal{E}^C = \{(v, v') | v = (s, t) \in \mathcal{V}^C, v' = (s', t') \in \mathcal{V}^C, \ s \in N_v \land t \in N_t\}.
\]

(5)

In addition to the above consistency, we can introduce an additional visual similarity to measure the difference of appearance between two correspondences. For example, we use the difference of color histograms [28] calculated locally around each keypoint. When such kind of visual similarity is considered, we can define the edges of a correspondence graph as follows:

\[
\mathcal{E}^C = \{(v, v') | v = (s, t) \in \mathcal{V}^C, v' = (s', t') \in \mathcal{V}^C, \ s \in N_v \land t \in N_t, sim(s, s', t, t') > \tau_a\}
\]

(6)

where \( sim(\cdot, \cdot, \cdot, \cdot) \) represents the visual similarity among the four points and \( \tau_a \) is a threshold.

As in the paper [19], we extend a correspondence graph by creating an edge between two nodes \( v \) and \( v' \) if there exists a node \( v'' \) which is consistent with both \( v \) and \( v' \). This procedure is called propagation and may result in a larger clique corresponding to an object or a larger part of an object.

3.3 Grouping by Maximal Cliques

Geometric consistency is shared by a subset of nodes of a correspondence graph in which every pair of nodes is connected, and these nodes or correspondences should be involved in a common, consistent region among two images. In other words, such a subset of nodes, i.e., a clique, implies an object or a part of an object because the background is not usually consistent. A maximal clique is a clique to which no other nodes can be added. To find all the maximal cliques of a correspondence graph, we adopt the method proposed in the literature [29]. We discard maximal cliques if they contain less than 5 nodes which are considered to be outliers. As shown in Fig. 3, while some nodes of a correspondence graph are densely connected with each other and form maximal cliques, there are nodes which are not connected with any other nodes. These isolated nodes are considered to be inconsistent with other correspondences, so each of them can be discarded as a mismatch.

3.4 Implementation Details

We use MSER [30] and affine invariant interest points [31] as keypoints, and compute SIFT [13] as visual descriptors from each point\(^1\). More than one descriptor can be extracted from a keypoint and some keypoints are detected very close to each other especially in textured regions. Because duplicate points should be removed to calculate the Delaunay

\(^1\)We use a code available at www.featurespace.org.
Fig. 3  Example of a correspondence graph. In this figure, all nodes are supposed to be placed at regular intervals on a circle, but they are not depicted. Each line segment connects two nodes representing a pair of consistent correspondences.

Fig. 4  Common visual patterns in Books pair [35]. (a) Original pair of images. (b) White mask regions indicate common patterns and line segments across the images indicate consistent correspondences by our method. Colors of line segments indicate different groups of correspondences, namely visual patterns (best viewed in color).

triangulation, we replace extracted keypoints with representatives obtained by the mean shift clustering [32] and allow them to have several descriptors. To find correspondences efficiently, we use LSH [33] to perform approximate nearest neighbor search.

When we consider color histograms to measure the visual consistency, the chi-square distance between histograms is used where a histogram for each keypoint is calculated within a window whose size is determined according to its scale. We can combine a pair of histograms of \( s, s' \) and \( t, t' \), and calculate the similarity as follows:

\[
\text{sim}(s, s', t, t') = e^{-\chi^2(h_{s}, h_{s'})}
\]

where \( h_{s} \) and \( h_{t} \) are the combined color histograms of \( s, s' \) and \( t, t' \), respectively, and \( \chi^2(\cdot, \cdot) \) is the chi-square distance between them. Here we simply define \( h_{s} \) by the normalized sum of histograms \( h_{s} \) and \( h_{t} \) as follows:

<table>
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<th>[17]</th>
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</table>

Fig. 5  Object detection (a) on the ETHZ Toys dataset, and (b) on the UIUC dataset.
\[ h_{s,s'} = \frac{h_s + h_{s'}}{||h_s + h_{s'}||} \]  

(8)

where color histograms \( h_s \) and \( h_{s'} \) are calculated locally around keypoints \( s \) and \( s' \), respectively (window sizes can be determined according to their scales). An integral histogram representation can be used for efficient calculation of histograms [34].

4. Experimental Results

To quantitatively evaluate our approach, we have conducted experiments involving common visual pattern detection, object detection, and object recognition. In all experiments, we used a same parameter setting as follows; the threshold for topological distances \( L = 2 \) and geometric distances \( D = 0.5 \sigma \) where \( \sigma \) is the scale of each keypoint (see Appendix for parameter tuning). The top 1,000 pairs of keypoints were left as initial correspondences, and at most 3 nearest neighbors were selected as matched counterparts of each keypoint. These initial correspondences were used as input to other methods.

4.1 Common Visual Pattern Detection

A direct application of our method is to detect visual patterns which commonly appear among a pair of images. We

Table 2 Areas under the curves on the ETHZ Toys and the UIUC dataset.

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Fig. 6 Object recognition on the Caltech 101 dataset. The symbol ○ indicates an outlier, which differs substantially from the rest of trials.

Fig. 7 Correspondences found on the ETHZ Toys dataset. Colors of line segments indicate different groups of correspondences, namely visual patterns (best viewed in color).
used the dataset which provides 6 pairs of images and also their masked images as ground truth [35]. This dataset is challenging due to the change in viewpoint, occlusion, and background clutter. Figure 4 shows an example of common visual patterns detected by our method. Our method can find common visual patterns as maximal cliques of a correspondence graph which is thought to be positive, and can remove mismatches by ignoring its small cliques.

We evaluate our method on the basis of the accuracy as the ratio of true positives and negatives to the total number of correspondences where a positive correspondence is true if each of the points is located within a corresponding region of ground truth. On the other hand, a negative correspondence is true if either or both of the points are located on the background, which means that a method can successfully remove a mismatch from initial correspondences. Because visual keypoints are typically extracted from region boundaries, the dilation operation is applied three times to ground truth segmentations. Note that, in this criterion, a true correspondence does not always mean an identical point in three-dimensional space, but we can evaluate the performance in terms of common pattern detection.

Table 1 shows this accuracy of our method and other methods [15]–[17], and just for reference, also shows our method combined with an additional visual consistency based on color histograms, which has ups and downs in comparison with the best methods. The threshold of visual similarity \( \tau \) was set to 0.5, which means that its impact is relatively low because correspondences are already checked by their visual descriptors. Our method achieves the best accuracy for four pairs of images and is comparable to other methods for the other two pairs. This is because each common pattern involves an object or a part of an object and the spatial neighborhood induced by relational graphs covers its shape. Because using color histograms removes not only mismatches but ambiguous correspondences, the result depends on the difficulty of matching keypoints.

4.2 Object Detection

In this experiment, we have applied our method to the task of object detection. The performance was evaluated using the ETHZ Toys dataset [36] and the UIUC dataset [37]. The former provides model images each of which contains an object taken from a different view and test images which contain one or more objects. The latter also provides training images of each object and test images.

As described above, our method (and other methods)
can be used to discard correspondences which do not involve a common object or part of an object. One can say that an object is detected from a test image if the number of remaining correspondences between the object and the test image exceeds a threshold value. If a threshold value is small, more objects can be detected and also the false positive rate increases. We therefore calculate both the detection rate and the false positive rate at each threshold. Figure 5 shows curves each of which is a plot of false positive rate versus detection rate, and Table 2 contains areas under the curves (AUC). The result of the method [17] is slightly better than that of ours, but both methods show a similar trend. Figure 7 shows examples of consistent correspondences between model and test images.

4.3 Object Recognition

We can conduct an experiment of object recognition in the same manner of object detection, and we determine whether two images contain a same object or not by thresholding the number of correspondences between them. We used the Caltech 101 dataset [38]. In a trial, 100 pairs of images are checked if they contain a same object or not, where 50 pairs of images are randomly chosen from same categories and 50 pairs chosen from different categories. We calculated both precision and recall while changing a threshold of the number of correspondences, and also calculated an area under the precision-recall curve (AUC) to evaluate the performance of recognition. We repeated these trials 50 times, and calculated the median, quartiles, minimum, and maximum of AUC. Figure 6 shows the result. When comparing medians, our method is slightly better than others. With respect to 50% of the trials indicated by each box, namely, located between the first and third quartiles, our method also achieved better results. Figure 8 shows examples of consist ent correspondences among pairs of images.

5. Conclusion

In this paper we have proposed an approach to grouping visual correspondences consistent with each other in terms of their spatial configuration. Our method is based on two graphs named relational graphs and correspondence graphs in which the geometric and topological consistency between two correspondences is evaluated according to a spatial neighborhood of each point. By enumerating maximal cliques of a correspondence graph, we can find groups of correspondences which involve common objects or parts of objects. Experimental results show that our method is comparable to or better than other methods in object detection and object recognition. In future work, we plan to integrate our method with a parts-based model.

References

Table A.1  Accuracy of common pattern detection when changing the values of parameters. (a) $L$, (b) $D/\sigma$, (c) computational time (msec.), (d) accuracy.

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<th>Mickeys</th>
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### Appendix: Parameter Tuning

We have conducted an experiment about the parameters $D$ and $L$. The same dataset described in Sect. 4.1 was used. Table A.1 shows the result where it is difficult to say which is best with respect to accuracy. However, the smaller values of $D$ and $L$ require less computational time, because the number of nodes of relational graphs decreases according to these values and a relatively small number of pairs of correspondences should be tested for the geometric consistency. Note that the computational time does not include both the extraction of visual descriptors and the initial matching between them.

If relational graphs grow to large sizes, they may contain almost all initial correspondences especially for the images of small sizes or the images containing a small number of descriptors. The geometric consistency induced by such relational graphs will make no sense.